Question 1 (20 marks) **Marks**

(a) For the function $f(x) = 4 - x^2$, find **4**

(i) $f(0)$ (ii) $f(-3)$

(iii) the domain (iv) the range

(b) Match the following equations with their loci. **8**

(i) $y = x^2$	(A) Semi-circle
(ii) $x^2 + y^2 = 16$	(B) Hyperbola
(iii) $y = 3^x$	(C) Vertical line
(iv) $y = \sqrt{4 - x^2}$	(D) Circle
(v) $x = -3$	(E) Line with gradient, $m = 2$
(vi) $y = 6$	(F) Exponential function
(vii) $xy = 9$	(G) Line parallel to the x–axis
(viii) $2x - y + 3 = 0$	(H) Parabola

(c) Sketch, on separate number planes, the above graphs **8**

in (b) from parts (i) to (iv).

Question 2 (20 marks) **Marks**

(a) Find the domain of the following functions : **8**

(i) $f(x) = \dfrac{1}{x^2 - 1}$ (ii) $f(x) = \sqrt{x^2 - 1}$

(iii) $f(x) = \dfrac{1}{x^2 + 1}$ (iv) $f(x) = 2^x$

(b) Are the following fuunctions odd, even or neither ? **6**

Justify your answer.

(i) $f(x) = \dfrac{x}{x^2 - 1}$ (ii) $f(x) = \dfrac{x - 1}{x^2 + 1}$

(iii) $f(x) = \dfrac{x^2}{x^2 - 1}$

(c) If $f(x) = \dfrac{1}{x}$, show that $\dfrac{f(x+h) - f(x)}{h} = \dfrac{-1}{x(x+h)}$. **3**

(d) Solve the following graphically or otherwise : **3**

$$\frac{x}{x-2} \le 4$$

Question 3 (11 marks) **Marks**

(a)

If $f(x) = \begin{cases} ax^2 + b & \text{for } x \le -2 \\ 2x - 1 & \text{for } -2 < x \le 3 \\ \dfrac{a}{x} & \text{for } x > 3 \end{cases}$

(i) Find a, b if $f(-2) = f(1)$ **3**

(ii) Find the range of $f(x)$ for these values of a and b **2**

(iii) Evaluate $f(-1) + 2f(2) - f(6)$ **2**

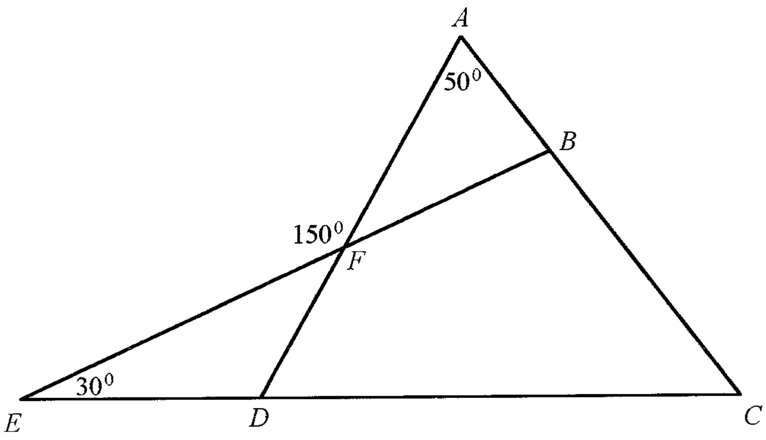
(b) Find the following limits : **4**

(i) $\lim\limits_{x \to 3} \dfrac{x^2 - 9}{x - 3}$

(ii) $\lim\limits_{x \to \infty} \dfrac{3x^2 - 5x + 1}{1 + 2x + 2x^2}$

Question 4 (12 marks) **Marks**

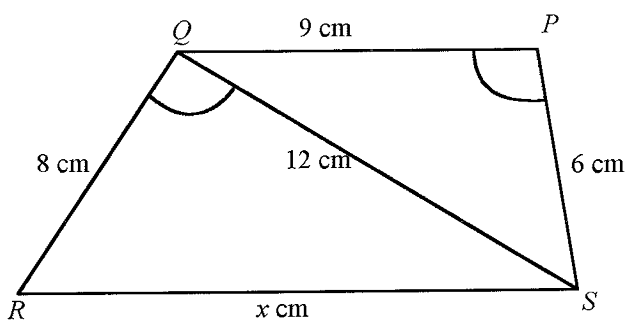
(a) **2**



Given that $\angle DAC = 50^0$, $\angle BEC = 30^0$ and $\angle AFE = 150^0$.

