# SOUTH SYDNEY HIGH SCHOOL

## MATHS – EXT 2 WORKSHEETS

## INTEGRATION

### EXERCISES SET 4J

### FURTHER PRACTICE ON INTEGRALS

This set should be used to consolidate the methods illustrated previously, and give practice in discriminating between them.

Find the following integrals.

1. $\int \frac{x\,dx}{x^2+4}$ 2. $\int \frac{x\,dx}{\sqrt{x^2+4}}$ 3. $\int \frac{5x+2}{x^2-4}\,dx$

4. $\int \sin x \cos^3 x\,dx$ 5. $\int \sin x \sec^3 x\,dx$ 6. $\int \cos^2 \frac{x}{2}\,dx$

7. $\int x \sin x\,dx$ 8. $\int x \sec^2 2x\,dx$ 9. $\int \tan^{-1} 2x\,dx$

10. $\int \frac{x^3 dx}{x^2+1}$ 11. $\int \frac{x\,dx}{(x+2)(x+4)}$ 12. $\int \frac{(x-1)(x+1)\,dx}{(x-2)(x-3)}$

13. $\int \frac{(2x-1)\,dx}{x^2+2x+3}$ 14. $\int \frac{x^3 dx}{2x-1}$ 15. $\int \frac{(1+x)\,dx}{\sqrt{1-x-x^2}}$

16. $\int \frac{dx}{x^2(1-x^2)^{\frac{1}{2}}}$ 17. $\int \frac{dx}{x\sqrt{a^2+x^2}}$ 18. $\int \frac{dx}{x\sqrt{a^2-x^2}}$

19. $\int \frac{dx}{x\sqrt{x^2-a^2}}$ 20. $\int \frac{x\,dx}{\sqrt{x+1}}$ 21. $\int \frac{\cos^{-1} x}{\sqrt{1-x^2}}\,dx$

22. $\int \sqrt{\frac{x+1}{x-1}}\,dx$ 23. $\int \frac{dx}{x(\log x)^3}$ 24. $\int \sec^4 3x\,dx$

25. $\int \frac{dx}{x^2(1-x)}$ 26. $\int \frac{dx}{x^2(1+x^2)}$ 27. $\int \frac{dx}{(1+x^2)^2}$

28. $\int \tan^3 x\,dx$ 29. $\int \frac{dx}{5+3\cos x}$ 30. $\int \frac{dx}{3+5\cos x}$

31. $\int \frac{\sin x\,dx}{5+3\cos x}$ 32. $\int \frac{dx}{1+\cos^2 x}$ 33. $\int \frac{dx}{\cos^2\frac{x}{2} - \sin^2\frac{x}{2}}$

34. $\int x^2 \sin x\,dx$ 35. $\int \frac{x^2 dx}{(x-1)(x-2)(x-3)}$ 36. $\int \frac{e^x\,dx}{e^x - 1}$

37. $\int \frac{dx}{3\sin^2 x + 5\cos^2 x}$ 38. $\int x^3 . e^{5x^4-7}\,dx$

39. $\int x^5 \log x\,dx$ 40. $\int \frac{(3x+2)\,dx}{x(x+1)^3}$ 41. $\int \log x^3\,dx$

42. $\int \frac{dx}{e^x+e^{-x}}$ 43. $\int (5x^3+7x-1)^{3/2}\cdot(15x^2+7)\,dx$

44. $\int \frac{dx}{(x^2+1)(x^2+4)}$ 45. $\int (x^2+x+1)^{-1}\,dx$ 46. $\int e^x \sin 2x\,dx$

47. $\int (x^2+x-1)^{-1}\,dx$ 48. $\int (x^2-x)^{-\frac{1}{2}}\,dx$ 49. $\int \frac{1-2x}{3+x}dx$

50. $\int x^3(4+x^2)^{-\frac{1}{2}}\,dx$ 51. $\int \frac{\sin 2x\,dx}{3\cos^2 x + 4\sin^2 x}$

