SOUTH SYDNEY HIGH SCHOOL MATHEMATICS

LOGARITHMS WORKSHEET

1. Simplify

(a) $\dfrac{a^4 \times a^7}{a^9}$ (b) $\dfrac{5a^{12} \times 3a^3}{15a^5}$

(c) $\left(2a^5\right)^4$ (d) $\dfrac{\left(3a^2\right)^4}{\left(2a^3\right)^2}$

(e) $\dfrac{a^{\frac{3}{4}} \times a^{-\frac{1}{2}}}{a^{\frac{1}{4}}}$ (f) $\dfrac{\left(a^{\frac{2}{3}}\right)^4 \times \left(a^{\frac{3}{4}}\right)}{\left(a^{\frac{3}{2}}\right)^3 \times a^{\frac{1}{4}}}$

2. Evaluate:

(a) $81^{\frac{1}{2}}$ (b) $81^{\frac{1}{4}}$ (c) $2^3 \times 64^{\frac{1}{6}}$

(d) $256^{\frac{1}{2}} \times 16^{-\frac{1}{2}}$ (e) $2187^{\frac{4}{7}}$

(f) $125^{-\frac{2}{3}}$ (g) $1000^{\frac{1}{3}} \times 16^{-\frac{3}{4}}$

3. Solve for a:

(a) $3^a = 81$ (b) $a^2 = 1024$

(c) $5^{2a} = 625$ (d) $10^{3x-2} = 10\ 000$

(e) $2^x = \frac{1}{4}$ (f) $5^{x+1} = \frac{1}{125}$

(g) $\left(\sqrt{3}\right)^x = 27$ (h) $\left(\frac{1}{\sqrt{5}}\right)^{4x} = 625^{2x-15}$

4. Simplify the following expressions using the tables of powers. Express your answer without an index.

(a) $9 \times 27 \times 27$ (b) $\dfrac{2 \times 16 \times 64 \times 128}{256}$

(c) $\dfrac{\sqrt{729}}{9}$ (d) $\sqrt{\dfrac{3125 \times 125}{625}}$

(e) $\dfrac{(81)^2 \times (9)^3}{(27)^4}$

5. Write in logarithm form:

(a) $5^3 = 125$ (b) $3^{-3} = \frac{1}{27}$ (c) $\sqrt{8} = 2^{\frac{3}{2}}$

6. Write in index form:

(a) $\log_{10} 1000 = 3$ (b) $\log_2 16 = 4$

(c) $\log_2 4\sqrt{2} = 2.5$ (d) $\log_5\left(\frac{1}{25}\right) = -2$

7. Solve the following logarithmic equations:

(a) $\log_2 64 = x$ (b) $\log_5 125 = x$

(c) $x = \log_3 \sqrt{27}$ (d) $\log_{10} 1\ 000\ 000 = x$

(e) $\log_2\left(\frac{1}{4}\right) = x$ (f) $\log_2 x = 3$

(g) $\log_5 x = 2$ (h) $\log_3 x = \frac{1}{2}$

(i) $\log_{10} x = -1$ (j) $\log_2 x = -3$

(k) $\log_8 x = -\frac{1}{3}$ (l) $\log_x 256 = 8$

(m) $\log_x\left(\frac{1}{9}\right) = -2$ (n) $\log_x 4 = \frac{2}{3}$

8. By first completing the following table of values:

x	−2	−1	0	0.5	1	1.5	2
y							

sketch the graph of $y = 3^x$.

9. Complete the following table for $y = 10^x$ and then sketch the curve.

x	−1	−0.5	0	0.2	0.5	0.8	1
y							

10. Rewrite $y = \log_3 x$ in index form and complete the following table:

x							
y	−2	−1	0	0.5	1	1.5	2

Sketch the curve $y = \log_3 x$.

11. Rewrite $y = \log_{10} x$ in index form and complete the following table:

x							
y	−1	−0.5	0	0.2	0.5	0.8	1

Sketch the curve $y = \log_{10} x$.

12. Simplify the following logarithmic expressions:

(a) $\log_a N + \log_a M$
(b) $\log_a N - \log_a M$
(c) $i \log_a N$
(d) $i \log_a N + j \log_a M$
(e) $\frac{1}{2}\log_a N$
(f) $\frac{1}{2}\log_a N - 2\log_a M$

13. Expand the following logarithmic expressions:

(a) $\log_a xy$ (b) $\log_a\left(\frac{x}{y}\right)$
(c) $\log_a\left(\frac{1}{y}\right)$ (d) $\log_a \sqrt{x}$
(e) $\log_a\left(\frac{x^2}{y}\right)$ (f) $\log_a\left[\frac{x^2\sqrt{y}}{z^4}\right]$

14. Use the laws of logarithms to evaluate the following expressions:

(a) $\log_2 64$ (b) $\log_4 64$
(c) $\log_3 \sqrt{3}$ (d) $\log_5 5\sqrt{5}$
(e) $\log_5\left(\frac{1}{\sqrt{5}}\right)$ (f) $\log_a a^3$
(g) $\log_a \sqrt{a}$ (h) $\log_a a + \log_a \sqrt{a}$
(i) $\log_a(\log_2 2)$
(j) $\log_x(x^2 - 2x) - \log_x(x-2)$
(k) $\log_5 40 - \log_5 8$
(l) $\log_3 54 - \log_3 2$

