Student name/number: _____

SOUTH SYDNEY HIGH SCHOOL

2002 PRELIMINARY FINAL EXAMINATION

Mathematics Extension 1

Total marks **(60)**

- Attempt Questions 1 – 5
- All questions are of equal value
- Topics: Further Trigonometry; Angle between lines; Internal and external division of lines; Quadratics & the parabola; Sequences & series; Financial Maths; Locus and the parabola.

General Instructions

- Working time – 2 hours
- Board-approved calculators may be used
- All necessary working should be shown in every question

Question 1

(a) The nth term of a sequence is $T_n = \dfrac{n^{-\frac{1}{n}}}{n+1}$, find the 4^{th} term, expressing your answer with a positive index. *(2m)*

(b) If $S_n = \dfrac{n}{n+1}$, find the value of $S_{n+1} - S_n$ as a single fraction. *(2m)*

(c) For the sequence $-4,\ -1,\ 2,\ \ldots$, find the

(i) common difference
(ii) 12^{th} term
(iii) sum to 12 terms in exact form. *(4m)*

(d) For the sequence $\dfrac{2}{3},\ \dfrac{1}{2},\ \dfrac{3}{8},\ \ldots$, find the

(i) common ratio
(ii) ninth term
(iii) sum to 9 terms in exact form. *(4m)*

Question 2

(a) (i) Find the gradients of the lines $2\sqrt{3}x - y + 4 = 0$ and $5\sqrt{3}x - 9y - 7 = 0$. *(2m)*

(ii) Find the acute angle between the lines in part (i). *(2m)*

(b) (i) Sketch the curve $y = x^2 + 6x - 16$, showing all the main features. *(3m)*

(ii) Find the minimum value of the function $y = x^2 + 6x - 16$. *(1m)*

(c) Find the rational number which exceeds its square by the greatest amount possible. *(2m)*

(d) Find the coordinates of the point that divides the line joining $(-1, 2)$ and $(3, -3)$ in the ratio $4 : -5$. *(2m)*

Question 3

(a) If $x=3$ is a root of the quadratic equation $x^2+kx-21=0$, find the other root. *(3m)*

(b) If α and β are the roots of the equation $x^2-3x+5=0$, find the value of :

(i) $\alpha+\beta$

(ii) $\alpha\beta$

(iii) $\alpha^2+\beta^2$

(iv) $\alpha^2\beta+\alpha\beta^2$

(v) $\dfrac{\alpha}{\beta}+\dfrac{\beta}{\alpha}$ *(6m)*

(c) Find the constants A, B and C if $3-4x-x^2 \equiv A(x+1)^2+B(x+1)+C$. *(3m)*

Question 4

(a) For each of the parabolas below, find the coordinates of its vertex and focus, the equation of its directrix and the length of the latus rectum. You are also required to sketch the parabolas.

(i) $x^2+24y=0$ *(2m)*

(ii) $(x+2)^2=8y+24$ *(3m)*

(b) Find the locus of the point $P(x,y)$ that moves according to the following rules:

(i) P is equidistant from the points $A(1,5)$ and $B(-2,-3)$, expressing your answer in general form. *(2m)*

(ii) P is equidistant from the point $(1,2)$ and the line $x=-3$. *(2m)*

(iii) $\angle RPQ=90^0$ where $R(-1,3)$ and $Q(3,-1)$, showing by algebraic methods that the locus is a circle. Find the centre and radius of this circle. *(3m)*

Question 5

(a) On 1st June 1990, Jonathan invested \$1000 in a managed fund that paid interest at a fixed rate of 12% per annum, compounded quarterly.

(i) How much would be in the managed fund after the payment of interest on 1st June 2002 if no additional deposits were made? *(2m)*

(ii) In fact, Jonathan added \$500 to the managed fund on 1st June each year beginning 1st June 1991.
How much was in his managed fund on 1st June 2002 after the payment of interest and his initial amount? *(2m)*

(b) A businessman borrows \$120 000 to purchase a business. The interest is calculated monthly at the rate 1.5% per month, and is compounded each month.
He intends to repay the loan with interest in four equal ***half-yearly*** instalments of \$$Q$ at the end of two years.

(i) How much does he owe at the end of the first month? *(1m)*

(ii) Write an expression involving Q for the total amount owed by him after 6 months, just after the first instalment of \$$Q$ has been paid. *(1m)*

(iii) Find an expression for the amount owed at the end of the second year and deduce that

$$Q = \frac{120000(1.015)^{24}\left(1.015^{6}-1\right)}{(1.015)^{24}-1}$$

(4m)

(iv) What is the total interest paid over the two year period? *(2m)*

END OF ASSESSMENT

T①

Peggy Ngan

a) $T_4 = \frac{4^{-\frac{1}{4}}}{4+1} = \frac{1}{4^{\frac{1}{4}}} \times \frac{1}{5} \checkmark = \frac{1}{5 \cdot 4^{\frac{1}{4}}}$ ✓

60/60

b) $S_{n+1} = \frac{n+1}{n+1+1} = \frac{n+1}{n+2}$

$\therefore S_{n+1} - S_n = \frac{n+1}{n+2} - \frac{n}{n+1}$

$= \frac{(n+1)^2 - n(n+2)}{(n+1)(n+2)}$ ✓

$= \frac{n^2+2n+1-n^2-2n}{(n+1)(n+2)}$

$= \frac{1}{(n+1)(n+2)}$ ✓

Excellent effort!

