South Sydney High

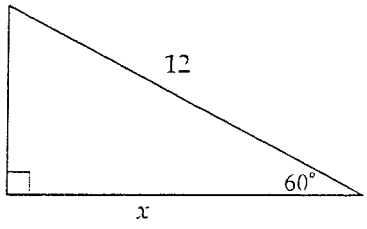
Mathematics

Trigonometry Mathematics Task

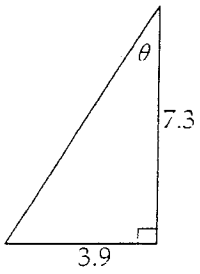
Term 3, 2003		**Date:**	Friday 15th August 2003
Time allowed: 80 minutes		**Assessment:**	Mathematics

Question1:

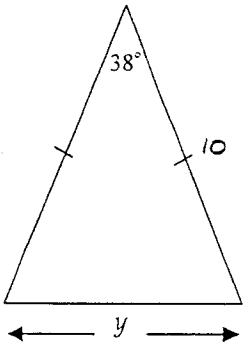
(a) Find the exact value of x:



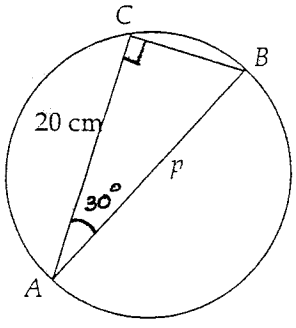
(b) Find the value of θ, to the nearest minute.



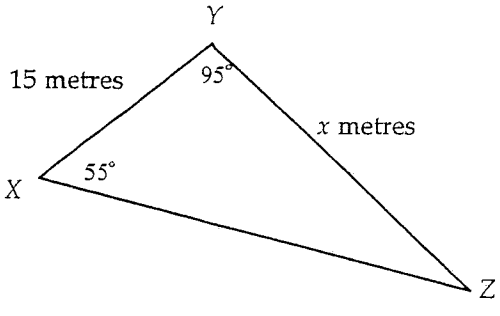
(c) Find the value of y, to 2 decimal places.



(d) AB is a diameter. Find p correct to the nearest integer.



(e)



(i) Find the value of x correct to 1 decimal place.

(ii) Calculate the area of triangle XYZ.

Question 2:

(a) If $\sin\theta = -\frac{5}{6}$, and $\cos\theta < 0$, find the exact ratio of $\cot\theta$.

(b) Find the value of x if $\operatorname{cosec}(2m - 40^\circ) = \sec 30^\circ$

(c) If $x = 3\sin A$ and $y = 4\cos A$, show that $16x^2 + 9y^2 = 144$

(d) Show that $(2\tan\alpha + 2)^2 = 4(\sec^2\alpha + 2\tan\alpha)$

(e) Solve, to the nearest minute, the equation $\cos x = 0.7$ for $0 \le x \le 360$.

(f) If $-180 \le \theta \le 360$, find θ if $\sin\theta = -0.7$

Question 3:

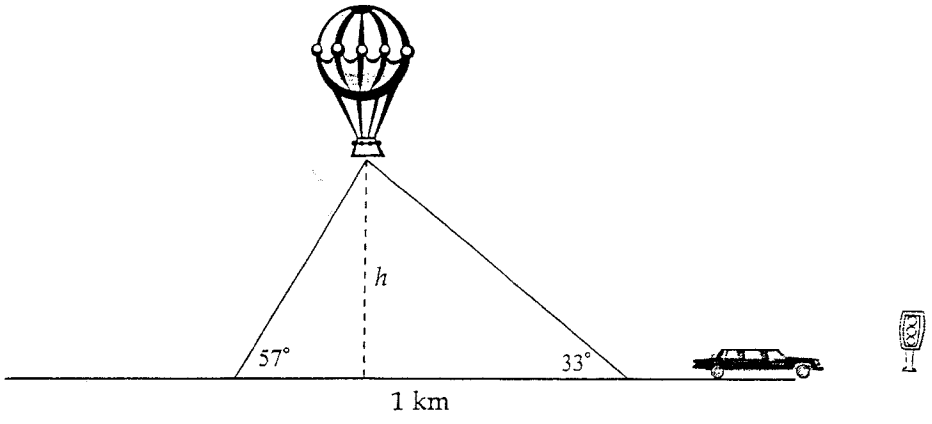
(a) (i) Sketch a neat graph of $y = \cos x$ for $0 \le x \le 360$.

(ii) On the same set of axes, sketch the graph of $y = \sin x$ for $0 \le x \le 360$.

(iii) Hence give the solution(s) to the equation $\cos x = \sin x$ for $0 \le x \le 360$.

Clearly show all solutions on your graph.

(b) From the ends of a straight horizontal road 1km long, a hot air balloon is observed to have angles of elevation of 57° and 33° respectively. Find, correct to the nearest metre, the height of the balloon above the road.



Question 4:

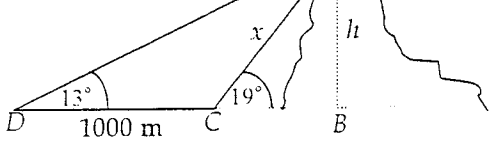
(a) (i) Find $\cot 240^0 + \sin 270^0$ as an exact value with a rational denominator.

(b) A boat sails 6km due north from the harbour H to A, and a 2nd boat sails 10km from H to B on a bearing of $120°$.

(i) What is the distance AB?

(ii) What is the bearing of B from A, correct to the nearest minute ?

(c) The angle of elevation of the top of a mountain from a point is $13°$. On advancing a distance 1000m towards the mounting, the angle of elevation is now $19°$



(i) Prove that $x = \dfrac{1000\sin 13^\circ}{\sin 6^\circ}$

(ii) Show that $h = x\sin 19^\circ$

(iii) Hence or otherwise find the height of the mountain to 3 significant figures.

