

SOUTH SYDNEY HIGH SCHOOL

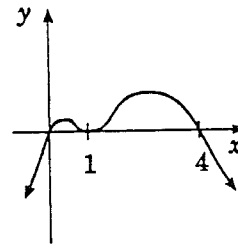
MATHEMATICS

POLYNOMIALS WORKSHEET

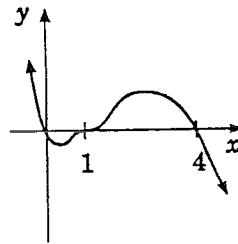
- For the following polynomials, find the degree, the leading term, leading coefficient and constant term:
 - $P(x) = 3x^4 - 7x + 2$
 - $P(x) = 4 - 2x^5$
 - $P(x) = 3x^5 - 2x^3 + 4x$
 - $P(x) = 7$
- Determine whether the following are polynomials in x :
 - $\frac{4}{x} - 3x + 2$
 - $\frac{5x^4 - x}{x}$
 - $\frac{3x}{4} + 2$
 - $4x^4 + \sqrt{x} - 2$
 - $(x+4)^2 - 2x + 3$
- Complete the following:
 - $\frac{4x^4 - 2x^2 + 3}{3x^4 - 5x^2 - 7} +$
 - $\frac{3x^2 - 4x + 1}{5x^2 + 2x - 5} +$
 - $\frac{4x^3 + 12x^2 + 3x + 7}{5x^3} + \frac{x + 3}{x + 3} +$
 - $\frac{4x^5 - 21x + 2}{3x^5 + 2x - 1} +$
 - $\frac{5x^3 + 2x^2 - x - 1}{3x^3 - x^2 - x + 5} -$
 - $\frac{4x^4 - 2x^2 + 1}{x^4 - 5x^2 - 7} -$
 - $\frac{4x^3}{3x^3 + 2x^2} - \frac{x + 2}{-1} -$
 - $\frac{5x^4}{3x^3 + x^2 - 5x + 2} - \frac{-2x^2 + 1}{-1} -$
- Find the degree of $P(x), Q(x)$ if:
 - $P(x) = 3x^2 - 4x + 1$ and $Q(x) = 4 - x + x^2$
 - $P(x) = 5x^4 - 2x - 1$ and $Q(x) = 3x - 5x^2 + x^3$
- Find the product of the following polynomials:
 - $2x^3 - 4$ and $5x^2 - x + 2$
 - $6x^2 - 4x$ and $x^3 + x - 1$
- Complete the following divisions, leaving your answer in the form: $\text{dividend} = \text{divisor} \times \text{quotient} + \text{remainder}$.
 - $(x^3 - 3x^2 + 7x - 5) \div (x - 3)$
 - $(-x^3 + 2x^2 - 5x - 1) \div (x + 2)$
 - $(x^5 + x^3 - x) \div (x^2 + x)$
 - $(x^4 - 1) \div (x^2 - 3)$
- If $P(x) = x^3 + 2x^2 - x + 1$ and $Q(x) = x - 1$, find
 - $P(x) + Q(x)$
 - $P(x) - Q(x)$
 - $P(x) \cdot Q(x)$
 - $\frac{P(x)}{Q(x)}$
- Find the remainder after the following divisions:
 - $(x^3 + 4x^2 + x - 1) \div (x + 2)$
 - $(x^3 - 4x + 2) \div (x - 1)$
- If $x^3 + 3x^2 - 7x + k$ is divided by $(x + 2)$ the remainder is 5. Find the value of k .
- When $3x^4 - 2x^3 + 2k$ is divided by $(x + 1)$, the remainder is 3. Find the value of k .
- If $x^2 - 7x + 9$ is divided by $(x - k)$, the remainder is 3. Find the value of k .
- If $(x - 5)$ is a factor of $x^3 + kx - 10$, find the value of k .