Find, without giving reasons, the size of $\angle ACE$

(b)



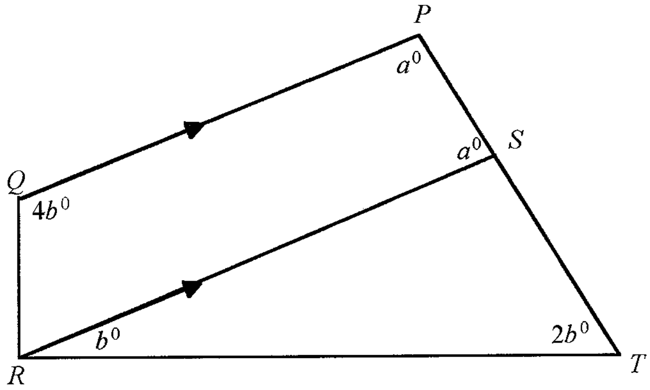
Given that $\angle QPS = \angle SQR$,

(a) Prove that $\Delta QPS \;|||\; \Delta SQR$ **3**

(b) Prove that $PQ \;||\; SR$ **2**

(c) Find the value of x. **1**

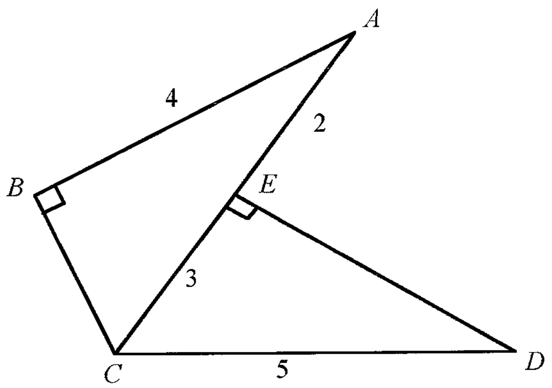
(c)



(a) Find, giving reasons, the values of a and b. **4**

(b) Prove that $\angle QPT$ is a right angle. **2**

(d)



Prove that $\Delta ABC \equiv \Delta CDE$. **3**

Half Yearly exam 1997.
3 unit
South Sydney High

Mira Chow

Excellent work!