15. Given $\log_{10}3 = 0.477$ and $\log_{10}5 = 0.699$, evaluate the following correct to three decimal places:

(a) $\log_{10} 15$ (b) $\log_{10}\left(\frac{5}{3}\right)$
(c) $\log_{10} \sqrt{3}$ (d) $\log_{10} 3\sqrt{3}$
(e) $\log_{10} 5\sqrt{3}$ (f) $\log_{10}\left(\frac{1}{3}\right)$
(g) $\log_{10} 0.2$ (h) $\log_{10} 45$
(i) $\log_{10} 75$ (j) $\log_{10} 5.4$

16. Given $\log_a 7 = 2.8$ and $\log_a 5 = 2.3$, evaluate the following correct to one decimal place:

(a) $\log_a 35$ (b) $\log_a\left(\frac{7}{5}\right)$
(c) $\log_a 1.4$ (d) $\log_a \sqrt{7}$
(e) $\log_a\left(\frac{1}{7}\right)$ (f) $\log_a\left(\frac{1}{25}\right)$
(g) $\log_a\left(\frac{1}{\sqrt{7}}\right)$ (h) $\log_a 175$
(i) $\log_a \sqrt{35}$ (j) $\log_a 9.8$

17. If $x = \log_n 2$ and $y = \log_n 3$, write the following in terms of x and y:

(a) $\log_n 6$ (b) $\log_n\left(\frac{2}{3}\right)$
(c) $\log_n 1.5$ (d) $\log_n \sqrt{2}$
(e) $\log_n \sqrt{6}$ (f) $\log_n 24$
(g) $\log_n 6\sqrt{2}$ (h) $\log_n 4.5$
(i) $\log_n\left(\frac{16}{3}\right)$ (j) $\log_n 0.5$

18. If $N = \log_a 10$ and $M = \log_a 3$, write simple logarithmic expressions for:

(a) $N + M$ (b) $N - M$
(c) $\frac{1}{2}M$ (d) $\frac{1}{2}M - \frac{1}{2}N$
(e) $2N + 3M$ (f) $4M - 2N$

19. Express y in terms of the other variables.

(a) $\log_2 y = \log_2 a + \log_2 b$
(b) $\log_2 y = \frac{1}{3}\log_2 a$
(c) $3\log_2 y = -\log_2 x$
(d) $\log_2 y = 1 - \frac{1}{2}\log_2 x$

20. Solve the following logarithmic equations for a:

(a) $\log_{10} a = \log_{10} 3 + \log_{10} 8$
(b) $\log_{10} a = \frac{1}{2}\log_{10} 81$
(c) $\log_{10} a = \log_{10} 72 - \log_{10} 9$
(d) $\log_{10} a = 3\log_{10} 2 - 2\log_{10} 5$
(e) $\log_{10} a = 1 - 2\log_{10} 5$
(f) $\log_{10} 6a - \log_{10}(a+4) = 1$
(g) $\log_2 a + \log_2(a+2) = 3$

21. Simplify the following to a single logarithm:

(a) $\frac{\log_{10} 5}{\log_{10} 3}$ (b) $\frac{\log_a N}{\log_a b}$

22. Calculate the following to four significant figures:

(a) $\log_2 5$ (b) $\log_2 10$
(c) $\log_2 \sqrt{5}$ (d) $\log_2\left(\frac{1}{5}\right)$
(e) $\log_2 0.1$

23. Solve the following for y, giving the answer correct to two decimal places:

(a) $3^x = 5$ (b) $3^x = 10$
(c) $3^{2x} = 5$ (d) $3^x = \sqrt{5}$
(e) $3^x = \left(\frac{1}{5}\right)$ (f) $3^x = 2^{3x-5}$
(g) $2^{3-x} = 5^{2x+1}$

24. Evaluate the following (all to the base 10):

(a) $\dfrac{\log 16}{\log 2}$ (b) $\dfrac{\log 81}{\log 27}$
(c) $\dfrac{\log 8}{\log\left(\frac{1}{4}\right)}$ (d) $\dfrac{\log 2}{\log(0.25)}$
(e) $\log 2 + \log\left(\frac{1}{4}\right)$ (f) $\log 2 - \log\left(\frac{1}{4}\right)$
(g) $\log 125 + \log 32 - \log\left(\frac{2}{5}\right)$

25. Evaluate:

(a) $\log_4(\log_2 16)$ (b) $\log_{10}\left(\log_{10} 10^{10}\right)$
(c) $2\log_{10}\left(\frac{16}{15}\right) + 3\log_{10}\left(\frac{5}{2}\right) + \log_{10}\left(\frac{9}{16}\right)$

