SOUTH SYDNEY HIGH SCHOOL

PRELIMINARY
HALF YEARLY
MATHEMATICS

MAY

2006

Time Allowed 1.5 Hours

Directions to Candidates

- Attempt ALL questions
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work.
- Board approved calculators maybe used.
- Start each question on a new page.

Question 1 (12 marks)

(a) Find the value of $4\pi\sqrt{\frac{a}{g}}$ if $a=4.1$ and $g=9.8$. Give answer correct to 2 significant figures.

(b) Simplify $\frac{x}{3}+\frac{3x-1}{2}$

(c) Solve $x+7\geq 3$ and graph the solution on the number line.

(d) Solve $x^2-2x-8=0$

(e) If $\frac{1}{3-\sqrt{8}}=a+b\sqrt{2}$ evaluate a and b.

(f) Evaluate $\left(5-\sqrt{2}\right)^2$

Question 2 (12 marks) (Start a new page)

(a) Evaluate correct to two decimal places $\sqrt{\frac{3^2+12^2}{231-12^2}}$.

(b) If $\sqrt{45}+\sqrt{80}=\sqrt{m}$, evaluate m.

(c) Factorise $2x^2+3x-2$.

(d) Solve the pair of simultaneous equations

$2x+y=7$

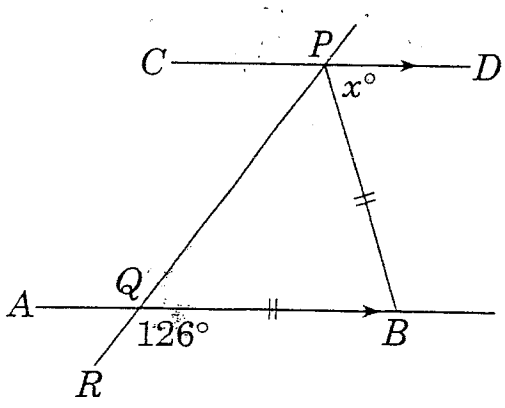
$x-2y=1$

(e) A merchant buys tea from a wholesaler and then sells it at a profit of 37.5%. If the merchant sells a packet of tea for \$3.08, what price does he pay to the wholesaler per packet of tea?

(f) Simplify the expression $4x-3(x+5)$.

Question 3 (12 marks) (Start a new page)

(a)



NOT TO SCALE

In the diagram, CD is parallel to AB, $PB=QB$, $\angle BQR=126^\circ$ and $\angle BPD=x^\circ$.
Copy this diagram on your page.
Find the value of x, giving complete reasons.

(b) Express $0.1\dot{4}\dot{5}$ as a basic fraction. Show all working.

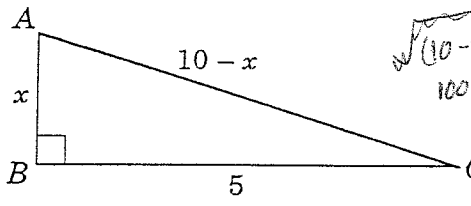
(c) Solve the equation $|3-2x|=9$

(d) Factorise fully $18x^2-2$.

(e) Solve $3x^2-4x-5=0$ Leave the answer as a basic surd.

Question 4 (12 marks) (Start a new page)

(a) In the diagram, $\angle ABC$ is a right angle.
Find the value of x.



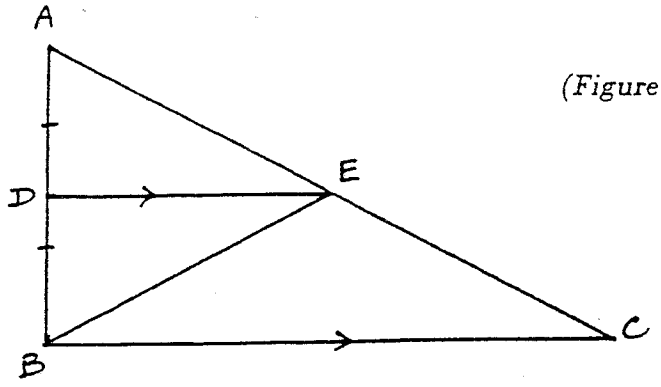
$\sqrt{(10-x)^2-5^2} = x^2$

$100-20x+x^2-25 = x^2$

$75-20x=0$

$-20x=-75$

(b)



(Figure not to scale)

The triangle ABC has a right angle at B. D is the mid point of AB. E lies on AC and DE is parallel to BC.

