NAME :

SOUTH SYDNEY HIGH SCHOOL
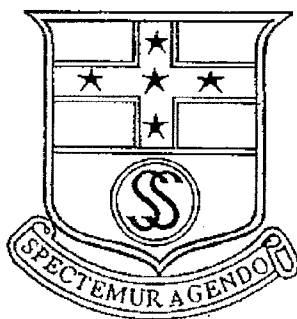
Year 11 Half-yearly Exam
May 2000

MATHEMATICS

Question 1 (10marks) (Start a NEW page)   **Marks**

(a)   Calculate :

   (i) $\dfrac{(3.24)^2}{5.73 - 2.84}$ correct to **two** decimal places   1

   (ii) $\dfrac{4.6 - 5.9}{4.6 \times 2.3}$ correct to **three** significant figures.   1

(b)   Find the value of k if $(7 - 3x)^2 = 49 + kx + 9x^2$ 1

(c)   Rationalise the denominator $\dfrac{4 - \sqrt{3}}{3\sqrt{3} + 5}$ 2

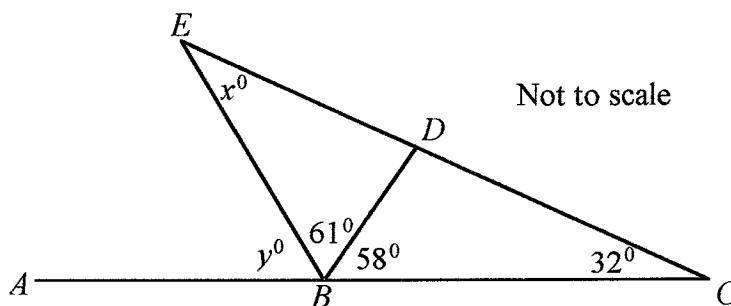(d)   For $|x - 2| \le 1$, solve for x and graph the solution on a number line.   3

(e)   Factorise fully $3m^2 - 12$ 2

Question 2 (10marks) (Start a NEW page)

(a)   Find a and b such that   2

$$\dfrac{2}{2 - \sqrt{3}} = a + \sqrt{b}$$

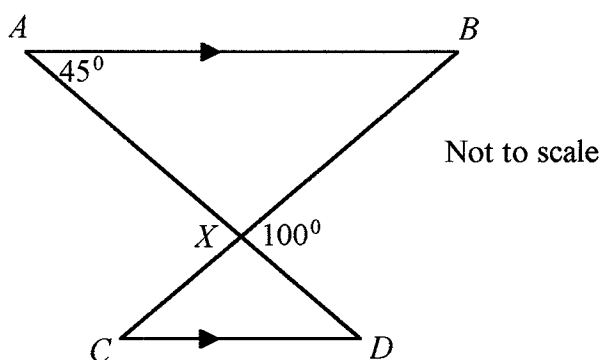(b)   Find the value of x and y giving reasons for your answer.   4

Not to scale

Question 2   (Continued)   **Marks**

(c)   Explain why $x^2 = 64$ has two solutions but $x^2 = -64$ has no solutions.   **2**

(d)   Given that $S_n = \frac{n}{2}[2a + (n-1)d]$, find S_n **2**
         when $n = 103, a = 5$ and $d = 1.2$.

Question 3 (8 marks)   (Start a NEW page)

(a)   Express $0.12\dot{8}$ as a simple fraction.   **3**

(b)   Solve $4(x - 5) = 3 - 2(x - 1)$.   **2**

(c)   **3**

Not to scale

$AB||CD$, $\angle BAX = 45^0$ and $\angle BXD = 100^0$.
Find, giving reasons, the size of $\angle XCD$.