WORKED SOLUTIONS

1. (a) $\dfrac{a^{11}}{a^9} = a^2$ (b) $\dfrac{15a^{15}}{15a^5} = a^{10}$

(c) $\left(2^4\right)a^{5\times4} = 16a^{20}$

(d) $\dfrac{\left(3^4\right)a^{2\times4}}{\left(2^2\right)a^{3\times2}} = \dfrac{81a^8}{4a^6} = \frac{81}{4}a^2$

(e) $\dfrac{a^{\frac{3}{4}-\frac{1}{2}}}{a^{\frac{1}{4}}} = \dfrac{a^{\frac{1}{4}}}{a^{\frac{1}{4}}} = 1$

(f) $\dfrac{a^{\frac{2}{3}\times4} \times a^{\frac{3}{4}}}{a^{\frac{3}{2}\times3} \times a^{\frac{1}{4}}} = \dfrac{a^{\frac{8}{3}} \times a^{\frac{3}{4}}}{a^{\frac{9}{2}} \times a^{\frac{1}{4}}}$

$= \dfrac{a^{\frac{8}{3}+\frac{3}{4}}}{a^{\frac{9}{2}+\frac{1}{4}}}$

$= \dfrac{a^{\frac{41}{12}}}{a^{\frac{19}{4}}}$

$= a^{\frac{41}{12}-\frac{19}{4}}$

$= a^{-\frac{4}{3}}$

2. (a) $\sqrt{81} = 9$ **Using table of powers**

(b) $\sqrt[4]{81} = 3$

(c) $8 \times \sqrt[6]{64} = 8 \times 2$
$= 16$

(d) $\sqrt{256} \times \dfrac{1}{\sqrt{16}} = 16 \times \frac{1}{4}$
$= 4$

(e) $\left(2187^{\frac{1}{7}}\right)^4 = \left(\sqrt[7]{2187}\right)^4$
$= 3^4$
$= 81$

(f) $125^{-\frac{2}{3}} = \dfrac{1}{125^{\frac{2}{3}}}$
$= \dfrac{1}{\left(125^{\frac{1}{3}}\right)^2}$
$= \dfrac{1}{\left(\sqrt[3]{125}\right)^2}$
$= \dfrac{1}{5^2}$
$= \frac{1}{25}$

(g) $\sqrt[3]{1000}\times\left(16^{\frac{1}{4}}\right)^{-3} = 10\times\frac{1}{\left(\sqrt[4]{16}\right)^3}$
$= 10\times\frac{1}{2^3}$
$= 10\times\frac{1}{8}$
$= \frac{5}{4}$ or $1\frac{1}{4}$

3. (a) $3^a = 3^4$ Using table of powers
$\therefore\ a = 4.$

(b) $a^2 = 1024$
$a = \sqrt{1024}$
$= 32.$

(c) $5^{2a} = 5^4$
$\therefore\ 2a = 4$
$a = 2.$

(d) $10^{3x-2} = 10^4$
$\therefore\ 3x - 2 = 4$
$\therefore\ 3x = 6$
$x = 2.$

(e) $2^x = 2^{-2}$
$\therefore\ x = -2.$

$\frac{1}{4} = \frac{1}{2^2} = 2^{-2}$

(f) $5^{x+1} = 5^{-3}$
$\therefore\ x + 1 = -3$
$\therefore\ x = -4.$

$\frac{1}{125} = \frac{1}{5^3} = 5^{-3}$

(g) $\left(3^{\frac{1}{2}}\right)^x = 3^3$
$\therefore\ 3^{\frac{1}{2}x} = 3^3$
$\therefore\ \frac{1}{2}x = 3,$
that is, $x = 6.$

(h) $\left(5^{-\frac{1}{2}}\right)^{4x} = \left(5^4\right)^{2x-15}$
$\therefore\ 5^{-2x} = 5^{4(2x-15)}$
$\therefore\ -2x = 8x - 60$
that is, $10x = 60$
$\therefore\ x = 6.$

4. (a) $3^2\times3^3\times3^3 = 3^8$
$= 6561.$

(b) $\frac{2\times2^4\times2^6\times2^7}{2^8} = \frac{2^{18}}{2^8}$
$= 2^{10}$
$= 1024.$

(c) $\frac{\sqrt{3^6}}{3^2} = \frac{3^3}{3^2} = 3$

(d) $\sqrt{\frac{3^6}{3^2}} = \sqrt{3^4} = 3^2 = 9$

(e) $\sqrt{\frac{5^5\times5^3}{5^4}} = \sqrt{\frac{5^8}{5^4}}$
$= \sqrt{5^4}$
$= 5^2$
$= 25.$

(f) $\frac{\left(3^4\right)^2\times\left(3^2\right)^3}{\left(3^3\right)^4} = \frac{3^8\times3^6}{3^{12}}$
$= \frac{3^{14}}{3^{12}}$
$= 3^2$
$= 9.$

5. (a) $\log_5 125 = 3$
(b) $\log_3\left(\frac{1}{27}\right) = -3$
(c) $\log_2\sqrt{8} = \frac{3}{2}$

