

SOUTH SYDNEY HIGH SCHOOL
YR 11 - 3U - MATHEMATICS
GEOMETRIC SERIES & SEQUENCES

EXERCISE 8.1

1. Are the following series geometric? If they are, find the common ratio
 - (a) $5 + 20 + 60 + \dots$
 - (b) $-4 + 3 - 2\frac{1}{4} + \dots$
 - (c) $\frac{3}{4} + \frac{3}{14} + \frac{3}{49} + \dots$
 - (d) $7 + 5\frac{5}{8} + 3\frac{1}{5} + \dots$
 - (e) $-14 + 42 - 168 + \dots$
 - (f) $1\frac{1}{3} + \frac{8}{9} + \frac{8}{27} + \dots$
 - (g) $5.7 + 1.71 + 0.513 + \dots$
 - (h) $2\frac{1}{4} - 1\frac{7}{20} + \frac{81}{100} + \dots$
 - (i) $63 + 9 + 1\frac{7}{8} + \dots$
 - (j) $-1\frac{7}{8} + 15 - 120 + \dots$

2. Find values of all pronumerals if these series are geometric
 - (a) $4 + 28 + x + \dots$
 - (b) $-3 + 12 + y + \dots$
 - (c) $2 + a + 72 + \dots$
 - (d) $\frac{1}{4} + t + \frac{1}{9} + \dots$
 - (e) $y + 2 + 6 + \dots$

3. Find the 6th term of
 - (a) $8 + 24 + 72 + \dots$
 - (b) $9 + 36 + 144 + \dots$
 - (c) $\frac{2}{3} + \frac{4}{9} + \frac{8}{27} + \dots$

4. Find the 9th term of the series
 - (a) $1 + 2 + 4 + \dots$
 - (b) $\frac{3}{4} - \frac{3}{8} + \frac{3}{16} - \dots$
 - (c) $4 + 12 + 36 + \dots$

5. Find the 8th term of the series
 - (a) $5 - 20 + 80 - \dots$
 - (b) $\frac{128}{81} + \frac{64}{27} + \frac{32}{9} + \dots$
 - (c) $-7 + 14 - 28 + \dots$

6. What is the 20th term of the series $9 + 81 + 729 + \dots$? Leave your answer in index form.

7. Find the 10th term of the series $3 + 1\frac{1}{2} + \frac{3}{4} + \dots$.

8. Which term of the series $4 + 20 + 100 + \dots$ is equal to 12 500?

9. Which term of the series $8 - 4 + 2 - \dots$ is equal to $\frac{1}{128}$?

10. Which term of the series $3 + 21 + 147 + \dots$ is equal to 352 947?

11. Which term of $\frac{7}{9} + \frac{14}{45} + \frac{28}{225} + \dots$ is equal to $\frac{224}{28125}$?

12. Which term of $54 + 18 + 6 + \dots$ is equal to $\frac{2}{243}$?

13. Find the value of n if the n th term of the series $-2 + 1\frac{1}{2} - 1\frac{1}{8} + \dots$ is equal to $-\frac{81}{128}$.

14. The first term of a geometric series is 7 and the 6th term is 1701. Find the common ratio.

15. The 4th term of a geometric series is -648 and the 5th term is 3888. Find the common ratio and the second term.

16. The third term of a geometric series is $\frac{2}{3}$ and the 5th term is $1\frac{3}{5}$. Find the common ratio and the first term.

17. Find the value of n for the first term of the series $5000 + 1000 + 200 + \dots$ that is less than 1.

18. Find the first term of the series $\frac{2}{7} + \frac{6}{7} + 2\frac{4}{7} + \dots$ that is greater than 100.

EXERCISE 8.1 ANSWERS

1. (a) No (b) Yes, $r = -\frac{5}{4}$ (c) Yes, $r = \frac{2}{7}$
 (d) No (e) No (f) No (g) Yes, $r = 0.3$
 (h) Yes, $r = -\frac{3}{5}$ (i) No (j) Yes, $r = -8$
 2. (a) $x = 196$ (b) $y = -48$ (c) $a = \pm 12$
 (d) $t = \pm \frac{1}{6}$ (e) $y = \frac{2}{3}$ 3. (a) 1944 (b) 9216
 (c) $\frac{64}{729}$ 4. (a) 256 (b) $\frac{3}{1024}$ (c) 26 244
 5. (a) -81 920 (b) 27 (c) 896 6. 9^{20} 7. $\frac{3}{3125}$
 8. 6th 9. 11th 10. 7th 11. 6th 12. 9th 13. 5
 14. 3 15. $r = -6$, $T_2 = -18$ 16. $a = \frac{1}{16}$,
 $r = \pm 2$ 17. $n = 7$ 18. $208\frac{2}{7}$ (7th term)