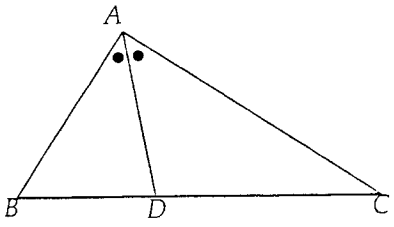
Question 5:

(a) (i) Prove that $\dfrac{\sin 120^\circ}{\tan 300^\circ} - \dfrac{\cos 240^\circ}{\cot 315^\circ} = \tan^2 240^\circ - \operatorname{cosec}^2 330^\circ$

(ii) Solve $2 - \cos x = 2\sin^2 x$ for $0 \le x \le 360$.

(b) (i) In triangle ABC, AD bisects $\angle BAC$. Use the sine rule to prove that:

$$\frac{AB}{AC} = \frac{BD}{DC}$$



THE END

South Sydney High - TRIGONOMETRY Mathematics Task

Eva Lee

Question 1

a) $\cos 60° = \frac{x}{12}$

$12\cos 60° = x$

$x = 6$ ✓

b) $\tan\theta = \frac{3.9}{7.3}$

$= 28°7'$ ✓

c) $\sin 19° = \frac{y/2}{10}$

$20\sin 19° = y$

$y = 6.51$ ✓

e i) $\angle Z = 30°$

$\frac{15}{\sin 30°} = \frac{x}{\sin 55°}$ ✓

$\frac{15\sin 55°}{\sin 30°} = x$

$x = 24.6\text{m}$ ✓

ii) $A = \frac{1}{2}ab\sin C$ ✓

$= \frac{1}{2} 24.6 \times 15 \times \sin 95°$

$= 183.80\text{m}^2$ ✓

Question 4

a i) $\cot 240° = \frac{1}{\tan 240°}$

$= \frac{1}{\tan 60}$

$= \frac{1}{\sqrt{3}}$ ✓

$\sin 270 = -1$

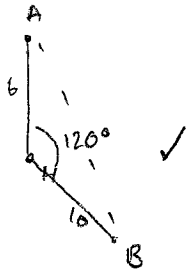
$\frac{1}{\sqrt{3}} - \frac{\sqrt{3}}{\sqrt{3}}$ ✓

$= \frac{1-\sqrt{3}}{\sqrt{3}}$

$= \frac{1-\sqrt{3}}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}}$

$= \frac{\sqrt{3}-3}{3}$ ✓

b i)



✓

$AB = 6^2 + 10^2 - 2(6\times10)\cos 120°$

$= 196$

$= 14\text{km}$ ✓

ii) $\frac{14}{\sin 120°} = \frac{10}{\sin A}$ ✓

$\sin A = \frac{10\sin 120°}{14}$

$= 38°13'$ ✓

c i) $\angle DAC = 6°$

$\frac{x}{\sin 13°} = \frac{1000}{\sin 6}$ ✓

$x = \frac{1000\sin 13}{\sin 6}$ ✓

i) In $\triangle ABC$

$\sin 19 = \frac{h}{x}$

$x\sin 19 = h$ ✓

ii) $\frac{1000\sin 13}{\sin 6} \times \sin 19 = h$ ✓

$h = 700\text{m}$ ✓

Question 5

a i) $\sin 120° = \frac{\sqrt{3}}{2}$ $\quad \tan 300 = -\sqrt{3}$ ✓

$\cos 240° = -\frac{1}{2}$ ✓ $\quad \cot 315° = \frac{1}{\tan 315} = -1$

$\tan^2 240° = 3$ $\quad \operatorname{cosec}^2 330 = \frac{1}{\sin^2 330} = 4$

$\therefore$ L.H.S $= \frac{\frac{\sqrt{3}}{2}}{-\sqrt{3}} - \frac{-\frac{1}{2}}{-1}$ R.H.S $= 3-4$

$\frac{\frac{\sqrt{3}}{2}}{-\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} - \frac{1}{2} = -1$

$= -\frac{3}{6} - \frac{1}{2} = -1$ ✓

$-\frac{1}{2} - \frac{1}{2} = -1$

$-1 = -1$ ✓

ii) $2 - \cos x = 2\sin^2 x$

$2 = 2\sin^2 x + \cos x$

$2 - 2(1-\cos^2 x) + \cos x$

$2 = 2 - 2\cos^2 x + \cos x$ ✓

$0 = -2\cos^2 x + \cos x$

$0 = 2\cos^2 x - \cos x$ ✓

$0 = \cos x(2\cos x - 1)$

$\therefore x = 0$ or $\frac{1}{2}$ ✓

$\therefore x = 90, 270, 60, 300$ ✓

bi) $\frac{AB}{\sin x} = \frac{AC}{\sin y}$.

$\frac{AB}{AC} = \frac{\sin x}{\sin y} = \frac{\sin(180-y)}{\sin y} = \frac{\sin y}{\sin y}$

$\frac{AB}{AC} = 1$ ✓ $= 1$.

$\frac{AD}{\sin y} = \frac{AD}{\sin x}$

$\therefore \frac{BD}{\sin\theta} = \frac{AD}{\sin y}$

$\frac{DC}{\sin\theta} = \frac{AD}{\sin x}$

$\therefore \frac{BD}{\sin\theta} = \frac{DC}{\sin\theta}$ ✓

$= \frac{BD}{DC} = \frac{\sin\theta}{\sin\theta}$ ✓

$= \frac{BD}{DC} = 1$ ✓

$\therefore \frac{BD}{DC} = \frac{AB}{AC}$ ✓