6. (a) $10^3 = 1000$
(b) $2^4 = 16$
(c) $2^{2.5} = 4\sqrt{2}$
(d) $5^{-2} = \frac{1}{25}$

7. Change to index form and solve the index equations as in Question 3.

(a) $2^x = 64$ Using table of powers
$\therefore\ 2^x = 2^6$
$\therefore\ x = 6.$

(b) $5^x = 125$
$\therefore\ 5^x = 5^3$
$\therefore\ x = 3.$

(c) $3^x = \sqrt{27}$
$\therefore\ 3^x = \left(3^3\right)^{\frac{1}{2}}$
$= 3^{\frac{3}{2}}$
$\therefore\ x = \frac{3}{2}.$

(d) $10^x = 1000\,000$
that is, $10^x = 10^6$
$\therefore\ x = 6.$

(e) $2^x = \frac{1}{4}$
$\therefore\ 2^x = 2^{-2}$
$\therefore\ x = -2.$

(f) $2^3 = x,$
that is, $x = 8.$

(g) $5^2 = x$
$\therefore\ x = 25.$

(h) $3^{\frac{1}{2}} = x$

$\therefore \quad x = \sqrt{3}.$

(i) $10^{-1} = x$

$\therefore \quad x = \frac{1}{10}.$

(j) $2^{-3} = x$

$\therefore \quad x = \frac{1}{2^3}$

$= \frac{1}{8}.$

(k) $8^{-\frac{1}{3}} = x$

$\therefore \quad x = \frac{1}{8^{\frac{1}{3}}}$

$= \frac{1}{\sqrt[3]{8}}$

$= \frac{1}{2}.$

(l) $x^8 = 256$

$\therefore \quad x^8 = 2^8$

that is, $x = 2.$

or
$x = (256)^{\frac{1}{8}}$
$= 2$

(m) $x^{-2} = \frac{1}{9}$

that is, $x^{-2} = 3^{-2}$

$\therefore \quad x = 3.$

or $x^{-2} = \frac{1}{9}$
$\therefore \quad x^2 = 9$
$\therefore \quad x = \sqrt{9}$
$= 3$

(n) $x^{\frac{2}{3}} = 4$

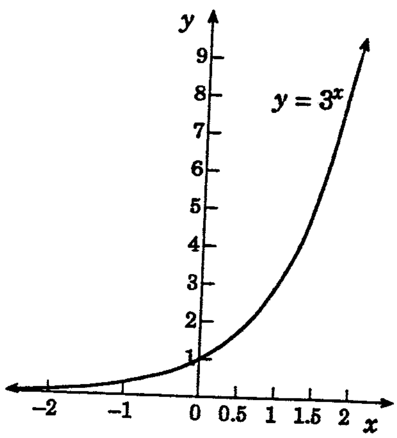
$\therefore \quad x^{\frac{2}{3}} = (8)^{\frac{2}{3}}$

$\therefore \quad x = 8.$

or $x^{\frac{2}{3}} = 4$
$\therefore \quad x = (4)^{\frac{3}{2}}$
$= \left(4^{\frac{1}{2}}\right)^3$
$= 2^3$
$= 8$

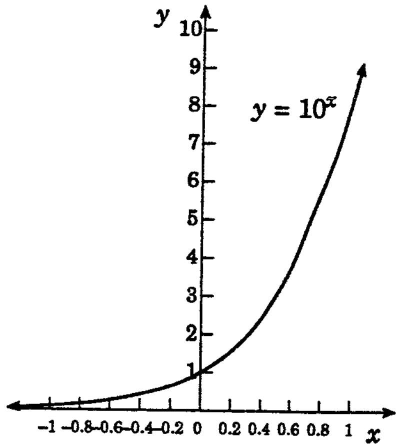
8. $y = 3^x$

x	−2	−1	0	0.5	1	1.5	2
y	$^1/_9$	$^1/_3$	1	≈1.7	3	≈5.2	9



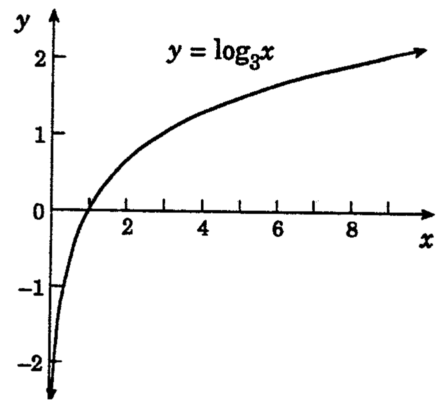
9. $y = 10^x$

x	−1	−0.5	0	0.2	0.5	0.8	1
y	0.1	≈0.3	1	≈1.6	≈3.2	≈6.3	10



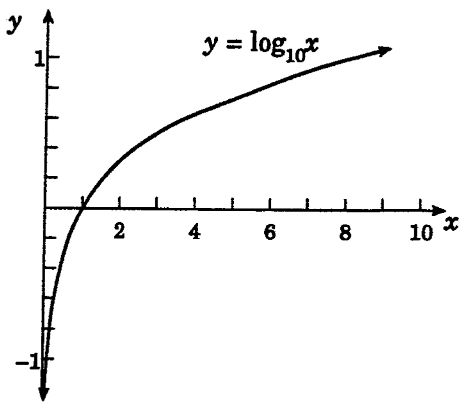
10. $y = \log_3 x$ [Rewrite $x = 3^y$]

