SOUTH SYDNEY HIGH SCHOOL

PRELIMINARY COURSE

MATHEMATICS

HALF YEARLY EXAMINATION

2008

Time Allowed—1.5 Hours

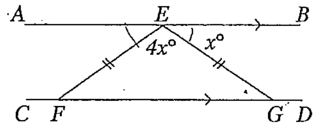
Directions to Candidates

- Attempt ALL questions
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work.
- Board approved calculators maybe used.
- Each question is to start on a new page.
- All questions are of equal value.

Question 1

(i) Evaluate $\dfrac{3{\cdot}216 \times 10^9}{4{\cdot}8 \times 10^{-6}}$.

(ii) In the diagram given below, $AB \parallel CD$ and $EF = EG$. Find the value of x, giving reasons.



(iii) Simplify $9\sqrt{7} + 2\sqrt{75} - 2\sqrt{63} + 4\sqrt{3}$.

(iv) Expand and simplify $(\sqrt{5} + 3\sqrt{2})^2$.

(v) Factorise $(x - y)^2 - (x + y)^2$.

(vi) Factorise $x^2 - 4x - 32$.

Question 2

(i) Solve $5x^2 + 6x - 3 = 0$ by the quadratic formula. (Round off to 2 decimal places)

(ii) Simplify $\dfrac{2}{3\sqrt{5}} + \dfrac{\sqrt{5}}{\sqrt{6}}$ to a single fraction with a rational denominator.

(iii) Given the equation of motion $v^2 = u^2 + 2as$, find u (correct to 1 decimal place) when $v = 56$, $a = 2$ and $s = 9$.

(iv) Solve simultaneously

$$3x - 4y = 9$$
$$6x + 2y = 3.$$

(v) Evaluate correct to 2 decimal places $\dfrac{13{\cdot}2 + 6{\cdot}7}{9{\cdot}1 - 4{\cdot}8}$.

Question 3

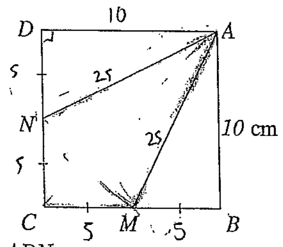
(i) Express $0{\cdot}2\dot{5}$ as a common fraction. Show all working.

(ii) Solve each of the following:

(a) $-3 \leqslant 1 - x < 4$;

(b) $|x + 1| = 4$.

(iii) $ABCD$ is a square of side length 10 cm. M is the midpoint of BC and N is the midpoint of CD.



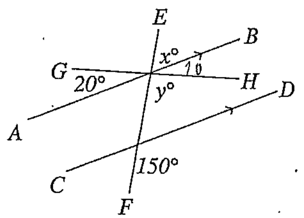
a. Prove that $\Delta ABM \equiv \Delta ADN$.

b. What type of quadrilateral is the figure $AMCN$? Give reasons.

c. Find the area of the quadrilateral $AMCN$.

Question 4

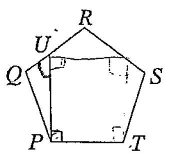
(i) In the diagram given below, $AB \parallel CD$. Find the values of x and y, giving reasons.



(ii) Find the value of x in the equation $\dfrac{2}{15} = \dfrac{1}{8} + \dfrac{1}{x}$.

(iii) Simplify $\dfrac{x^3 - 8}{3x^2 + 6x + 12} \div \dfrac{x^3 - 2x^2}{3}$.

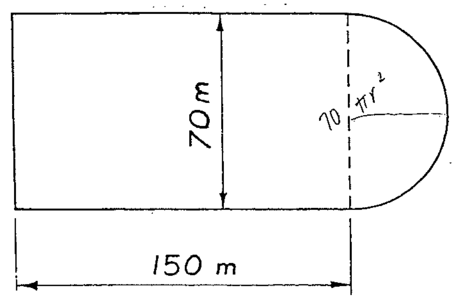
(vi)



$PQRST$ is a regular pentagon and $PU \perp PT$. Find the size of $\angle QUP$, in degrees. Give reasons.

Question 5

(i) A sporting field is in the shape of a rectangle with a semicircle at one end as shown in the diagram.

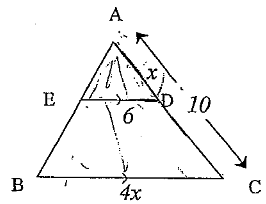


Using the approximate value $\dfrac{22}{7}$ for π, find:

(a) the area of the entire field;

(b) the total cost of fencing the boundary of the field at a cost of \$30 per metre.

