South Sydney High School

HIGHER SCHOOL CERTIFICATE ASSESSMENT TASK 3

HALF YEARLY

2004
MATHEMATICS EXTENSION 2

Time allowed – 1½ hour

Name: ....

General Instructions

- Write using blue or black pen
- All necessary working should be shown for every question
- Approved calculators may be used
- Begin each question on a new page clearly marked "Question 1", "Question 2"
- Write your name on every sheet of paper handed in.
- Attempt all questions
- All questions are of equal value

Total marks (**60**)

Question 1 (15 marks)

(a) For the ellipse $\frac{x^2}{12}+\frac{y^2}{4}=1$, find the eccentricity, the coordinates of the foci S and S' and the equation of the directrices. **4**

Sketch the curve and indicate on your diagram the foci and the directrices. **2**

(b) Sketch the graph of $y=\cos^2\frac{x}{2}$ from $0 \le x \le 4\pi$. **3**

(c) The point $P(x_1, y_1)$ lies on the hyperbola of equation $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

(i) Find the equation of the normal at P. **2**

(ii) The normal at P meets the x-axis at G, and N is the foot of the perpendicular from P to the x-axis. **4**

Show that $NG : ON = b^2 : a^2$, where O is the origin (0, 0).

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Question 2 (15 marks) Begin a new page

(a) Reduce the complex expression $\frac{(2-i)(8+3i)}{(3+i)}$ in the form $a+ib$, where a, b are real numbers **3**

(b) Find the radius and centre of the circle $x^2+y^2-6x+3y+5=0$. **3**

(c) The point $R(x_1, y_1)$ lies on the curve with equation $\sqrt{x}+\sqrt{y}=\sqrt{a}$.
Find the equation of the tangent to the curve at the point R. **3**

(d) The point $P(a\sec\theta, b\tan\theta)$ is a point on the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$.

(i) Show that the equation of the tangent to the hyperbola at P is given by $\frac{x\sec\theta}{a}-\frac{y\tan\theta}{b}=1$ **2**

(ii) The tangent to the hyperbola at P meets the asymptotes at A and B. Prove that P is the midpoint of the interval AB. **4**

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Question 3 (15 marks) Begin a new page

(a) (i) Show that $\sin x + \sin 3x = 2\sin 2x \cos x$. **3**

(ii) Hence or otherwise, find all solutions of $\sin x + \sin 2x + \sin 3x = 0$ for $0 \le x \le 2\pi$. **4**

(b) $P(a\cos\theta, b\sin\theta)$ lies on the ellipse $\dfrac{x^2}{a^2} + \dfrac{y^2}{b^2} = 1, a > b > 0$.

The tangent and the normal at P cut the y-axis at A and B respectively, and S is the focus of the ellipse.

(i) Show that $\angle ASB = 90^\circ$. **5**

(ii) Hence show that A, P, S and B are concyclic and state the location of the centre of the circle through A, P, S and B. **3**

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Question 4 (15 marks) Begin a new page

(a)

(i) Show that the equation $\frac{x^2}{29-k}+\frac{y^2}{4-k}=1$, where k is a real number, represents

(α) an ellipse if $k<4$. **3**

(β) a hyperbola if $4<k<29$ **2**

(ii) Show that the foci of each ellipse in part (α) are independent of the value of k. **3**

(c) The polynomial $P(z)=z^4+bz^2+d$ has a double root α where b and d are real numbers and $d \neq 0$.

(i) Prove that $P'(z)$ is an odd function. **2**

(ii) Prove that -α is also a double root of $P(z)$. **1**

(iii) Prove that $d=\frac{b^2}{4}$. **2**

(iv) For what values of b does $P(z)$ have a double root equal to $\sqrt{3}\,i$. **1**

(v) For what values of b does $P(z)$ have real roots? **1**