x	$^1/_9$	$^1/_3$	1	≈1.7	3	≈5.2	9
y	−2	−1	0	0.5	1	1.5	2



11. $y = \log_{10} x$

x	0.1	≈0.3	1	≈1.6	≈3.2	≈6.3	10
y	−1	−0.5	0	0.2	0.5	0.8	1



12. (a) $\log_a NM$
(b) $\log_a\left(\frac{N}{M}\right)$
(c) $\log_a N^i$
(d) $\log_a N^i M^j$
(e) $\log_a \sqrt{N}$
(f) $\log_a\left(\frac{\sqrt{N}}{m^2}\right)$

13. (a) $\log_a x + \log_a y$
(b) $\log_a x - \log_a y$
(c) $\log_a y^{-1} = -\log_a y$
(d) $\log_a x^{\frac{1}{2}} = \frac{1}{2}\log_a x$
(e) $\log_a x^2 - \log_a y = 2\log_a x - \log_a y$
(f) $\log_a x^2 + \log_a \sqrt{y} - \log_a z^4$
$= 2\log_a x + \log_a y^{\frac{1}{2}} - 4\log_a z$
$= 2\log_a x + \frac{1}{2}\log_a y - 4\log_a z$

14. (a) $\log_2 2^6 = 6\log_2 2$
$= 6$

Using $\log_a a = 1$

(b) $\log_4 64 = \log_4 4^3$
$= 3\log_4 4$
$= 3$

(c) $\log_3 \sqrt{3} = \log_3 3^{\frac{1}{2}}$
$= \frac{1}{2}\log_3 3$
$= \frac{1}{2}$

(d) $\log_5 5\sqrt{5} = \log_5 5^{1.5}$
$= 1.5\log_5 5$
$= 1.5$

(e) $\log_5\left(\frac{1}{5^{\frac{1}{2}}}\right) = \log_5 5^{-\frac{1}{2}}$
$= -\frac{1}{2}\log_5 5$
$= -\frac{1}{2}$

(f) $\log_a a^3 = 3\log_a a$
$= 3$

(g) $\log_a a^{\frac{1}{2}} = \frac{1}{2}\log_a a$
$= \frac{1}{2}$

(h) $\log_a a\sqrt{a} = \log_a a^{1.5}$
$= 1.5\log_a a$
$= 1.5$

(i) $\log_a 1$ $[\log_2 2 = 1]$
$= 0$

(j) $\log_x\left(\frac{x^2-2x}{x-2}\right) = \log_x\left(\frac{x\cancel{(x-2)}}{\cancel{(x-2)}}\right)$
$= \log_x x$
$= 1$

(k) $\log_5\left(\frac{40}{8}\right) = \log_5 5$
$= 1$

(l) $\log_3\left(\frac{54}{2}\right) = \log_3 27$
$= \log_3 3^3$
$= 3\log_3 3$
$= 3$

15. (a) $\log_{10} 3 + \log_{10} 5 = 0.477 + 0.699$
$= 1.176$

(b) $\log_{10} 5 - \log_{10} 3 = 0.699 - 0.477$
$= 0.222$

(c) $\log_{10} 3^{\frac{1}{2}} = \frac{1}{2}\log_{10} 3$
$= \frac{1}{2}(0.477)$
$= 0.239$

(d) $\log_{10} 3\sqrt{3} = \log_{10} 3^{1.5}$
$= 1.5\log_{10} 3$
$= 1.5(0.477)$
$= 0.716$

(e) $\log_{10} 5 + \log_{10} \sqrt{3} = 0.699 + \frac{1}{2}(0.477)$
$= 0.938$

(f) $\log_{10} 3^{-1} = -\log_{10} 3$
$= -0.477$

(g) $\log_{10} \frac{1}{5} = -\log_{10} 5^{-1}$
$= -\log_{10} 5$
$= -0.699$

(h) $9 \times 5 = 45$

$\log_{10} 9 + \log_{10} 5 = \log_{10} 3^2 + \log_{10} 5$
$= 2\log_{10} 3 + \log_{10} 5$
$= 2(0.477) + 0.699$
$= 1.653$

(i) $\log_{10} 25 + \log_{10} 3 = \log_{10} 5^2 + \log_{10} 3$
$= 2\log_{10} 5 + \log_{10} 3$
$= 2(0.699) + 0.477$
$= 1.875$

(j) $\log_{10} 5.4 = \log_{10}\left(\frac{27}{5}\right)$
$= \log_{10} 27 - \log_{10} 5$
$= \log_{10} 3^3 - \log_{10} 5$
$= 3\log_{10} 3 - \log_{10} 5$
$= 3(0.477) - 0.699$
$= 0.732$