EXERCISE 8.2

- Find the sum of 10 terms of the series
 - $6 + 24 + 96 + \dots$
 - $3 + 15 + 75 + \dots$
- Find the sum of 8 terms of the series
 - $-1 + 7 - 49 + \dots$
 - $8 + 24 + 72 + \dots$
- Find the sum of 15 terms of the series
 - $4 + 8 + 16 + \dots$
 - $\frac{3}{4} - \frac{3}{8} + \frac{3}{16} - \dots$
- Evaluate
 - $2 + 10 + 50 + \dots + 6250$
 - $18 + 9 + 4\frac{1}{2} + \dots + \frac{9}{64}$
 - $3 + 21 + 147 + \dots + 7203$
 - $\frac{3}{4} + 2\frac{1}{4} + 6\frac{3}{4} + \dots + 182\frac{1}{4}$
 - $-3 + 6 - 12 + \dots + 384$
- Evaluate
 - $\sum_{n=1}^8 2^{n-1}$
 - $\sum_{n=1}^6 \frac{1}{3^n}$
 - $\sum_{n=2}^{10} 5^{n-3}$
 - $\sum_{r=1}^8 \frac{1}{2^{r-1}}$
 - $\sum_{k=1}^7 4^{k+1}$
- Find the
 - 9th term
 - sum of 9 terms of the series $7 + 14 + 28 + \dots$
- Find the sum of 30 terms of the series $1.09 + 1.09^2 + 1.09^3 + \dots$, correct to 2 decimal places.
- Find the sum of 25 terms of the series $1 + 1.12 + 1.12^2 + \dots$, correct to 2 decimal places.
- Find the value of n if the sum of n terms of the series $11 + 33 + 99 + \dots$ is equal to 108 251.
- How many terms of the series $\frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots$ give a sum of $\frac{1023}{1024}$?
- The common ratio of a geometric series is 4 and the sum of the first 5 terms is 3069. Find the first term.

- Find the number of terms needed to be added for the sum to exceed 1 000 000 in the series $4 + 16 + 64 + \dots$
- Lucia currently earns \$25 000. Her wage increases by 5% each year. Find
 - her wage after 6 years
 - her total earnings (before tax) in 6 years
- Write down an expression for the series $2 - 10 + 50 - \dots + 2(-5^{k-1})$
 - in sigma notation
 - as a sum of n terms
- Find the sum of the first 10 terms of the series $3 + 7 + 13 + \dots + [2^n + (2n - 1)] + \dots$

ANSWERS:**EXERCISE 8.2**

- (a) 2097 150 (b) 7324 218 2. (a) 720 600 (b) 26 240 3. (a) 131 068 (b) $\frac{32769}{65536}$ 4. (a) 7812 (b) $35\frac{55}{64}$ (c) 8403 (d) 273 (e) 255 5. (a) 255 (b) $\frac{364}{729}$ (c) 97 656.2 (d) $1\frac{127}{128}$ (e) 87 376
- (a) 1792 (b) 3577 7. 148.58 8. 133.33
- $n = 9$ 10. 10 terms 11. $a = 9$ 12. 10 terms
- (a) \$33 502.39 (b) \$203 550.21
- (a) $\sum_{n=1}^k 2(-5)^{n-1}$
- (b) $S_k = -\frac{(-5)^k - 1}{3}$ 15. 2146