13. Show that $(x - 1)$ is a factor of $x^3 - 2x^2 - 5x + 6$, and hence solve the equation $x^3 - 2x^2 - 5x + 6 = 0$.
14. Factorise $2x^3 - 5x^2 - 4x + 3$.
15. Solve for x , the equation $x^3 + 2x^2 - 9x - 18 = 0$.
16. If $x = -3$ and $x = 1$ are roots of the equation $x^3 - 2x^2 - ax + b$, find the values of a and b , and hence find the third root of the equation.
17. Sketch the graphs of the following polynomials:
- $y = (x - 4)(x - 1)(x + 2)$
 - $y = x(x - 2)^2(4 - x)$
 - $y = (3 - x)(x + 1)^3(4 - x)$
 - $y = x^2(1 - x)^2$
 - $y = (1 + x)^3$
18. Factorise the following and then sketch their graphs, showing clearly the zeros of each function:
- $y = x^3 - 4x^2 + x + 6$
 - $y = x^3 - x^2 - x + 1$
 - $y = x^3 - 3x^2 + 3x - 1$
 - $y = -x^3 + 12x + 16$
19. Match the polynomial function to the correct sketch:
- $y = x(x - 1)^2(4 - x)$
 - $y = x^2(x - 1)(x + 4)$
 - $y = x(x - 1)^3(4 - x)$
 - $y = x(x + 1)(4 + x)$

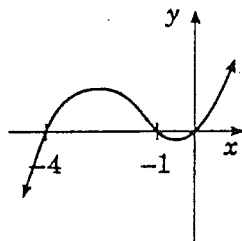
(iii)



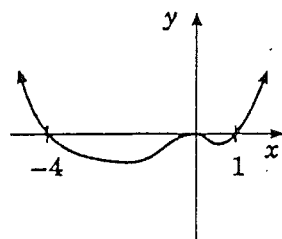
(iv)



(i)



(ii)



WORKED SOLUTIONS

1. (a) $P(x) = 3x^4 - 7x + 2$
deg: 4, lead t: $3x^4$, lead c = 3
const t: 2
- (b) $P(x) = 4 - 2x^5$
deg: 5, lead t: $-2x^5$, lead c = -2
const t: 4
- (c) $P(x) = 3x^5 - 2x^3 + 4x$
deg: 5, lead t: $3x^5$, lead c = 3
const t: 0
- (d) $P(x) = 7$
deg: 0, lead t: 7, lead c = 7
const t: 7
2. (a) $\frac{4}{x} - 3x + 2 = 4x^{-1} - 3x + 2$
 \therefore not polynomial [power of -1]
- (b) $\frac{5x^4 - x}{x} = 5x^3 - 1$
 \therefore polynomial.
- (c) $\frac{3x}{4} + 2 = \frac{3}{4}x + 2$
 \therefore polynomial.
- (d) $4x^4 + \sqrt{x} - 2 = 4x^4 + x^{\frac{1}{2}} - 2$
 \therefore not polynomial [power of $\frac{1}{2}$]
- (e) $(x+4)^2 - 2x + 3$
 $= x^2 + 8x + 16 - 2x + 3$
 $= x^2 + 6x + 19$
 \therefore polynomial
3. (a) $\frac{4x^4 - 2x^2 + 3}{3x^4 - 5x^2 - 7} + \frac{7x^4 - 7x^2 - 4}{7x^4 - 7x^2 - 4}$
- (b) $\frac{3x^2 - 4x + 1}{5x^2 + 2x - 5} + \frac{8x^2 - 2x - 4}{8x^2 - 2x - 4}$
- (c) $\frac{4x^3 + 12x^2 + 3x + 7}{5x^3 + 0x^2 + x + 3} + \frac{9x^3 + 12x^2 + 4x + 10}{9x^3 + 12x^2 + 4x + 10}$
- (d) $\frac{4x^5 - 21x + 2}{3x^5 + 2x - 1} + \frac{7x^5 - 19x + 1}{7x^5 - 19x + 1}$
- (e) $\frac{5x^3 + 2x^2 - x - 1}{3x^3 - x^2 - x - 5} - \frac{2x^3 + 3x^2 + 4}{2x^3 + 3x^2 + 4}$
- (f) $\frac{4x^4 - 2x^2 + 1}{x^4 - 5x^2 - 7} - \frac{3x^4 + 3x^2 + 8}{3x^4 + 3x^2 + 8}$
- (g) $\frac{4x^3 + 0x^2 - x + 2}{3x^3 + 2x^2 + 0x - 1} - \frac{x^3 - 2x^2 - x + 3}{x^3 - 2x^2 - x + 3}$
- (h) $\frac{5x^4 + 0x^3 - 2x^2 + 0x + 1}{3x^3 + x^2 - 5x + 2} - \frac{5x^4 - 3x^3 - 3x^2 + 5x - 1}{5x^4 - 3x^3 - 3x^2 + 5x - 1}$
4. (a) $P(x) \cdot Q(x)$
 $= (3x^2 - 4x + 1)(4 - x + x^2)$
 $= \dots + 3x^4 + \dots$
 \therefore degree is 4
[only need leading term]
- (b) $P(x) \cdot Q(x)$
 $= (5x^4 - 2x - 1)(3x - 5x^2 + x^3)$
 $= \dots + 5x^7 + \dots$
 \therefore degree is 7.
5. (a) $(2x^3 - 4)(5x^2 - x + 2)$
 $= 10x^5 - 2x^4 + 4x^3 - 20x^2 + 4x - 8$
- (b) $(6x^2 - 4x)(x^3 + x - 1)$
 $= 6x^5 + 6x^3 - 6x^2 - 4x^4 - 4x^2 + 4x$
 $= 6x^5 - 4x^4 + 6x^3 - 10x^2 + 4x$
6. (a) $x - 3 \overline{) \begin{array}{r} x^3 - 3x^2 + 7x - 5 \\ x^3 - 3x^2 \\ \hline 0 + 7x - 5 \\ 7x - 21 \\ \hline 16 \end{array}}$
 $\therefore x^3 - 3x^2 + 7x - 5$
 $= (x - 3)(x^2 + 7) + 16$
- (b) $x + 2 \overline{) \begin{array}{r} -x^3 + 2x^2 - 5x - 1 \\ -x^3 - 2x^2 \\ \hline 4x^2 - 5x \\ 4x^2 + 8x \\ \hline -13x - 1 \\ -13x - 26 \\ \hline 25 \end{array}}$
 $\therefore -x^3 + 2x^2 - 5x - 1$
 $= (x + 2)(-x^2 + 4x - 13) + 25$