16. (a) $\log_a 7 + \log_a 5 = 2.8 + 2.3$
$= 5.1$

(b) $\log_a 7 - \log_a 5 = 2.8 - 2.3$
$= 0.5$

(c) $\log_a 1.4 = \log_a\left(\frac{7}{5}\right)$
$= 0.5$ [from (b)]

(d) $\log_a \sqrt{7} = \log_a 7^{\frac{1}{2}}$
$= \frac{1}{2}\log_a 7$
$= \frac{1}{2}(2.8)$
$= 1.4$

(e) $\log_a\left(\frac{1}{7}\right) = \log_a 7^{-1}$
$= -\log_a 7$
$= -2.8$

(f) $\log_a\left(\frac{1}{5^2}\right) = \log_a 5^{-2}$
$= -2\log_a 5$
$= -2(2.3)$
$= -4.6$

(g) $\log_a\left(\frac{1}{7^{\frac{1}{2}}}\right) = \log_a 7^{-\frac{1}{2}}$
$= -\frac{1}{2}\log_a 7$
$= -\frac{1}{2}(2.8)$
$= -1.4$

(h) $\log_a 175 = \log_a 25 + \log_a 7$
$175 = 25 \times 7$ $= \log_a 5^2 + \log_a 7$
$= 2\log_a 5 + \log_a 7$
$= 2(2.3) + 2.8$
$= 7.4$

(i) $\log_a 35^{\frac{1}{2}} = \frac{1}{2}\log_a 35$
$= \frac{1}{2}\log_a(5 \times 7)$
$= \frac{1}{2}[\log_a 5 + \log_a 7]$
$= \frac{1}{2}[2.8 + 2.3]$
$= 2.6$ [one dec. place]

(j) $\log_a 9.8 = \log_a\left[9\frac{4}{5}\right]$
$= \log_a\left[\frac{49}{5}\right]$
$= \log_a 49 - \log_a 5$
$= \log_a 7^2 - \log_a 5$
$= 2\log_a 7 - \log_a 5$
$= 2(2.8) - 2.3$
$= 3.3$

17. (a) $\log_n(3 \times 2) = \log_n 3 + \log_n 2$
$= y + x.$
(or $x + y$)

(b) $\log_n\left(\frac{2}{3}\right) = \log_n 2 - \log_n 3$
$= x - y.$

(c) $\log_n\left(\frac{3}{2}\right) = \log_n 3 - \log_n 2$
$= y - x.$

(d) $\log_n 2^{\frac{1}{2}} = \frac{1}{2}\log_n 2$
$= \frac{1}{2}x.$

(e) $\log_n 6^{\frac{1}{2}} = \frac{1}{2}\log_n 6$
$= \frac{1}{2}(x + y)$ [from (a)].

(f) $\log_n(6 \times 4) = \log_n 6 + \log_n 4$
$= \log_n 6 + \log_n 2^2$
$= \log_n 6 + 2\log_n 2$
$= x + y + 2x$
$= 3x + y.$

(g) $\log_n\left(6 \times \sqrt{2}\right) = \log_n 6 + \log_n 2^{\frac{1}{2}}$
$= \log_n 6 + \frac{1}{2}\log_n 2$
$= x + y + \frac{1}{2}x$
$= \frac{3}{2}x + y.$

(h) $\log_n\left(4\frac{1}{2}\right) = \log_n\left(\frac{9}{2}\right)$
$= \log_n 9 - \log_n 2$
$= \log_n 3^2 - \log_n 2$
$= 2\log_n 3 - \log_n 2$
$= 2y - x.$

(i) $\log_n\left(\frac{16}{3}\right) = \log_n 16 - \log_n 3$
$= \log_n 2^4 - \log_n 3$
$= 4\log_n 2 - \log_n 3$
$= 4x - y.$

(j) $\log_n\left(\frac{1}{2}\right) = \log_n 2^{-1}$
$= -\log_n 2$
$= -x.$

18. (a) $N + M = \log_a 10 + \log_a 3$
$= \log_a(10 \times 3)$
$= \log_a 30.$

(b) $N - M = \log_a 10 - \log_a 3$
$= \log_a\left(\frac{10}{3}\right).$

(c) $\frac{1}{2}M = \frac{1}{2}\log_a 10$
$= \log_a 10^{\frac{1}{2}}$
$= \log_a \sqrt{10}.$

(d) $\frac{1}{2}M-\frac{1}{2}N=\frac{1}{2}[\log_a 3-\log_a 10]$
$=\frac{1}{2}\log_a \frac{3}{10}$
$=\log_a \sqrt{0.3}.$

(e) $2N+3M=2\log_a 10+3\log_a 3$
$=\log_a 10^2+\log_a 3^3$
$=\log_a 100+\log_a 27$
$=\log_a(100\times 27)$
$=\log_a 2700.$

(f) $4M-2N=4\log_a 3-2\log_a 10$
$=\log_a 3^4-\log_a 10^2$
$=\log_a 81-\log_a 100$
$=\log_a\left(\frac{81}{100}\right)$
$=\log_a 0.81.$