(i) Copy this diagram onto your page. Prove that triangle ADE is a right angle.

(ii) Prove that triangle AED is congruent to triangle BED.

(iii) Prove that $BE = EC$.

(c) Solve the equation $\dfrac{2x}{x-5}=\dfrac{3}{5}$

(d) Simplify $\dfrac{k^2+k-20}{k^2-16}$

Question 5 (12marks) (Start a new page)

(a)

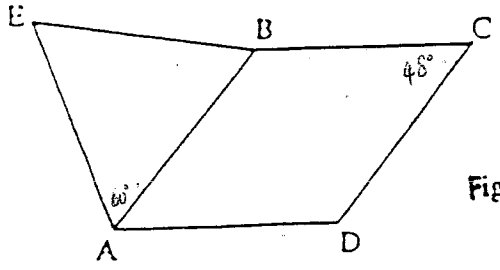
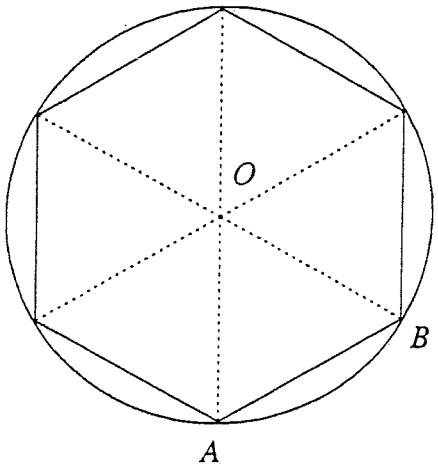


Figure not scale.

$ABCD$ is a rhombus with $\angle BCD = 48°$.
ABE is an equilateral triangle

(i) On your page, draw a neat sketch showing this information.
(ii) Find the size of $\angle EAD$ giving reasons for your answer.
(iii) Find the size of $\angle EDA$ giving reasons for your answer.

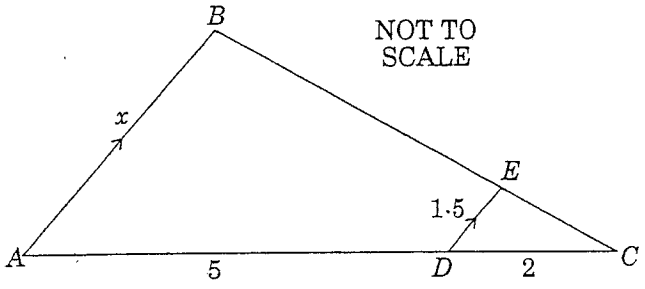
(b)



A regular hexagon is drawn inside a circle with centre O. So that its vertices lie on the circumference as shown in the diagram. The circle has radius 1cm.

(i) Prove that ΔOAB is equilateral.
(ii) Find the area of ΔAOB.

(c)



In the diagram, AB is parallel to DE, AD is 5 cm, DC is 2 cm and DE is 1·5 cm.

Find the length of AB.

SGH '06 YR 11 MATHS H/YRLY SOLUTIONS

Q1 (12 marks)

(a) $4\pi\sqrt{\frac{a}{g}} = 4\times\pi\times\sqrt{\frac{4.1}{9.8}}$

$= 8.128$

$= \underline{8.1}$ (2 s.f.)