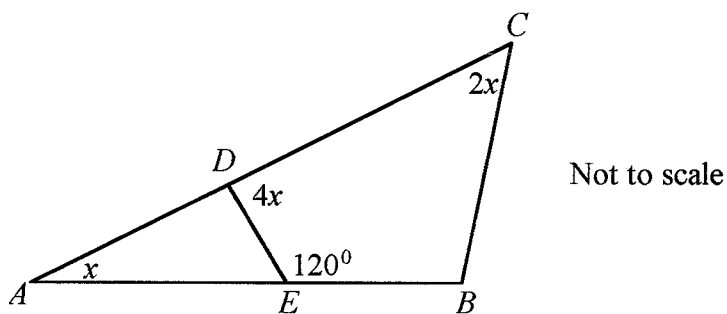**Question 4 (10marks)** (Start a NEW page)

(a)   Simplify : $\dfrac{x^2 - 9}{4x - 12} \times \dfrac{2x + 14}{x^2 + 6x - 7}$ **3**

(b)   Solve these equations simultaneously :   **2**

         $3x - 2y = 5$ and $2x + y = 15$

Question 4   (Continued)   **Marks**

(c)   Simplify $\dfrac{x-5}{3} - \dfrac{x+2}{2}$ **2**

(d)   Find the value of x in the diagram below.   **3**

Not to scale

Question 5 (10 marks) (Start a NEW page)

(a)   Solve the equation $3x^2 + 7x - 1 = 0$ **2**

(b)   Solve $|3n - 5| = 4n - 1$ **3**

(c)   Factorise fully $3t^4 - 3$ **2**

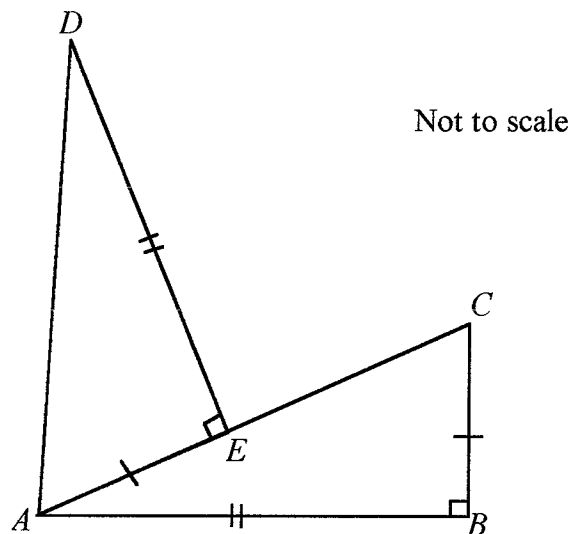(d)   Solve the equations for x and y **3**

         $x + y - 3 = 0$ and $y = 2x^2$

Question 6 (10marks) (Start a NEW page)   **Marks**

(a)

Not to scale

$AB = DE$, $BC = AE$, $\angle ABC = \angle DEA = 90^0$.

Prove that   (i) $\triangle ABC \equiv \triangle DEA$ **3**

              (ii) AD is perpendicular to AB **2**

(b)   The sum of a number and it's square is 12. Write down a equation   **3**
       and solve it to find the numbers.