52. $\int \frac{x^2dx}{1-x^4}$ 53. $\int \frac{dx}{\sin x \cos x}$ 54. $\int \log\sqrt{x-1}\,dx$

55. $\int \frac{dx}{e^x-1}$ 56. $\int \frac{\sec^2 x\,dx}{\tan^2 x - 3\tan x + 2}$

57. $\int \frac{(x+1)\,dx}{(x^2-3x+2)^{\frac{1}{2}}}$ 58. $\int \sin 2x \cos x\,dx$ 59. $\int \frac{x\,dx}{1+x^3}$

60. $\int x\tan^{-1}x\,dx$ 61. $\int (1+3x+2x^2)^{-1}\,dx$ 62. $\int (9-x^2)^{\frac{1}{2}}\,dx$

63. $\int (9+x^2)^{\frac{1}{2}}\,dx$ 64. $\int x(9+x^2)^{\frac{1}{2}}\,dx$ 65. $\int \sec^2 x \tan^3 x\,dx$

66. $\int x^2 e^{-x}\,dx$ 67. $\int x\,e^{x^2}\,dx$ 68. $\int \sin x \tan x\,dx$

69. $\int \sin^4 x \cos^3 x\,dx$ 70. $\int \frac{(x^3+1)\,dx}{x^3-x}$ 71. $\int \log(x+\sqrt{x^2-1})\,dx$

72. $\int \frac{dx}{(x+1)^{\frac{1}{2}}+(x+1)}$

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*Evaluate the following definite integrals,* leaving results in irrational form.

73. $\int_0^4 \frac{x\,dx}{\sqrt{x+4}}$ 74. $\int_1^2 \frac{dx}{x(1+x^2)}$ 75. $\int_1^2 \frac{\log x}{x}\,dx$

76. $\int_0^1 \cos^{-1}x\,dx$ 77. $\int_1^2 \frac{(x+1)\,dx}{\sqrt{-2+3x-x^2}}$ 78. $\int_0^{\pi/2} \frac{dx}{\cos^2 x + 2\sin^2 x}$

79. $\int_0^1 x\sqrt{1-x^2}\,dx$ 80. $\int_2^4 x \log x\,dx$ 81. $\int_1^2 \frac{dx}{x^2+5x+4}$

82. $\int_0^{\pi/2} (1+\frac{1}{2}\sin x)^{-1}dx$ 83. $\int_0^1 x^2e^{-x}\,dx$ 84. $\int_0^1 \frac{7+x\,dx}{1+x+x^2+x^3}$

85. $\int_0^1 \frac{e^{-2x}\,dx}{e^{-x}+1}$ 86. $\int_0^{a/2} \frac{y}{a-y}\,dy$ 87. $\int_0^a \frac{(a-x)^2dx}{a^2+x^2}$

88. $\int_0^1 \frac{(x+3)\,dx}{(x+2)(x+1)^2}$ 89. $\int_0^1 \frac{x^2dx}{x^6+1}$ 90. $\int_0^{\pi} \cos^2 mx\,dx$, m integral

91. $\int_{\pi/4}^{\pi/2} x\sin 2x\,dx$ 92. $\int_0^{a/2} x^2\sqrt{a^2-x^2}\,dx$ 93. $\int_0^{\pi/4} \sec^2 x \tan x\,dx$

94. $\int_0^1 (x+2)(x^2+4x+5)^{\frac{1}{2}}dx$ 95. $\int_1^2 x(\log x)^2dx$

96. $\int_3^4 \frac{x^2+4}{x^2-1}\,dx$ 97. $\int_1^4 \frac{x^2+4}{x(x+2)}\,dx$ 98. $\int_0^{\pi/2} \frac{\cos x\,dx}{5-3\sin x}$

99. $\int_0^1 \frac{dx}{(4-x^2)^{\frac{3}{2}}}$ 100. $\int_0^{\pi/2} 2\sin\theta\cos\theta(3\sin\theta - 4\sin^3\theta)\,d\theta$