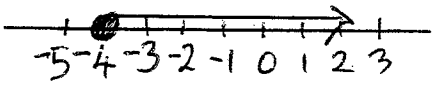
(b) $\frac{x}{3} + \frac{3x-1}{2}$

$= \frac{2x+3(3x-1)}{6}$

$= \frac{11x-3}{6}$

(c) $x+7 \geq 3$

$x \geq -4$



(d) $x^2-2x-8=0$

$(x-4)(x+2)=0$

$\underline{x=4 \text{ or } -2}$

(e) $\frac{1}{3-\sqrt{8}} = \frac{1}{3-\sqrt{8}} \times \frac{3+\sqrt{8}}{3+\sqrt{8}}$

$= \frac{3+\sqrt{8}}{1}$

$= a+b\sqrt{2}$

So $\underline{a=3,\ b=2}$

(f) $(5-\sqrt{2})^2$

$= (5-\sqrt{2})(5-\sqrt{2})$

$= 25-10\sqrt{2}+\sqrt{4}$

$= \underline{27-10\sqrt{2}}$

22 (12 marks)

a) $\sqrt{\frac{3^2+12^2}{231-12^2}} = \sqrt{\frac{153}{87}}$

$= 1.3261$

$= \underline{1.33}$ (2dp)

(b) $\sqrt{45} + \sqrt{80}$

$= 3\sqrt{5} + 4\sqrt{5}$

$= 7\sqrt{5}$

$= \sqrt{245}$

$\underline{m = 245}$

c) $2x^2+3x-2$

$2x$ -1

x 2

$-x + 3x$

$(2x-1)(x+2)$

(d) $2x + y = 7$ — (1)

$x - 2y = 1$ — (2)

(1)×2 $4x + 2y = 14$ — (3)

(2)+(3) $5x = 15$

$x = 3$

sub (1) $6 + y = 7$

$y = 1$

$\underline{\text{Soln } (3, 1)}$

e) 137.5% is \$3.08

1% is $\frac{3.08}{137.5}$

100% is $\frac{3.08}{137.5} \times 100$

Original price is \$2.24

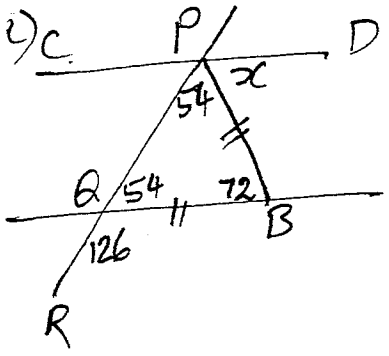
f) $4x - 3(x+5)$

$= 4x - 3x - 15$

$= \underline{x - 15}$

2 (12 marks)

a) c.



$\angle PQB = 54^\circ$ (Straight $\angle$)

$\angle QPB = 54^\circ$ (Isosceles $\triangle$) 1

$\angle PBQ = 72^\circ$ ($\angle$'s in $\triangle$) 1

$x = 72^\circ$ (Alternate $\angle$'s) 1

(b) Let $x = 0.1454545....$

$100x = 14.545454...$

$x = 0.145454....$ 1

$-99x = 14.4$

$x = \frac{14.4}{99}$ 1

$x = \frac{144}{990}$

$x = \frac{8}{55}$ 1

c) $|3-2x| = 9$

$3-2x = 9$ or $3-2x = -9$

$-2x = 6$ $\quad -2x = -12$

$x = -3$ 1 $\quad x = 6$ 1

(d) $18x^2 - 2 = 2(9x^2 - 1)$ 1

$= 2(3x-1)(3x+1)$ 1

e) $3x^2 - 4x - 5 = 0$

$x = \frac{-b \pm \sqrt{b^2-4ac}}{2a}$

$x = \frac{4 \pm \sqrt{16+60}}{6}$ 1

$x = \frac{4 \pm \sqrt{76}}{6}$

$x = \frac{4 \pm 2\sqrt{19}}{6} = \frac{2 \pm \sqrt{19}}{3}$ 1

Q4 (12 marks)

(a) $a^2+b^2=c^2$

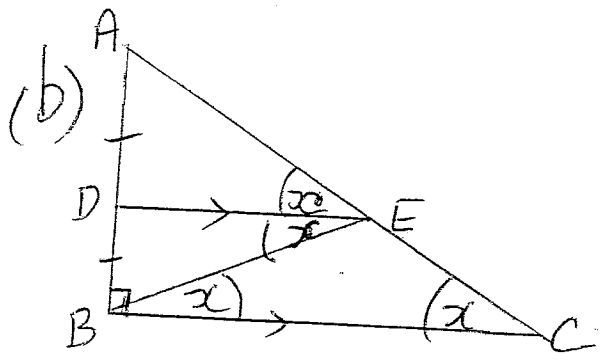
$x^2+5^2=(10-x)^2$ 1

$x^2+25=100-20x+x^2$

$20x=75$ 1

$x=3.75$

(b)