EXERCISE 8.3

1. Find the limiting sum (sum to infinity) of
 - (a) $6 + 3 + 1\frac{1}{2} + \dots$
 - (b) $-5 - 1 - \frac{1}{5} - \dots$
 - (c) $320 + 80 + 20 + \dots$
 - (d) $100 - 50 + 25 - \dots$
 - (e) $-\frac{2}{5} + \frac{6}{35} - \frac{18}{245} + \dots$
2. Write each recurring decimal as a fraction
 - (a) 0.4
 - (b) $0.\dot{2}5$
 - (c) $2.\dot{8}$
 - (d) $0.2\dot{3}$
 - (e) $1.0\dot{1}5$
3. Find the difference between the limiting sum and the sum of 6 terms of the series
 - (a) $56 - 28 + 14 - \dots$
 - (b) $72 + 24 + 8 + \dots$
 - (c) $1\frac{1}{4} + \frac{15}{16} + \frac{45}{64} + \dots$
4. A frog jumps 0.5 m. It then jumps 0.1 m and on each subsequent jump travels 0.2 m of the previous distance. Find the total distance through which the frog jumps.
5. A tree 3 m high grows 1 m taller one year, $\frac{1}{3}$ m taller the following year, and so on, each year growing taller by $\frac{1}{3}$ of the last year's. Find the tree's ultimate height.
6. An 8 cm seedling grows by $4\frac{4}{5}$ cm the first week it is planted. It then keeps growing by $\frac{3}{5}$ of the previous week's growth. How tall will it grow?
7. A geometric series has a limiting sum of 9 and second term 2. Find the values of a and r .
8. The series $x + \frac{x}{4} + \frac{x}{16} + \dots$ has a sum to infinity of $\frac{7}{8}$. Find the value of x .
9. An object falls 0.5 m in the first second. Then, each second after, it falls $\frac{5}{8}$ of its previous fall. Find how far it will fall.
10. Mary bounces a ball, dropping it from 1.5 m on its first bounce. It then rises up to $\frac{2}{5}$ of its height on each bounce. Find the distance through which the ball travels.
11. (a) For what values of k does the limiting sum exist for the series $k + k^2 + k^3 + \dots$?
 (b) Find the limiting sum of the series when $k = -\frac{2}{3}$.
 (c) Find the value of k if the series has a limiting sum of 3.
12. Evaluate
 - (a) $\sum_{r=1}^{\infty} (\frac{1}{5})^{r-1}$
 - (b) $\sum_1^{\infty} (\frac{2}{7})^r$
 - (c) $\sum_3^{\infty} 5(\frac{1}{3})^{n-1}$
 - (d) $\sum_2^{\infty} (-\frac{1}{2})^{r-1}$
 - (e) $\sum_1^{\infty} 2(-\frac{9}{10})^r$
13. (a) For what values of p will the sum to infinity exist for the series $1 - 2p + 4p^2 - \dots$?
 (b) Find the value of p if the sum to infinity is $\frac{7}{8}$.

EXERCISE 8.3

1. (a) 12 (b) $-6\frac{1}{2}$ (c) $426\frac{2}{3}$ (d) $66\frac{2}{3}$ (e) $-\frac{7}{25}$
2. (a) $\frac{4}{5}$ (b) $\frac{25}{30}$ (c) $2\frac{2}{5}$ (d) $\frac{7}{30}$ (e) $1\frac{1}{100}$
3. (a) $\frac{7}{12}$ (b) $\frac{4}{27}$ (c) $\frac{30643}{30000}$ 4. 0.625 m 5. 4.5 m
6. 20 cm 7. $a = 3, r = \frac{2}{3}$ or $a = 6, r = \frac{1}{3}$
8. $x = \frac{21}{32}$ 9. 3 m 10. 3.5 m 11. (a) $|k| < 1$
 (b) $-\frac{2}{5}$ (c) $k = \frac{2}{3}$ 12. (a) $1\frac{1}{2}$ (b) $\frac{2}{3}$ (c) $1\frac{1}{2}$ (d) $-\frac{1}{3}$
 (e) $-\frac{18}{19}$ 13. (a) $|p| < \frac{1}{2}$ (b) $p = \frac{1}{4}$

$$\begin{aligned}
 14. S_{\infty} - S_n &= \frac{a}{1-r} - \frac{a(1-r^n)}{1-r} \\
 &= \frac{a - a(1-r^n)}{1-r} \\
 &= \frac{a - a + ar^n}{1-r} \\
 &= \frac{ar^n}{1-r} \\
 15. a = 3, r = \frac{2}{3} \text{ or } a = 2, r = \frac{2}{3}
 \end{aligned}$$

14. Show that in any geometric series the difference between the limiting sum and the sum of n terms is $\frac{ar^n}{1-r}$.
15. The limiting sum of a geometric series is 5 and the second term is $1\frac{1}{3}$. Find the first term and the common ratio of the series.