SET 4J (page 130) (ANSWERS)

1. $\frac{1}{2}\log(x^2+4)$
2. $\sqrt{x^2+4}$
3. $3\log(x-2) + 2\log(x+2)$
4. $-\frac{1}{4}\cos^4 x$
5. $\frac{1}{2}\sec^2 x$
6. $\frac{1}{2}[x + \sin x]$
7. $-x\cos x + \sin x$
8. $\frac{1}{2}x\tan 2x + \frac{1}{4}\log\cos 2x$
9. $x\tan^{-1}2x - \frac{1}{4}\log(1+4x^2)$
10. $\frac{1}{2}x^2 - \frac{1}{2}\log(1+x^2)$
11. $2\log(x+4) - \log(x+2)$
12. $x - 3\log(x-2) + 8\log(x-3)$
13. $\log(x^2+2x+3) - \frac{3}{\sqrt{2}}\tan^{-1}(\frac{x+1}{\sqrt{2}})$
14. $\frac{1}{6}x^3 + \frac{1}{8}x^2 + \frac{1}{8}x + \frac{1}{16}\log(2x-1)$
15. $\frac{1}{2}\sin^{-1}(\frac{2x+1}{\sqrt{5}}) - \sqrt{1-x-x^2}$
16. $-\frac{\sqrt{1-x^2}}{x}$
17. $-\frac{1}{a}\log\left[\frac{\sqrt{a^2+x^2}+a}{x}\right]$ or $-\frac{1}{a}\log\left[\frac{x}{\sqrt{a^2+x^2}-a}\right]$
18. $-\frac{1}{a}\log\left[\frac{a+\sqrt{a^2-x^2}}{x}\right]$ or $-\frac{1}{a}\log\left[\frac{x}{a-\sqrt{a^2-x^2}}\right]$
19. $\frac{1}{a}\sec^{-1}\frac{x}{a}$
20. $\frac{2}{3}x^{3/2} - x + 2x^{1/2} - 2\log(1+x^{1/2})$
21. $-\frac{1}{2}(\cos^{-1}x)^2$
22. $\sqrt{x^2-1} + \log[x+\sqrt{x^2-1}]$
23. $\frac{-1}{2(\log x)^2}$
24. $\frac{1}{3}\tan 3x + \frac{1}{9}\tan^3 3x$
25. $\log x - \frac{1}{x} - \log(1-x)$
26. $-\frac{1}{x} - \tan^{-1}x$
27. $\frac{1}{2}\tan^{-1}x + \frac{x}{2(1+x^2)}$
28. $\frac{1}{2}\tan^2 x + \log\cos x$
29. $\frac{1}{2}\tan^{-1}(\frac{\tan x/2}{2})$
30. $\frac{1}{4}\log(\frac{2+\tan x/2}{2-\tan x/2})$
31. $-\frac{1}{3}\log(5+3\cos x)$
32. $\frac{1}{\sqrt{2}}\tan^{-1}(\frac{\tan x}{\sqrt{2}})$
33. $\log(\sec x + \tan x) = \log\tan(\frac{x}{2}+\frac{\pi}{4})$
34. $-x^2\cos x + 2x\sin x + 2\cos x$
35. $\frac{1}{2}\log(x-1) - 4\log(x-2) + \frac{9}{2}\log(x-3)$
36. $\log(e^x-1)$
37. $\frac{1}{\sqrt{15}}\tan^{-1}(\sqrt{\frac{3}{5}}\tan x)$
38. $\frac{1}{20}e^{5x^4-7}$
39. $\frac{x^6}{6}\log x - \frac{x^6}{36}$
40. $2\log x - 2\log(x+1) + \frac{2}{x+1} - \frac{1}{2(x+1)^2}$
41. $3[x\log x - x]$
42. $\tan^{-1}(e^x)$
43. $\frac{2}{5}(5x^3+7x-1)^{5/2}$
44. $\frac{1}{3}[\tan^{-1}x - \frac{1}{2}\tan^{-1}\frac{x}{2}]$