19. (a) $\log_2 y=\log_2 ab$
$\therefore\quad y=ab.$

(b) $\log_2 y=\log_2 a^{\frac{1}{3}}$
$=\log_2 \sqrt[3]{a}$
$\therefore\quad y=\sqrt[3]{a}.$

(c) $\log_2 y^3=\log_2 x^{-1}$
$\therefore\quad y^3=x^{-1}$
$=\frac{1}{x}$
$\therefore\quad y=\sqrt[3]{\frac{1}{x}}$ or $\frac{1}{\sqrt[3]{x}}.$

(d) $\log_2 y=\log_2 2-\frac{1}{2}\log_2 x$

Note use of $1=\log_2 2$

$=\log_2 2-\log_2 x^{\frac{1}{2}}$
$=\log_2 2-\log_2 \sqrt{x}$
$=\log_2\left(\frac{2}{\sqrt{x}}\right)$
$\therefore\quad y=\frac{2}{\sqrt{x}}.$

20. (a) $\log_{10} a=\log_{10}(3\times 8)$
$\therefore\quad a=3\times 8$
$=24$

(b) $\log_{10} a=\log_{10} 81^{\frac{1}{2}}$
$=\log_{10}\sqrt{81}$
$\therefore\quad a=\sqrt{81}$
$=9$

(c) $\log_{10} a=\log_{10}\left(\frac{72}{9}\right)$
$=\log_{10} 8$
$\therefore\quad a=8$

(d) $\log_{10} a=\log_{10} 2^3-\log_{10} 5^2$
$=\log_{10} 8-\log_{10} 25$
$=\log_{10}\left(\frac{8}{25}\right)$
$\therefore\quad a=\frac{8}{25}$

(e) $\log_{10} a=\log_{10} 10-\log_{10} 5^2$
$=\log_{10} 10-\log_{10} 25$
$=\log_{10}\left(\frac{10}{25}\right)$
$\therefore\quad a=\frac{10}{25}$
$=\frac{2}{5}.$

(f) $\log_{10}\left(\frac{6a}{a+4}\right)=\log_{10} 10$
$\therefore\quad \frac{6a}{a+4}=10$
$\therefore\quad 6a=10(a+4)$
$=10a+40$
$\therefore\quad 4a=-40$
$a=-10.$

(g) $\log_2\left(\frac{a}{a+2}\right)=3\times 1$
$=3\log_2 2$
$=\log_2 2^3$
$=\log_2 8$
$\therefore\quad \frac{a}{a+2}=8$
$\therefore\quad a=8(a+2)$
$=8a+16$
$\therefore\quad 7a=-16$
$a=-\frac{16}{7}$

Note the use of $1=\log_{10}10$ and $1=\log_2 2$.

21. (a) $\log_3 5$
(b) $\log_b N$

22. (a) $\frac{\log_{10} 5}{\log_{10} 2}=\frac{0.698\,97}{0.301\,03}$
$=2.321\,9281$
≈ 2.322
[to four significant figures]

(b) $\frac{\log_{10} 10}{\log_{10} 2}=\frac{1}{\log_{10} 2}$
$=3.321\,9281$
≈ 3.322
[to four significant figures]

(c) $\log_2 5^{\frac{1}{2}}=\frac{1}{2}\log_2 5$
$=\frac{1}{2}(2.321\,9281)$ [from (a)]
$=1.160\,964$
≈ 1.161
[to four significant figures]

(d) $\log_2\left(\frac{1}{5}\right) = \log_2 5^{-1}$
$= -\log_2 5$
$= -2.322$ [from (a)]

(e) $\log_2 0.1 = \log_2\left(\frac{1}{10}\right)$
$= \log_2 10^{-1}$
$= -\log_2 10$
$= -3.322$ [from (b)]

23. (a) $3^x = 5$
$\log_{10} 3^x = \log_{10} 5$ [Taking $\log_{10}$ of each side]
$\therefore\ x\log_{10} 3 = \log_{10} 5$
$\therefore\ x = \dfrac{\log_{10} 5}{\log_{10} 3}$
$= 1.464\,9735,$
≈ 1.46
[to 2 dec. places]

On calculator
5 [LOG] [÷] 3 [LOG] [=]

or express $3^x = 5$ in log form.

$x = \log_3 5$
$= \dfrac{\log_{10} 5}{\log_{10} 3}$
≈ 1.46
[to two decimal places]

(b) $3^x = 10$
$\log_{10} 3^x = \log_{10} 10$ [Take logs of both sides.]
$\therefore x\log_{10} 3 = 1$
$x = \dfrac{1}{\log_{10} 3}$
$= 2.095\,9033$
≈ 2.10
[to two decimal places]

(c) $3^{2x} = 5$
$\log_{10} 3^{2x} = \log_{10} 5$
$\therefore 2x\log_{10} 3 = \log_{10} 5$
$2x = \dfrac{\log_{10} 5}{\log_{10} 3}$
$x = \dfrac{\log_{10} 5}{2\log_{10} 3}$
$= 0.732\,4867$
≈ 0.73
[to two decimal places]