Question 1

a: (i) $f(x) = 4 - x^2$

$f(0) = 4 - (0)^2$

$\underline{f(0) = 4}$ ✓

(ii) $f(x) = 4 - x^2$

$f(-3) = 4 - (-3)^2$

$f(-3) = 4 - 9$

$\underline{f(-3) = -5}$ ✓

(iii) $y = 4 - x^2$

$y = -x^2 + 4$

Let $y = 0 \Rightarrow 0 = 4 - x^2$

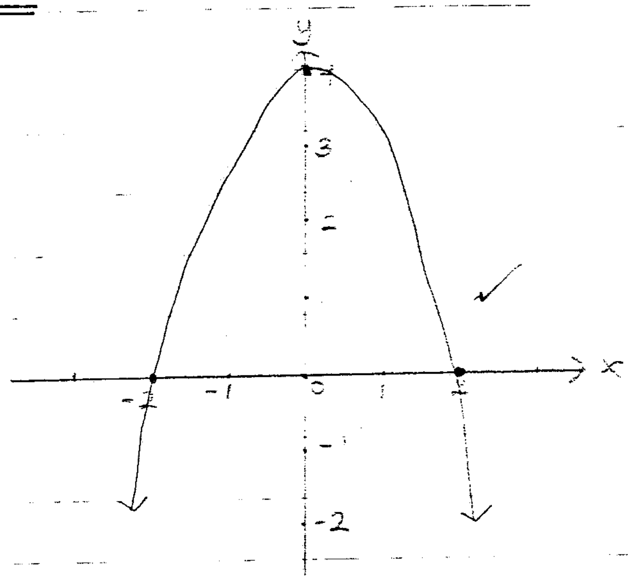
$0 = (2 - x)(2 + x)$

$x = +2$

$x = -2$

Let $x = 0 \Rightarrow y = 4 - (0)^2$

$y = 4$

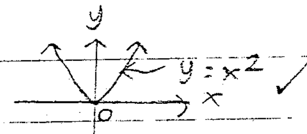


✓

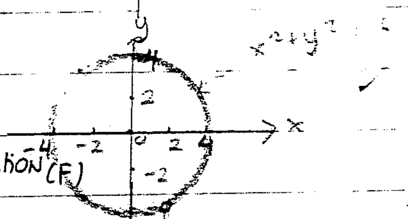
Domain : all real x ✓

(iv) Range : $y \leq 4$ ✓

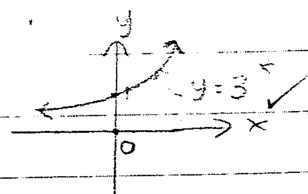
b. (i) $y = x^2$ ✓ → parabola (H)



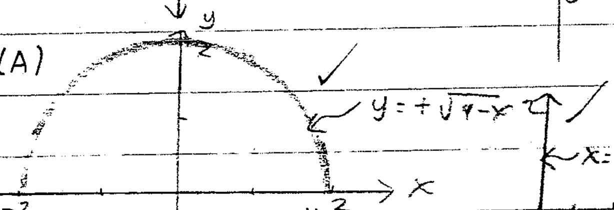
(ii) $x^2 + y^2 = 16$ ✓ → circle (D)



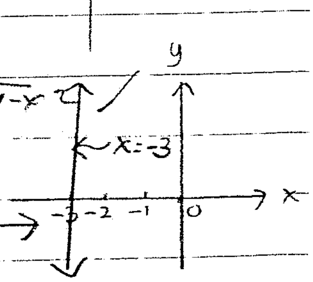
(iii). $y = 3^x$ ✓ → Exponential function (F)



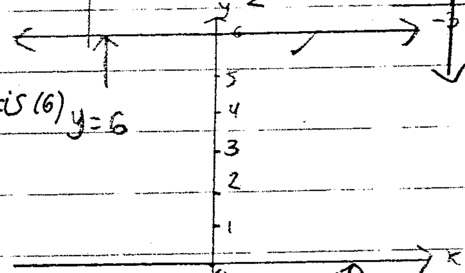
(iv). $y = \sqrt{4 - x^2}$ ✓ → semi circle (A)



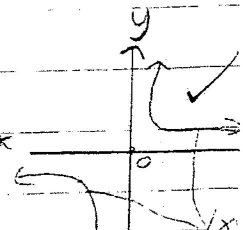
(v). $x = -3$ ✓ → verticalline (C)



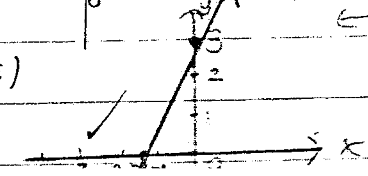
(vi). $y = 6$ ✓ → line parallel to x-axis (G) $y = 6$