(i) $\angle ADE = 90°$ (Corresp. $\angle$'s) 1

(ii) $AD = DB$ (Given)

$\angle ADE = \angle BDE$ ($90°$) 1

DE (Common)

$\therefore \triangle AED \equiv \triangle BED$ (SAS) 1

(iii) $\angle AED = \angle DEB = x$

(Corresp $\angle$'s in $\equiv \triangle$'s) 1

$\angle DEB = \angle EBC = x$

(Alternate $\angle$'s) 1

$\angle AED = \angle ECB$

(Corresp $\angle$'s) 1

So $\triangle BEC$ is isosceles

and $BE = EC$

c) $\frac{2x}{x-5} = \frac{3}{5}$ 1

$10x = 3(x-5)$

$10x = 3x-15$

$7x = -15$ 1

$x = \frac{-15}{7}$

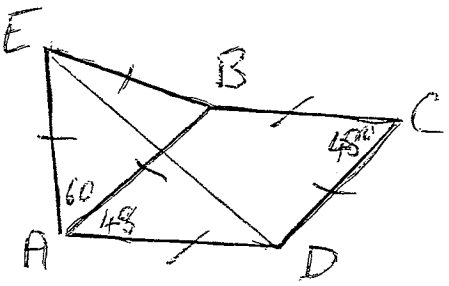
$x = -2\frac{1}{7}$

d) $\frac{k^2+k-20}{k^2-16} = \frac{(k+5)(k-4)}{(k-4)(k+4)}$ 1

$= \frac{k+5}{k+4}$ 1

5. (12 marks)

a) (i)



(ii) $\angle EAB = 60^\circ$ (Equilateral $\triangle$)

$\angle BAD = 48^\circ$ (Opp $\angle$s in rhombus)

$\angle EAD = 108^\circ$

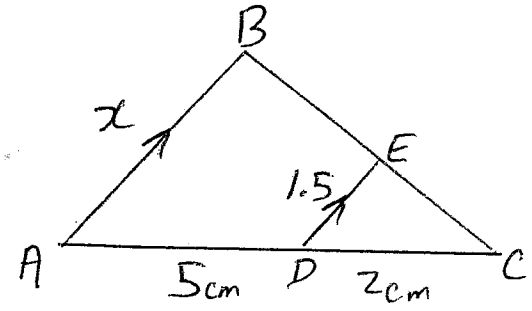
(iii) $\triangle EDA$ is isosceles (Since $EA = AD$)

$\angle EDA$ & $\angle AED$ are base angles.

So $2 \times \angle EDA + 108 = 180$

$\angle EDA = 36^\circ$

c)



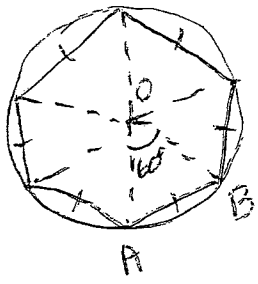
Triangles are similar.
So corresponding sides are in ratio

$$\frac{x}{1.5} = \frac{7}{2}$$

$$x = \frac{7}{2} \times 1.5$$

$$x = 5.25\text{cm}$$

(b)



(i) $\angle AOB = 60^\circ$

Centre of regular hexagon

$OA = OB$ (radii)

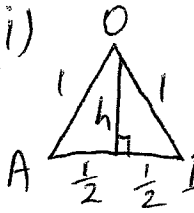
So $\triangle OAB$ is isosceles

Base angles $= 120^\circ$

$\therefore$ Each angle $= 60^\circ$

So $\triangle OAB$ is equilateral

(ii)



Using Pythagoras'

$$h^2 = 1^2 - \frac{1}{2}^2$$

$$h^2 = \frac{3}{4}$$

$$h = 0.87$$

$$A = \frac{1}{2}bh$$

$$= \frac{1}{2} \times 1 \times 0.866$$

$$= 0.433 \text{ sq cm}$$