(d) $3^x = \sqrt{5}$
$\log_{10} 3^x = \log_{10} \sqrt{5} = \log_{10} 5^{\frac{1}{2}}$
$\therefore x\log_{10} 3 = \frac{1}{2}\log_{10} 5$
$\therefore\ x = \dfrac{\frac{1}{2}\log_{10} 5}{\log_{10} 3}$
≈ 0.73
[to two decimal places]

(e) $3^x = 5^{-1}$
$\log_{10} 3^x = \log_{10} 5^{-1}$
$\therefore x\log_{10} 3 = -\log_{10} 5$
$\therefore\ x = \dfrac{-\log_{10} 5}{\log_{10} 3}$
$= -1.46$ [from (a)]

(f) $\log_{10} 3^x = \log_{10} 2^{3x-5}$
$\therefore x\log_{10} 3 = (3x-5)\log_{10} 2$
$= 3x\log_{10} 2 - 5\log_{10} 2$
$\therefore\ 3x\log_{10} 2 - x\log_{10} 3 = 5\log_{10} 2$
$\therefore\ x(3\log_{10} 2 - \log_{10} 3) = 5\log_{10} 2$
$\therefore\ x = \dfrac{5\log_{10} 2}{3\log_{10} 2 - \log_{10} 3}$
$= 3.533\,4753$
$\approx 3.53.$
[to two decimal places]

Calculator
2 [LOG] [×] 5 [÷] [(] 2 [LOG] [×] 3 [−] 3 [LOG] [)] [=]

(g) $2^{3-x} = 5^{2x+1}$
$\therefore\ \log_{10} 2^{3-x} = \log_{10} 5^{2x+1}$
$\therefore (3-x)\log_{10} 2 = (2x+1)\log_{10} 5$
$\therefore\ 3\log_{10} 2 - x\log_{10} 2$
$= 2x\log_{10} 5 + \log_{10} 5$
$\therefore\ 3\log_{10} 2 - \log_{10} 5$
$= 2x\log_{10} 5 + x\log_{10} 2$
$= x(2\log_{10} 5 + \log_{10} 2)$
$\therefore\ x = \dfrac{3\log_{10} 2 - \log_{10} 5}{2\log_{10} 5 + \log_{10} 2}$
$= 0.120\,1433$
≈ 0.12
[to two decimal places]

Calculator

[[] 2 [LOG] [×] 3 [−] 5 [LOG] []] [÷]

[[] 5 [LOG] [×] 2 [−] 2 [LOG] []] [=]

24. (a) $\frac{\log 2^4}{\log 2} = \frac{4\log 2}{\log 2}$
$= 4$

(b) $\frac{\log 3^4}{\log 3^3} = \frac{4\log 3}{3\log 3}$
$= \frac{4}{3}$

(c) $\frac{\log 2^3}{\log 2^{-2}} = \frac{3\log 2}{-2\log 2}$
$= -\frac{3}{2}$

(d) $\frac{\log 2}{\log\left(\frac{1}{4}\right)} = \frac{\log 2}{\log 2^{-2}}$
$= \frac{\log 2}{-2\log 2}$
$= \frac{1}{-2}$
$= -\frac{1}{2}$

(e) $\log_2\left(2\times\frac{1}{4}\right) = \log_2\left(\frac{1}{2}\right)$
$= \log_2 2^{-1}$
$= -\log_2 2$
$= -1$

or $1+\log_2\left(\frac{1}{2^2}\right)$
$= 1+\log_2 2^{-2}$
$= 1-2\log_2 2$
$= 1-2 = -1$

(f) $\log_2 2 - \log_2\left(\frac{1}{4}\right) = 1-\log_2 2^{-2}$
$= 1+2\log_2 2$
$= 1+2$
$= 3$

(g) $\log_{10}\left(125\times 32\div\frac{2}{5}\right)$
$= \log_{10}\left(125\times \overset{16}{\cancel{32}}\times\frac{5}{\cancel{2}_1}\right)$
$= \log_{10} 10\,000$
$= \log_{10} 10^4$
$= 4\log_{10} 10 = 4$

25. (a) $\log_4\left(\log_2 2^4\right) = \log_4(4\log_2 2)$
$= \log_4 4$
$= 1$

Remember
$\log_a a = 1$.

(b) $\log_{10}(10\log_{10} 10) = \log_{10} 10$
$= 1$

(c) $\log_{10}\left(\frac{16}{15}\right)^2 + \log_{10}\left(\frac{5}{2}\right)^3 + \log_{10}\left(\frac{9}{16}\right)$
$= \log_{10}\left[\left(\frac{16}{15}\right)^2\times\left(\frac{5}{2}\right)^3\times\left(\frac{9}{16}\right)\right]$
$= \log_{10} 10$
$= 1$

$\frac{16}{15}\times\frac{16}{15}\times\frac{5}{2}\times\frac{5}{2}\times\frac{5}{2}\times\frac{9}{16}$
$= 10$ after cancelling.