$$(c) \quad x^2 + x \overline{) \begin{array}{r} x^3 - x^2 + 2x - 2 \\ x^5 + 0x^4 + x^3 + 0x^2 - x \\ \underline{x^5 + x^4} \\ -x^4 + x^3 \\ \underline{-x^4 - x^3} \\ 2x^3 + 0x^2 \\ \underline{2x^3 + 2x^2} \\ -2x^2 - x \\ \underline{-2x^2 - 2x} \\ x \end{array}}$$

$$\therefore x^5 + x^3 - x = (x^2 + x)(x^3 + x^2 + 2x - 2) + x$$

$$(d) \quad (x^4 - 1) + (x^2 - 3)$$

$$x^2 - 3 \overline{) \begin{array}{r} x^4 + 0x^3 + 0x^2 + 0x - 1 \\ \underline{x^4 - 3x^2} \\ 3x^2 - 1 \\ \underline{3x^2 - 9} \\ 8 \end{array}}$$

$$\therefore x^4 - 1 = (x^2 - 3)(x^2 + 3) + 8$$

7. $p(x) = x^3 + 2x^2 - x + 1$
 $q(x) = x - 1$

(a) $p(x) + q(x)$
 $\therefore x^3 + 2x^2 - x + 1 + \underline{x - 1}$
 $x^3 + 2x^2$

$$\therefore p(x) + q(x) = x^3 + 2x^2$$

(b) $p(x) - q(x)$
 $\therefore x^3 + 2x^2 - x + 1 - \underline{x - 1}$

$$\therefore p(x) - q(x) = x^3 + 2x^2 - 2x + 2$$

(c) $p(x) \cdot q(x)$
 $= (x^3 + 2x^2 - x + 1)(x - 1)$
 $= x^4 - x^3 + 2x^3 - 2x^2 - x^2 + x + x - 1$
 $= x^4 + x^3 - 3x^2 + 2x - 1$