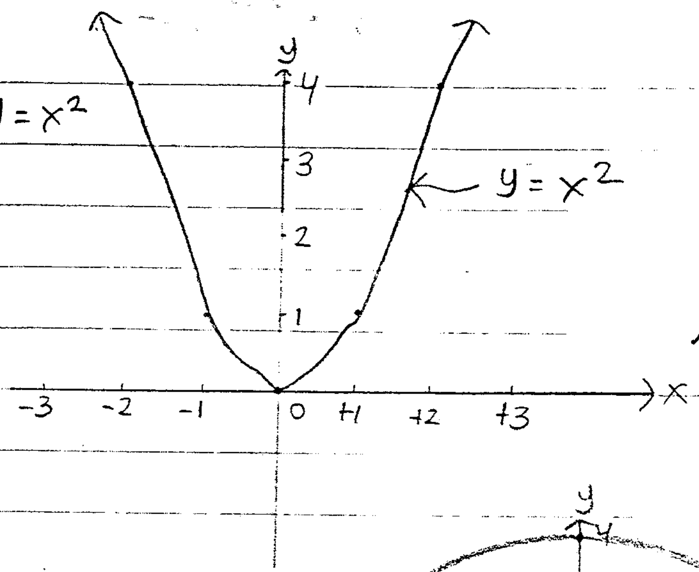
(vii). $xy = 9$ ✓ → hyperbola (B)



(viii). $2x - y + 3 = 0$ ✓ → Line with gradient, $m = 2$ (E)