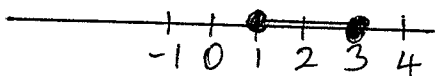(c)   Solve $19 \le 13 - 3x$ **2**

End of paper

1] a)(i) 3.63 (ii) -0.123

b) $(7-3x)^2 = 49 + kx + 9x^2$

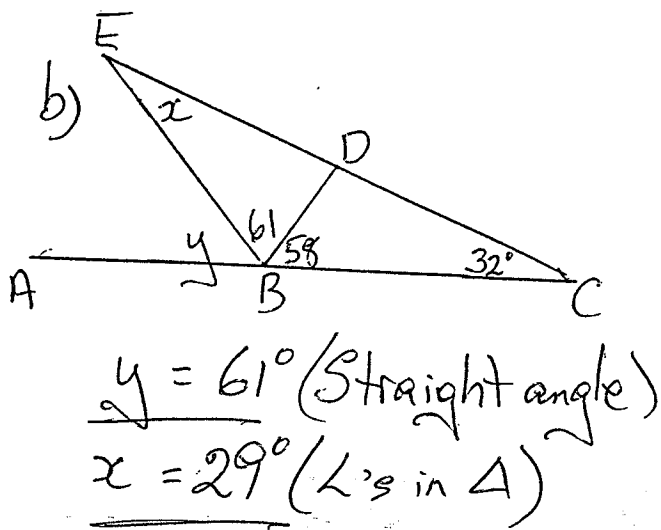$49 - 42x + 9x^2 = 49 + kx + 9x^2$

$\underline{-42 = k}$

c) $\dfrac{(4-\sqrt{3})}{(3\sqrt{3}+5)} \times \dfrac{(3\sqrt{3}-5)}{(3\sqrt{3}-5)}$

$= \dfrac{12\sqrt{3} - 20 - 9 + 5\sqrt{3}}{27 - 25}$

$= \dfrac{17\sqrt{3} - 29}{2}$

d) $|x-2| \leq 1$

$-1 \leq x-2 \leq 1$

$1 \leq x \leq 3$

e) $3m^2 - 12 = 3(m^2 - 4)$

$\qquad\qquad = 3(m+2)(m-2)$

2] a) $\dfrac{2}{2-\sqrt{3}} = \dfrac{2}{(2-\sqrt{3})} \times \dfrac{(2+\sqrt{3})}{(2+\sqrt{3})}$

$= \dfrac{4 + 2\sqrt{3}}{4 - 3}$

$= 4 + 2\sqrt{3}$

$= a + \sqrt{b}$

$= 4 + \sqrt{12}$

So $\underline{a = 4, \quad b = 12}$

b)

$\underline{y = 61°}$ (Straight angle)

$\underline{x = 29°}$ ($\angle$'s in $\triangle$)

c) $x^2 = 64$

$x = \pm\sqrt{64}$

$x = \pm 8$

$x^2 = -64$

$x = \pm\sqrt{-64}$

Can't find square root
of a negative number

d) $S_n = \dfrac{n}{2}[2a + (n-1)d]$

$S_n = \dfrac{103}{2}[2\times 5 + (103-1)\times 1.2]$

$= \dfrac{103}{2} \times 132.4$

$= \underline{6818.6}$

3] a) Let $x = 0.1\dot{2}\dot{8}$

$\qquad = 0.1282828$

So $100x = 12.8282828$

$\qquad x = 0.1282828$

$99x = 12.7$

$x = \dfrac{12.7}{99}$

$x = \dfrac{127}{990}$

___

b) $4(x-5) = 3 - 2(x-1)$

$4x - 20 = 3 - 2x + 2$

$4x - 20 = 5 - 2x$

$6x = 25$

$x = 4\frac{1}{6}$

c) $108\% = \$1485$

$1\% = \dfrac{1485}{108}$

$100\% = \dfrac{1485 \times 100}{108}$

$\qquad = \$1375$

Old rate $= \$1375$

d)
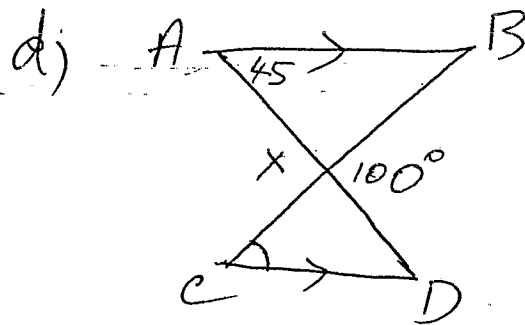
Find $\angle XCD$

$\angle CXD = 80°$ (Straight $\angle$)

$\angle XDC = 45°$ (Alternate

$\angle XCD = 55°$ ($\angle$'s in $\triangle$)