(d) $\frac{p(x)}{q(x)}$

$$\therefore x - 1 \overline{) \begin{array}{r} x^2 + 3x + 2 \\ x^3 + 2x^2 - x + 1 \\ \underline{x^3 - x^2} \\ 3x^2 - x \\ \underline{3x^2 - 3x} \\ 2x + 1 \\ \underline{2x - 2} \\ 3 \end{array}}$$

$$\therefore p(x) + q(x) = (x^2 + 3x + 2) + 3$$

8. (a) Let $P(x) = x^3 + 4x^2 + x - 1$
 Dividing by $(x + 2)$, \therefore find $P(-2)$
 $\therefore P(-2) = (-2)^3 + 4(-2)^2 + (-2) - 1$
 $= -8 + 16 - 2 - 1 = 5$
 \therefore remainder is 5.

(b) Let $P(x) = x^3 - 4x + 2$
 Dividing by $(x - 1)$, \therefore find $P(1)$
 $\therefore P(1) = 1^3 - 4(1) + 2$
 $= 1 - 4 + 2 = -1$
 \therefore remainder is -1 .

9. Let $P(x) = x^3 + 3x^2 - 7x + k$
 Dividing by $(x + 2)$, $\therefore P(-2) = 5$
 $\therefore P(-2) = (-2)^3 + 3(-2)^2 - 7(-2) + k = 5$
 $-8 + 12 + 14 + k = 5$
 $18 + k = 5$
 $k = 5 - 18$
 $k = -13$
 \therefore value of k is -13 .

10. Let $P(x) = 3x^4 - 2x^3 - 2k$
 Dividing by $(x + 1)$, $\therefore P(-1) = 3$
 $\therefore P(-1) = 3(-1)^4 - 2(-1)^3 + 2k = 3$
 $3 + 2 + 2k = 3$
 $5 + 2k = 3$
 $2k = 3 - 5$
 $2k = -2$
 $k = -1$
 \therefore value of k is -1 .

11. Let $P(x) = x^2 - 7x + 9$
 Dividing by $(x - k)$, $\therefore P(k) = 3$
 $\therefore P(k) = k^2 - 7k + 9 = 3$,
 that is, $k^2 - 7k + 6 = 0$
 $(k - 6)(k - 1) = 0$
 $k = 6$ or 1
 $\therefore k$ is 6 or 1.

12. Let $P(x) = x^3 + kx - 10$
 Dividing by $(x - 5)$, $\therefore P(5) = 0$
 $\therefore P(5) = 5^3 + 5k - 10 = 0$
 $125 + 5k - 10 = 0$
 $5k + 115 = 0$
 $5k = -115$
 $\frac{5k}{5} = \frac{-115}{5}$
 $k = -23$
 \therefore value of k is -23 .

13. Let $P(x) = x^3 - 2x^2 - 5x + 6$
 If $(x - 1)$ is a factor, then $P(1) = 0$
 $\therefore P(1) = 1^3 - 2(1)^2 - 5(1) + 6$
 $= 1 - 2 - 5 + 6$
 $= 0$
 \therefore as $P(1) = 0$, then $(x - 1)$ is a factor of $P(x)$.

$$\begin{array}{r} x^2 - x - 6 \\ x-1 \overline{) x^3 - 2x^2 - 5x + 6} \\ \underline{x^3 - x^2} \\ -x^2 - 5x \\ \underline{-x^2 + x} \\ -6x + 6 \\ \underline{-6x + 6} \\ 0 \end{array}$$

- \therefore as $x^3 - 2x^2 - 5x + 6 = 0$
 $\therefore (x - 1)(x^2 - x - 6) = 0$
 $(x - 1)(x - 3)(x + 2) = 0$
 $\therefore x = 1, 3, -2$,
 \therefore the roots of the equation are $x = 1, 3$ and -2 .