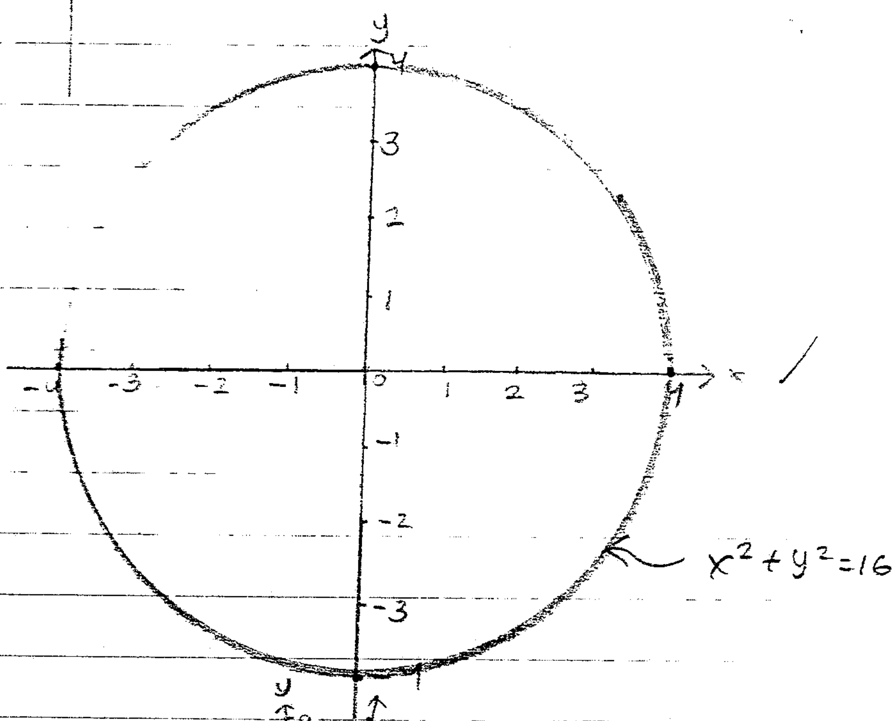


c. (i). $y = x^2$

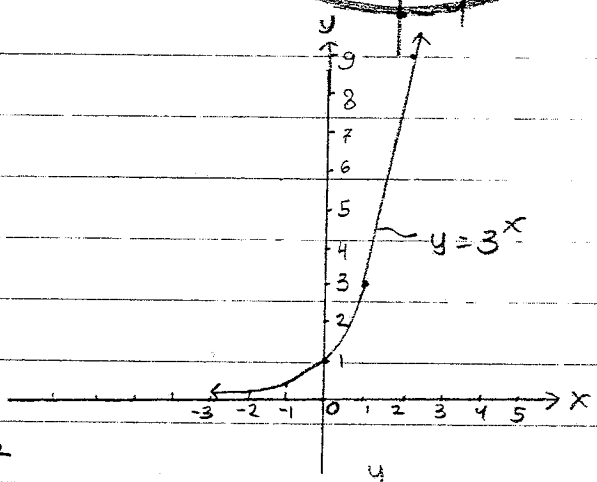


(ii). $x^2 + y^2 = 16$

$x^2 + y^2 = 4$

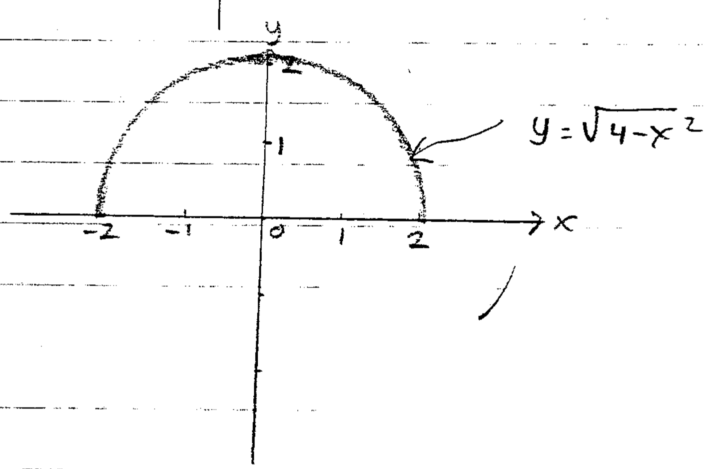


(iii). $y = 3^x$



x	-2	-1	0	1	2
y	$\frac{1}{9}$	$\frac{1}{3}$	1	3	9

(iv). $y = +\sqrt{4-x^2}$



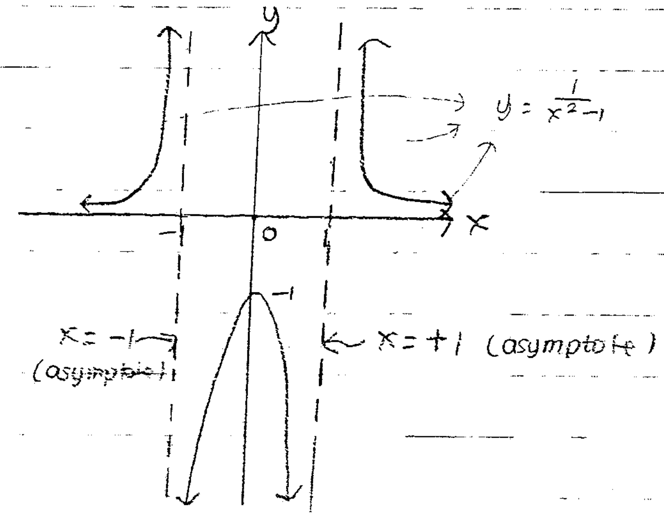
Question 2

a. (i). $f(x) = \frac{1}{x^2-1} \rightarrow y = \frac{1}{x^2-1}$

Domain : $x^2 \neq 1$

$\underline{\underline{x \neq \pm 1}}$ ✓

When $x = 0$, $y = -1$



(ii). $f(x) = \sqrt{x^2-1}$

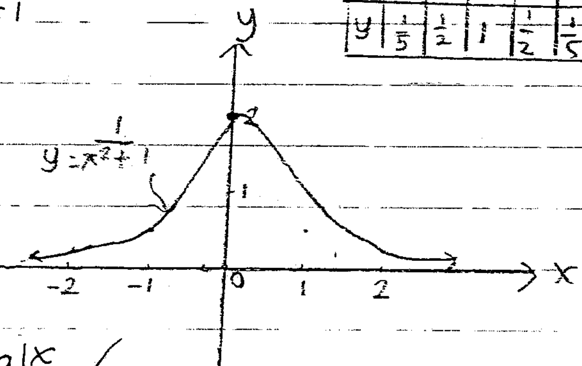
Domain : $x^2 - 1 \geq 0$

$\underline{\underline{x \leq -1 \text{ OR } x \geq 1}}$ ✓

(iii). $f(x) = \frac{1}{x^2+1}$

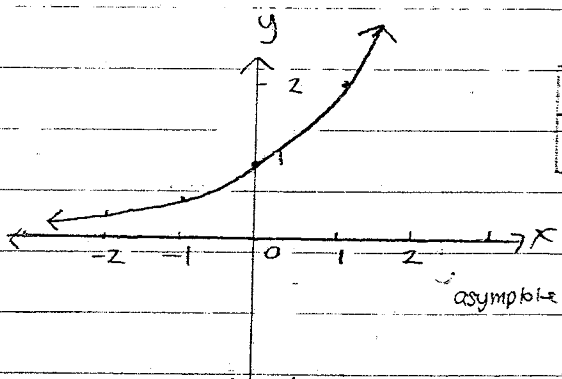
$y = \frac{1}{x^2+1}$

x	-2	-1	0	1	2
y	$\frac{1}{5}$	$\frac{1}{2}$	1	$\frac{1}{2}$	$\frac{1}{5}$



Domain : all real x ✓

(iv). $f(x) = 2^x$



x	-2	-1	0	1	2	3
y	$\frac{1}{4}$	$\frac{1}{2}$	1	2	4	8

Domain : all real x ✓

b. (i). $f(x) = \frac{x}{x^2-1}$

$f(-x) = \frac{(-x)}{(-x)^2-1}$

$= \frac{-x}{x^2-1}$

$= -f(x)$

$\therefore$ It is an odd function ✓

(ii). $f(x) = \frac{x-1}{x^2+1}$

$f(-x) = \frac{(-x)-1}{(-x)^2+1}$