(ii) In the diagram given below (All lengths are in cm.)



(a) Prove that the triangles AED and ABC are similar.

(b) Hence or otherwise, find the exact value of x.

(iii) For the function $f(x) = x + \dfrac{1}{x}$:

(a) Show that $f(\frac{a}{b}) = f(\frac{b}{a})$.

(b) Find x such that $f(x) = -2$.

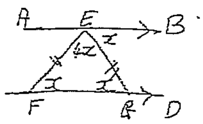
END OF EXAMINATION

SSH PRELIM HALF YEARLY MATHS EXAM 2008

Theo Q2

ESTION 1 (12 marks)

6.7×10^{14} (1)



$\angle EGF = x^\circ$ (Alternate ∠'s)
$\angle EFG = x^\circ$ (base ∠ in isosceles Δ)
$x + x + x = 180$ (Angles in Δ)
$6x = 180$
$x = 30^\circ$ (3)

) $9\sqrt{7} + 2\sqrt{75} - 2\sqrt{63} + 4\sqrt{3}$
$= 9\sqrt{7} + 10\sqrt{3} - 6\sqrt{7} + 4\sqrt{3}$
$= 3\sqrt{7} + 14\sqrt{3}$ (2)

) $(\sqrt{5} + 3\sqrt{2})^2$
$= \sqrt{25} + 6\sqrt{10} + 9\sqrt{4}$
$= 23 + 6\sqrt{10}$ (2)

) $(x-y)^2 - (x+y)^2$
$= [(x-y)-(x+y)][(x-y)+(x+y)]$
$= [x-y-x-y][x-y+x+y]$
$= [-2y][2x]$
$= -4xy$ (2)

vi) $x^2 - 4x - 32$

$x \quad -8$
$x \quad 4$
$-8x$
$4x$
$-4x$

$= (x-8)(x+4)$ (2)

QUESTION 2 (12 marks)

i) $5x^2 + 6x - 3 = 0$
$a = 5,\ b = 6,\ c = -3$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-6 \pm \sqrt{36+60}}{10}$

$= \dfrac{-6 \pm \sqrt{96}}{10}$

$= 0.38$ or -1.58 (2)

ii) $\dfrac{2}{3\sqrt{5}} + \dfrac{\sqrt{5}}{\sqrt{6}}$

$= \dfrac{2\sqrt{6} + 15}{3\sqrt{30}}$

$= \dfrac{(2\sqrt{6} + 15)}{3\sqrt{30}} \times \dfrac{\sqrt{30}}{\sqrt{30}}$

$= \dfrac{2\sqrt{180} + 15\sqrt{30}}{90}$

$= \dfrac{2 \times \sqrt{9 \times 20} + 15\sqrt{30}}{90}$

$= \dfrac{6\sqrt{20} + 15\sqrt{30}}{90}$

$= \dfrac{12\sqrt{5} + 15\sqrt{30}}{90}$

$= \dfrac{3(4\sqrt{5} + 5\sqrt{30})}{90} = \dfrac{4\sqrt{5} + 5\sqrt{30}}{30}$ (3)

iii) $v^2 = u^2 + 2as$
$56^2 = u^2 + 2 \times 2 \times 9$
$3136 = u^2 + 36$
$3100 = u^2$
$\pm 55.7 = u$ (2)

iv) $3x - 4y = 9$ — (1)
$6x + 2y = 3$ — (2)
(1)×2 $(6x - 8y = 18)$ — (3)
(2)−(3) $10y = 15$
$y = -1\tfrac{1}{2}$
Sub (2) $6x - 3 = 3$
$6x = 6$
$x = 1$

Solution $x = 1,\ y = -1\tfrac{1}{2}$ (3)

v) 4.63 (2)

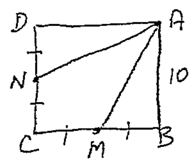
QUESTION 3 (12 marks)

i) Let $x = 0.2\dot{5}$
$= 0.25555\ldots$
$10x = 2.5555\ldots$
$-(\ x = 0.2555\ldots)$
$9x = 2.3$
$x = \dfrac{2.3}{9}$
$x = \dfrac{23}{90}$ (3)

ii) a) $-3 < 1 - x < 4$
$-3-1 < -x < 4-1$
$-4 < -x < 3$
$4 > x > -3$ (2)

b) $|x+1| = 4$
$x + 1 = 4$ or $x + 1 = -4$
$x = 3$ $\quad x = -5$ (2)

iii)