14. Let $P(x) = 2x^3 - 5x^2 - 4x + 3$
 Now find a , where $P(a) = 0$,
 that is, try
 $P(1) = 2(1)^3 - 5(1)^2 - 4(1) + 3$
 $= 2 - 5 - 4 + 3$
 $\neq 0$

Try

$$\begin{aligned} P(-1) &= 2(-1)^3 - 5(-1)^2 - 4(-1) + 3 \\ &= -2 - 5 + 4 + 3 \\ &= 0 \end{aligned}$$

$\therefore P(-1) = 0$, $\therefore (x + 1)$ is a factor.

$$\begin{array}{r} 2x^2 - 7x + 3 \\ x+1 \overline{) 2x^3 - 5x^2 - 4x + 3} \\ \underline{2x^3 + 2x^2} \\ -7x^2 - 4x \\ \underline{-7x^2 - 7x} \\ 3x + 3 \\ \underline{3x + 3} \\ 0 \end{array}$$

$$\begin{aligned} \therefore 2x^3 - 5x^2 - 4x + 3 &= (x + 1)(2x^2 - 7x + 3) \\ &= (x + 1)(2x - 1)(x - 3). \end{aligned}$$

15. Let $P(x) = x^3 + 2x^2 - 9x - 18$
 Try $P(1) = 1^3 + 2(1)^2 - 9(1) - 18 \neq 0$
 $P(-1) = (-1)^3 + 2(-1)^2 - 9(-1) - 18 \neq 0$
 $P(2) = (2)^3 + 2(2)^2 - 9(2) - 18$
 $= 8 + 8 - 18 - 18 \neq 0$
 $P(-2) = (-2)^3 + 2(-2)^2 - 9(-2) - 18$
 $= -8 + 8 + 18 - 18 = 0$,
 \therefore as $P(-2) = 0$, $(x + 2)$ is a factor,

$$\begin{array}{r} x^2 - 9 \\ x+2 \overline{) x^3 + 2x^2 - 9x - 18} \\ \underline{x^3 + 2x^2} \\ -9x - 18 \\ \underline{-9x - 18} \\ 0 \end{array}$$

$$\begin{aligned} \text{As } x^3 + 2x^2 - 9x - 18 &= 0 \\ (x + 2)(x^2 - 9) &= 0 \\ (x + 2)(x - 3)(x + 3) &= 0 \\ \therefore x &= -2, 3, -3. \end{aligned}$$

16. Let $P(x) = x^3 - 2x^2 - ax + b$
 As $x = -3$ is a root, $P(-3) = 0$
 $\therefore P(-3) = (-3)^3 - 2(-3)^2 - a(-3) + b$
 $= 0$
 $-27 - 18 + 3a + b = 0$
 $-45 + 3a + b = 0$
 $\therefore 3a + b = 45$.
 Also, as $x = 1$ is a root, $P(1) = 0$.
 $\therefore P(1) = (1)^3 - 2(1)^2 - a(1) + b = 0$
 $1 - 2 - a + b = 0$
 $-1 - a + b = 0$
 $\therefore a - b = -1$

$$\therefore 3a + b = 45 \quad (1)$$

$$a - b = -1 \quad (2)$$

$$(1) + (2)$$

$$4a = 44$$

$$a = 11$$

Substitute $a = 11$ in (2)

$$11 - b = -1$$

$$-b = -1 - 11$$

$$-b = -12$$

$$\therefore b = 12,$$

$$\therefore a = 11, b = 12,$$

$$\therefore P(x) = x^3 - 2x^2 - 11x + 12.$$

Now as $x = -3, x = 1$ are roots,

$\therefore (x + 3), (x - 1)$ are factors,

$\therefore (x + 3)(x - 1)$ is also a factor,

$\therefore (x^2 + 2x - 3)$ is a factor,

$$\begin{array}{r} x^3 - 2x^2 - 11x + 12 \\ x^2 + 2x - 3 \overline{) } \\ \underline{x^3 + 2x^2 - 3x} \\ -4x^2 - 8x + 12 \\ \underline{-4x^2 - 8x + 12} \\ 0 \end{array}$$

$$\begin{aligned} \therefore x^3 - 2x^2 - 11x + 12 \\ &= (x^2 + 2x - 3)(x - 4) \\ &= (x + 3)(x - 1)(x - 4), \end{aligned}$$

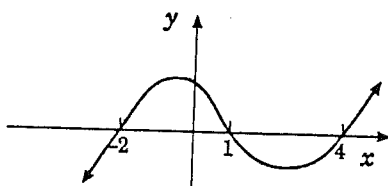
$$\therefore \text{as } P(x) = 0$$

$$(x + 3)(x - 1)(x - 4) = 0,$$

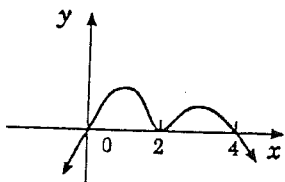
$$\therefore x = -3, 1, 4$$

\therefore the other root is $x = 4$.