$= \frac{-x-1}{x^2+1}$ ✓

$\therefore$ Neither odd nor even functions.

(iii). $f(x) = \frac{x^2}{x^2-1}$

$f(-x) = \frac{(-x)^2}{(-x)^2-1}$

$= \frac{x^2}{x^2-1}$

$= f(x)$

$\therefore$ It is an even function

c. $f(x) = \frac{1}{x}$

$f(x+h) = \frac{1}{x+h}$

$$\frac{f(x+h) - f(x)}{h} = \frac{-1}{x(x+h)}$$

LHS : $\frac{f(x+h) - f(x)}{h}$

$$= \frac{\left(\frac{1}{x+h}\right) - \frac{1}{x}}{h}$$

$$= \frac{\frac{x-(x+h)}{x(x+h)}}{h}$$

$$= \frac{\frac{x-x-h}{x^2+xh}}{h} = \frac{\cancel{x}-\cancel{x}-h}{x^2+xh} \times \frac{1}{h} = \frac{\overset{-1}{\cancel{-h}}}{x^2+xh} \times \frac{1}{\cancel{h}} = \frac{-1}{x^2+xh}$$

$$= \frac{-1}{x(x+h)}$$

$= RHS$

d. $\frac{x}{x-2} \leq 4$

$\frac{x-4(x-2)}{x-2} \leq 0 \qquad x-2 \neq 0$

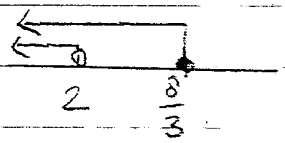
$\frac{x-4x+8}{x-2} \leq 0 \qquad x \neq 2$

$\frac{-3x+8}{x-2} \leq 0$

• $-3x+8 \geq 0 \quad \cap \quad x-2 \leq 0$

$-3x \geq -8 \qquad x \leq 2$

$x \leq +\frac{8}{3} \qquad \boxed{x < 2}$



Test :

$3 \rightarrow \frac{3}{3-2} \leq 4$

$3 \leq 4$ (TRUE)

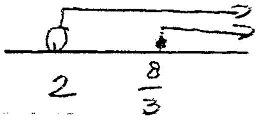
$1 \rightarrow \frac{1}{-1} \leq 4$

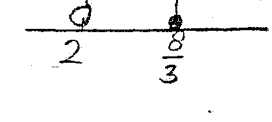
$-1 \leq 4$ (TRUE)