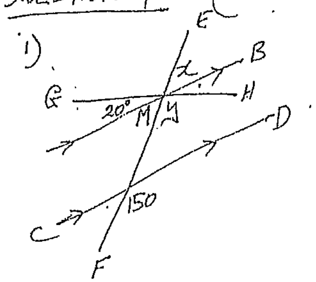
a) Prove $\triangle ABM \equiv \triangle ADN$
$AB = AD$ (Sides of squ
$\angle B = \angle D$ (Angles of sq
$BM = DN$ (Midpts - give
$\therefore \triangle ABM \equiv \triangle ADN$
(SAS)

b) $AN = AM$ (Corresponding sides of ≡ Δs
∴ AMCN is a kite (2 pa of adjacent sides equ (1)

c) Area of AMCN
= Area Square − 2 Area Tri
$= (L \times W) - 2 \times (\tfrac{1}{2}bh)$
$= (10 \times 10) - (2 \times \tfrac{1}{2} \times 5 \times$
$= 100 - 50$
$= 50$ sq cm (2)

QUESTION 4 (12 marks)

i)



$\angle BMH = 20°$ (Vertically Opp)

$y + 20 = 150°$ (Corresp $\angle$'s)

$\underline{y = 130°}$

$y + 20 + x = 180°$ (Straight $\angle$)

$130 + 20 + x = 180$

$\underline{x = 30°}$ 4

ii) $\frac{2}{15} = \frac{1}{8} + \frac{1}{x}$

$\frac{2}{15} = \frac{x+8}{8x}$

$16x = 15(x+8)$

$16x = 15x + 120$

$\underline{x = 120}$ 2

iii) $\frac{x^3-8}{3(x^2+2x+4)} \div \frac{x^3-2x^2}{3}$

$= \frac{\cancel{(x-2)}\cancel{(x^2+2x+4)}}{\cancel{3}\cancel{(x^2+2x+4)}} \times \frac{\cancel{3}}{x^2\cancel{(x-2)}}$

$= \frac{1}{x^2}$

$= \underline{x^{-2}}$ 3

iv)

$\angle Q = 108°$ ($\angle$ in regular pentagon).

$90 + \angle UPQ = 108°$ ($\angle$ in Pentag)

$\angle UPQ = 18°$

$\angle Q + \angle UPQ + \angle QUP = 180°$ ($\angle$'s in Δ)

$108 + 18 + \angle QUP = 180$

$126 + \angle QUP = 180$

$\underline{\angle QUP = 54°}$ 3

QUESTION 5 (12 marks)

i)

a) Area $= L \times W + \frac{1}{2}\pi r^2$

$= 150 \times 70 + \frac{1}{2} \times \frac{22}{7} \times 35^2$

$= \underline{12425 \text{ m}^2}$ 2

b) Perimeter $= 150 + 70 + 150 + \frac{1}{2}$ circle

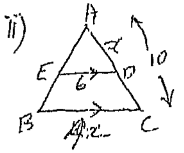
$= 370 + \frac{1}{2} \times 2 \times \frac{22}{7} \times 35$

$= 480 \text{ m}$

Cost $= 480 \times 30$

$= \underline{\$14400}$ 2

ii)



a) $\angle A$ Common

$\angle AED = \angle ABC$ (Corresp $\angle$'s)

$\angle ADE = \angle ACB$ (" ")

$\therefore \Delta AED \parallel\!\mid \Delta ABC$ (AAA) 2

b) Corresponding sides are in ratio

$\frac{6}{4x} = \frac{x}{10}$

$60 = 4x^2$

$15 = x^2$

$\underline{\sqrt{15} = x}$ 2

iii) $f(x) = x + \frac{1}{x}$

a) $f\left(\frac{a}{b}\right) = \frac{a}{b} + \frac{1}{\left(\frac{a}{b}\right)}$

$= \underline{\frac{a}{b} + \frac{b}{a}}$

$f\left(\frac{b}{a}\right) = \frac{b}{a} + \frac{1}{\left(\frac{b}{a}\right)}$

$= \underline{\frac{b}{a} + \frac{a}{b}}$

$\therefore f\left(\frac{a}{b}\right) = f\left(\frac{b}{a}\right)$ 2

b) $-2 = x + \frac{1}{x}$

$-2x = x^2 + 1$

$0 = x^2 + 2x + 1$

$0 = (x+1)^2$

$\underline{x = -1}$ 2