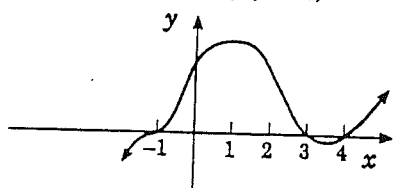
17. (a) $y = (x - 4)(x - 1)(x + 2)$



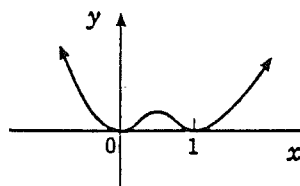
(b) $y = x(x - 2)^2(4 - x)$



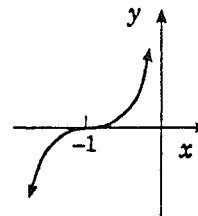
(c) $y = (3 - x)(x + 1)^3(4 - x)$



(d) $y = x^2(1 - x)^2$



(e) $y = (1 + x)^3$



18. (a) $y = x^3 - 4x^2 + x + 6$

Let $y = P(x)$

$$\therefore P(x) = x^3 - 4x^2 + x + 6.$$

$$\begin{aligned} \text{Try } P(1) &= 1^3 - 4(1)^2 + 1 + 6 \\ &= 1 - 4 + 1 + 6 \neq 0 \end{aligned}$$

$$\begin{aligned} P(-1) &= (-1)^3 - 4(-1)^2 - 1 + 6 \\ &= -1 - 4 - 1 + 6 = 0 \end{aligned}$$

\therefore as $P(-1) = 0$, $x + 1$ is a factor,

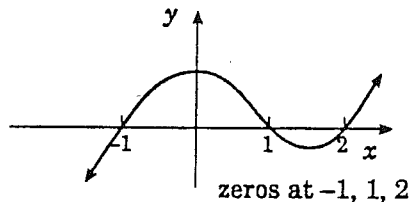
$$\begin{array}{r} x^3 - 4x^2 + x + 6 \\ x + 1 \overline{) } \\ \underline{x^3 + x^2} \\ -5x^2 + x \\ \underline{-5x^2 - 5x} \\ 6x + 6 \\ \underline{6x + 6} \\ 0 \end{array}$$

$$\begin{aligned} \therefore P(x) &= x^3 - 4x^2 + x + 6 \\ &= (x + 1)(x^2 - 5x + 6) \\ &= (x + 1)(x - 3)(x - 2) \\ \therefore y &= (x + 1)(x - 3)(x - 2) \end{aligned}$$

Substitute $x = 4$ in y ,

$$\begin{aligned} \text{that is, } y(4) &= (4 + 1)(4 - 3)(4 - 1) \\ &= (5)(1)(3) > 0 \end{aligned}$$

\therefore above the x -axis at $x = 4$.



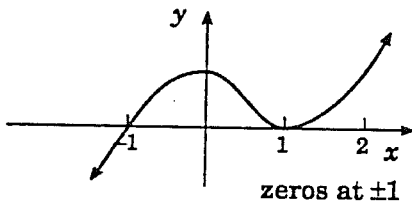
(b) $y = x^3 - x^2 - x + 1$.
 Let $y = P(x)$
 $\therefore P(x) = x^3 - x^2 - x + 1$.
 Try $P(1) = 1^3 - 1^2 - 1 + 1$
 $= 0$
 \therefore as $P(1) = 0$, $x - 1$ is a factor,

$$\begin{array}{r} x^2 - 1 \\ x-1 \overline{) x^3 - x^2 - x + 1} \\ \underline{x^3 - x^2} \\ -x + 1 \\ \underline{-x + 1} \\ 0 \end{array}$$