45. $\frac{2}{\sqrt{3}}\tan^{-1}(\frac{2x+1}{\sqrt{3}})$
46. $\frac{e^x}{5}(\sin 2x - 2\cos 2x)$
47. $\frac{1}{\sqrt{5}}\log(\frac{2x+1-\sqrt{5}}{2x+1+\sqrt{5}})$
48. $\log[(x-\frac{1}{2}) + \sqrt{x^2-x}]$
49. $-2x + 7\log(3+x)$
50. $\frac{1}{3}(x^2-8)\sqrt{4+x^2}$
51. $\log(3+\sin^2 x)$
52. $\frac{1}{4}\log(1+x) - \frac{1}{4}\log(1-x) - \frac{1}{2}\tan^{-1}x$
53. $\log\tan x$ or $-\log(\operatorname{cosec} 2x + \cot 2x)$
54. $\frac{1}{2}(x-1)\log(x-1) - \frac{1}{2}x$
55. $\log(e^x-1) - x$
56. $\log(\frac{\tan x - 2}{\tan x - 1})$
57. $\sqrt{x^2-3x+2} + \frac{5}{2}\log[x - \frac{3}{2} + \sqrt{x^2-3x+2}]$
58. $-\frac{2}{3}\cos^3 x$
59. $\frac{1}{6}\log(1-x+x^2) - \frac{1}{3}\log(1+x) + \frac{1}{\sqrt{3}}\tan^{-1}(\frac{2x-1}{\sqrt{3}})$
60. $\frac{1}{2}[x^2\tan^{-1}x + \tan^{-1}x - x]$
61. $\log\frac{1+2x}{1+x}$
62. $\frac{1}{2}[x\sqrt{9-x^2} + 9\sin^{-1}\frac{x}{3}]$
63. $\frac{1}{2}[x\sqrt{9+x^2} + 9\log[x+\sqrt{9+x^2}]]$
64. $\frac{1}{3}(9+x^2)^{3/2}$
65. $\frac{1}{4}\tan^4 x$
66. $-e^{-x}(x^2+2x+2)$
67. $\frac{1}{2}e^{x^2}$
68. $\log(\sec x + \tan x) - \sin x$
69. $\frac{1}{5}\sin^5 x - \frac{1}{7}\sin^7 x$
70. $x + \log(x-1) - \log x$
71. $x\log(x+\sqrt{x^2-1}) - \sqrt{x^2-1}$
72. $2\log[1+\sqrt{x+1}]$
73. $\frac{16}{3}(2-\sqrt{2})$
74. $\frac{1}{2}\log(\frac{8}{5})$
75. $\frac{1}{2}(\log 2)^2$
76. $1$
77. $\frac{5\pi}{2}$
78. $\frac{\pi\sqrt{2}}{4}$
79. $\frac{1}{3}$
80. $14\log 2 - 3$
81. $\frac{1}{3}\log(\frac{5}{4})$
82. $\frac{2\pi}{3\sqrt{3}}$
83. $2 - \frac{5}{e}$
84. $\frac{3}{2}\log 2 + \pi$
85. $\log(\frac{e+1}{2e}) - \frac{1}{e} + 1$
86. $\frac{a}{2}(\log 4 - 1)$
87. $a(1 - \log 2)$
88. $1 + \log(\frac{3}{4})$
89. $\frac{\pi}{12}$
90. $\frac{\pi}{2}$
91. $\frac{1}{4}(\pi-1)$
92. $\frac{(4\pi-3\sqrt{3})a^4}{192}$
93. $\frac{1}{2}$
94. $\frac{5\sqrt{5}}{3}(2\sqrt{2}-1)$
95. $2(\log 2)^2 - 2\log 2 + \frac{3}{4}$
96. $1 + \frac{5}{2}\log\frac{6}{5}$
97. $3$
98. $\frac{1}{3}\log(\frac{5}{2})$
99. $\frac{1}{4\sqrt{3}}$
100. $\frac{2}{5}$