• $-3x+8 \leq 0 \quad \cap \quad x-2 \geq 0$

$-3x \leq -8 \qquad x \geq 2$

$\boxed{x \geq \frac{8}{3}} \qquad x > 2$





Question 3

a. (i). $f(-2) = f(1)$

$ax^2+b = 2x-1$

$a(-2)^2+b = 2(1)-1$

$4a+b = 2-1$

$\boxed{4a+b = 1}$

$b = 1-4a$

$4a = 1-b$

$a = \frac{1-b}{4}$

(ii). $x=-2$ $x=4$

$f(x) = 4a+b$ $f(x) = \frac{a}{4}$

The range of f(x) between $4a+b$ and $\frac{a}{4}$

(iii). $f(-1) = 2x-1$

$= 2(-1)-1$

$= -2-1 = -3$

$f(2) = 2x-1$

$= 2(2)-1 = 4-1 = 3$

$f(6) = \frac{a}{x} = \frac{1-b}{6}$

$f(-1)+2f(2)-f(6)$

$-3+2(3) - \frac{1-b}{6}$

$-3+6 - \frac{1-b}{6}$

$3 - \frac{1-b}{6}$

Question 4

a. $\angle DFE = \angle AFE = 150°$ (vertically opposite $\angle$).

$\angle EFD = 180° - 150° = 30°$ (straight line).

$\angle EDF = 180° - 30° - 30°$ (sum of $\angle$ in Δ)

$= 120°$

$\angle EDF = \angle DAC + \angle ACE$ (exterior $\angle$ of ΔADC)

$120° = 50° + \angle ACE$

$\therefore \angle ACE = 120° - 50° = 70°$

b. (i). $\Delta QPS, \Delta SQR$

$QP : SQ = 9 : 12 = 3 : 4$ ✓

$\angle QPS = \angle SQR$ (given)

$PS : RQ = 6 : 8 = 3 : 4$ ✓

$\therefore \Delta QPS /// \Delta RQS$ (the two sides are in proportio or in the same ratio, include one angle which is equal).

(ii). Since $\Delta QPS /// \Delta SQR$ (proportional to the $\angle$s and sides in the same ratio → proven)

$\angle PQS = \angle QSR$

$\therefore PQ // SR$ (alternate $\angle$s are equal)

(iii). $\frac{SP}{RQ} = \frac{SQ}{RS}$

$\frac{6}{8} = \frac{12}{x}$

$6x = 96$

c. (a.) $a^\circ + a^\circ = 180^\circ$ (cointerior ∠s of QP // RS)

$2a^\circ = 180^\circ$

$a = 90^\circ$ ✓

$a^\circ = b^\circ + 2b^\circ$ (exterior ∠s of △ RST)

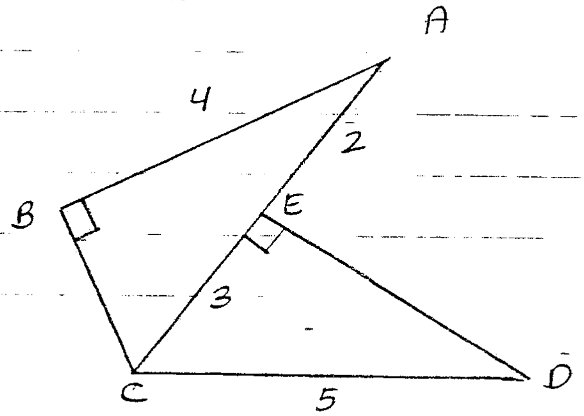
$90^\circ = 3b^\circ$

$b = 30^\circ$ ✓

(b). Since $a = 90^\circ$ (has been proven),

$\therefore \angle QPT = a = 90^\circ$

$\therefore$ ∠QPT is a right angle. ✓



d. $ED^2 = \sqrt{CD^2 - CE^2}$ (Phytagoras' Theorem)

$= \sqrt{25 - 9}$ ✓

$= \sqrt{16}$

$ED = 4$ $AB = 4$ $\therefore ED = AB$

△ ABC, △ DEC

R : ∠ABC = ∠DEC (given) ✓

H : AC = DC (proven)

S : AB = ED (proven) ✓

$\therefore$ △ABC ≡ △DEC (RHS Test)

$AC = AE + EC$

$= 2 + 3 = 5$

$CD = 5$

$\therefore AC = CD$