$$\begin{aligned} \therefore P(x) &= x^3 - x^2 - x + 1 \\ &= (x-1)(x^2 - 1) \\ &= (x-1)(x-1)(x+1) \\ &= (x-1)^2(x+1) \end{aligned}$$

$$\therefore y = (x-1)^2(x+1).$$

Substitute $x = 2$ in y ,
 that is, $y(2) = (2-1)^2(2+1)$
 $= (1)^2(3) > 0$,
 \therefore above x -axis at $x = 2$,
 that is, $y > 0$ when $x = 2$,
 (that is, $y > 0$ for $x > 1$).



(c) $y = x^3 - 3x^2 + 3x - 1$
 Let $y = P(x)$
 $\therefore P(x) = x^3 - 3x^2 + 3x - 1$.
 Try $P(1) = 1^3 - 3(1)^2 + 3(1) - 1$
 $= 1 - 3 + 3 - 1$
 $= 0$

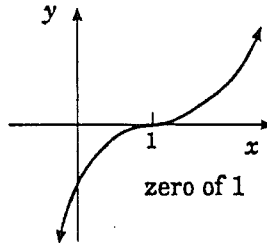
\therefore as $P(1) = 0$, $(x - 1)$ is a factor,

$$\begin{array}{r} x^2 - 2x + 1 \\ x-1 \overline{) x^3 - 3x^2 + 3x - 1} \\ \underline{x^3 - x^2} \\ -2x^2 + 3x \\ \underline{-2x^2 + 2x} \\ x - 1 \\ \underline{x - 1} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \text{as } P(x) &= x^3 - 3x^2 + 3x - 1 \\ &= (x-1)(x^2 - 2x + 1) \\ &= (x-1)(x-1)(x-1) \\ &= (x-1)^3 \end{aligned}$$

$$\therefore y = (x-1)^3$$

Substitute $x = 2$ in y ,
 that is, $y(2) = (2-1)^3$
 $= 1^3 > 0$,
 \therefore above x -axis at $x = 2$,
 $\therefore y > 0$ when $x > 1$.



(d) $y = -x^3 + 12x + 16$
 Let $y = P(x)$

$$\therefore P(x) = -x^3 + 12x + 16.$$

$$\text{Try } P(1) = -1^3 + 12(1) + 16$$

$$\neq 0$$

$$P(-1) = -(-1)^3 + 12(-1) + 16$$

$$\neq 0$$

$$P(2) = -2^3 + 12(2) + 16$$

$$= -8 + 24 + 16$$

$$\neq 0$$

$$P(-2) = -(-2)^3 + 12(-2) + 16$$

$$= 8 - 24 + 16$$

$$= 0,$$

\therefore as $P(-2) = 0$, $(x + 2)$ is a factor,

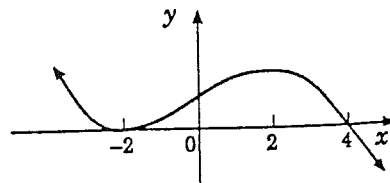
$$\begin{array}{r} -x^2 + 2x + 8 \\ x+2 \overline{) -x^3 + 0x^2 + 12x + 16} \\ \underline{-x^3 - 2x^2} \\ 2x^2 + 12x \\ \underline{2x^2 + 4x} \\ 8x + 16 \\ \underline{8x + 16} \\ 0 \end{array}$$

$$\begin{aligned} \therefore \text{as } P(x) &= -x^3 + 12x + 16 \\ &= (x+2)(-x^2 + 2x + 8) \\ &= -(x+2)(x^2 - 2x - 8) \\ \therefore y &= -(x+2)(x+2)(x-4) \\ &= -(x+2)^2(x-4) \end{aligned}$$

\therefore zeros at $-2, 4$

Substitute $x = 0$ in y ,
 that is, $y(0) = -(0)^2(-4)$
 $= -4(-4) > 0$,

\therefore below axis when $x = 0$.



19. (a) (iii)
 (b) (ii)
 (c) (iv)
 (d) (i)