4] a) $\dfrac{x^2-9}{4x-12} \times \dfrac{2x+14}{x^2+6x-7} = \dfrac{(x+3)(x-3)}{\underset{2}{4}(x-3)} \times \dfrac{2(x+7)}{(x+7)(x-1)}$

$$= \dfrac{x+3}{2(x-1)}$$

b) $3x - 2y = 5$ —— ①
     $2x + y = 15$ —— ②

② ×2 $4x + 2y = 30$ —— ③

① + ③ $7x \qquad = 35$

$$x = 5$$

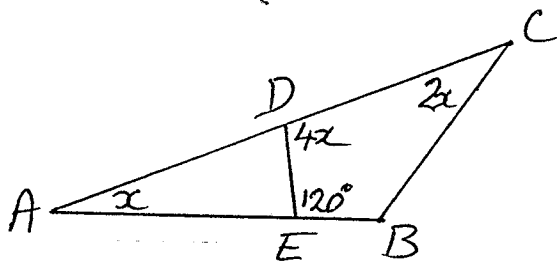Sub in ① $10 + y = 15$
$$y = 5$$

Solution $(5, 5)$

c) $\dfrac{x-5}{3} - \dfrac{x+2}{2}$

$$= \dfrac{2(x-5) - 3(x+2)}{6}$$

$$= \dfrac{2x - 10 - 3x - 6}{6}$$

$$= \dfrac{-x - 16}{6}$$

d)

One solution
$\angle DEA = 60°$ (Straight $\angle$)
$x + 60 = 4x$ (Exterior $\angle$)
$60 = 3x$
$$20° = x$$

5]

a) $3x^2 + 7x - 1 = 0$

$a = 3, b = 7, c = -1$

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \dfrac{-7 \pm \sqrt{49 - -12}}{6}$

$= \dfrac{-7 \pm \sqrt{61}}{6}$

$\left(x = 0.135 \quad \text{or} \quad x = -2.468 \right)$

b) $|3n - 5| = 4n - 1$

$3n - 5 = 4n - 1$ or $3n - 5 = -4n + 1$

$-4 = n$ $7n = 6$

                        $n = \dfrac{6}{7}$

Test

when $n = -4$ when $n = \dfrac{6}{7}$

Eq^n false.   Eq^n true

So one solution

$n = \dfrac{6}{7}$

c) $3t^4 - 3 = 3(t^4 - 1)$

$= 3(t^2 + 1)(t^2 - 1)$

$= 3(t^2 + 1)(t + 1)(t - 1)$

d) $x + y - 3 = 0$ ——①

      $y = 2x^2$ ——②

Sub ② in ①

$x + 2x^2 - 3 = 0$

$2x^2 + x - 3 = 0$

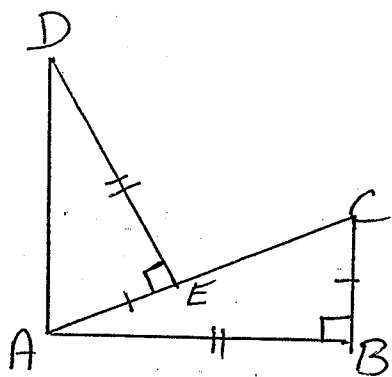$(2x + 3)(x - 1) = 0$

$x = -\dfrac{3}{2}, 1$

Sub in ②

$y = \dfrac{9}{2}, 2$

Sol^{ns} $\left(-\dfrac{3}{2}, \dfrac{9}{2} \right)$

$\& \; (1, 2)$

6] a)

i) Prove $\triangle ABC \equiv \triangle DEA$

$AB = DE$ (Given)

$\angle DEA = \angle CBA$ (Given $90°$)

$AE = CB$ (Given)

$\therefore \triangle ABC \equiv \triangle DEA$ (SAS)

ii) Prove $AD \perp AB$.

Let $\angle ACB = x = \angle DAE$ (Corresponding angles in $\equiv \triangle$'s

$\therefore \angle CAB = 90 - x°$ ($\angle$'s in $\triangle$)

So $\angle DAB = \angle DAE + \angle CAB$

$= x + 90 - x$

$= 90°$

So $AD \perp AB$

b) Let $x =$ the number

$x + x^2 = 12$

$x^2 + x - 12 = 0$

$(x + 4)(x - 3) = 0$

$x = -4$ or 3

c) $19 \leq 13 - 3x$

$6 \leq -3x$

$-2 \geq x$