c) i, $d = -1 - (-4)$

$= 3$ ✓

ii, $T_{12} = a + (12-1)d$

$= -4 + 11(3)$

$= 29$ ✓

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iii, $S_{12} = \frac{12}{2}(-4+29)$ ✓

$= 150$ ✓

d) i, $r = \frac{1}{2} \div \frac{2}{3}$

$= \frac{3}{4}$ ✓

ii, $T_9 = ar^{(9-1)}$

$= ar^8$

$= [redacted] \left(\frac{2}{3}\right)\left(\frac{3}{4}\right)^8$

$= \frac{2187}{32768}$ ✓

iii, $S_9 = \dfrac{\frac{2}{3}\left[1-\left(\frac{3}{4}\right)^9\right]}{1-\frac{3}{4}}$

$= \left[\dfrac{2}{3} \times \dfrac{242461}{262144}\right] \div \dfrac{1}{4}$

$= \left[\dfrac{1}{3} \times \dfrac{242461}{131072}\right] \times 4$

$= \dfrac{969844}{393216}$

$= \dfrac{484922}{196608}$

$= \dfrac{242461}{98304}$

(2)

a) i, $2\sqrt{3}x - y + 4 = 0$

$m_1 = \frac{-2\sqrt{3}}{-1}$

$= \underline{2\sqrt{3}}$ ✓

$5\sqrt{3} - 9y - 7 = 0$

$m_2 = \frac{-5\sqrt{3}}{-9}$

$= \frac{5\sqrt{3}}{9}$ ✓

ii, $\tan\theta = \left| \frac{2\sqrt{3} - \frac{5\sqrt{3}}{9}}{1 + 2\sqrt{3}\left(\frac{5\sqrt{3}}{9}\right)} \right|$ ✓

$\theta = \underline{30°}$ ✓

12

b) i, $y = x^2 + 6x - 16$

Put $y = 0$,

$x^2 + 6x - 16 = 0$

$(x+8)(x-2) = 0$

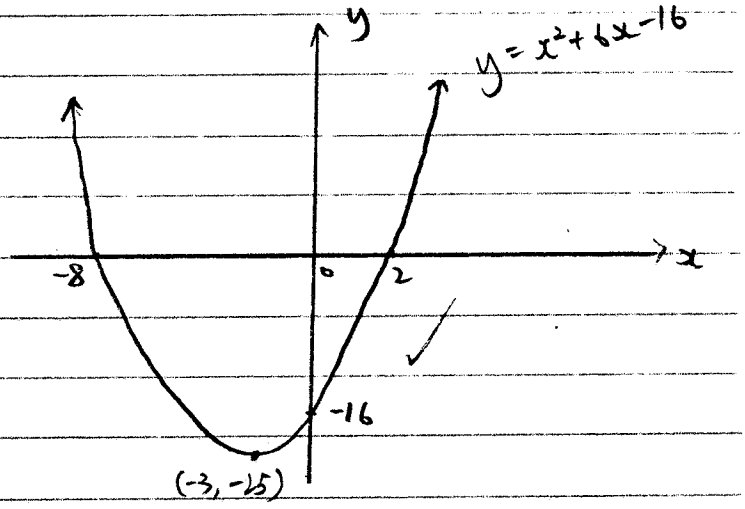
$\therefore x = -8$ or 2 ✓

Put $x = 0$, $y = -16$

axis of symmetry : $x = \frac{-6}{2}$

$x = -3$ ✓

Put $x = -3$, $y = -25$



✓

ii, from graph, min value : $\underline{y = -25}$ ✓

c) Let x be the rational number,

$x > x^2$

$x^2 - x < 0$

$x(x-1) < 0$ ✓

$\therefore \underline{0 < x < 1}$ $x = \underline{\frac{1}{2}}$ ✓

$\frac{1}{2} = 0.5$

$(\frac{1}{2})^2 = 0.25$

$d = 0.25$

d) $P\left(\frac{4(3)-5(-1)}{4-5}, \frac{4(-3)-5(2)}{4-5}\right)$ ✓

$= P(-17; 22)$ ✓

(3)

a) Put $x=3$.

$3^2 + k(3) - 21 = 0$

$3k = 12$

$k = 4$ ✓

Put $k=4$, Let α be other root

$-k = 3 + \alpha$

$-4 = 3 + \alpha$ ✓

$\alpha = -7$ ∴ it's -7 ✓

b) i. $\alpha + \beta = 3$ ✓

ii. $\alpha\beta = 5$ ✓

iii. $\alpha^2 + \beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= (3)^2 - 2(5)$

$= -1$ ✓

iv. $\alpha^2\beta + \alpha\beta^2 = \alpha\beta(\alpha+\beta)$

$= 5(3)$ ✓

$= 15$ ✓

v. $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$ ✓

$= \frac{-1}{5}$ ✓

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c) $A(x+1)^2 + B(x+1) + C$

$= A(x^2+2x+1) + Bx + B + C$

$= Ax^2 + (2A+B)x + A + B + C$

By comparing coefficients, $-x^2 - 4x + 3$

$A = -1$, $2A + B = -4$, $A + B + C = 3$

$2(-1) + B = -4$ $(-1) + (-2) + C = 3$

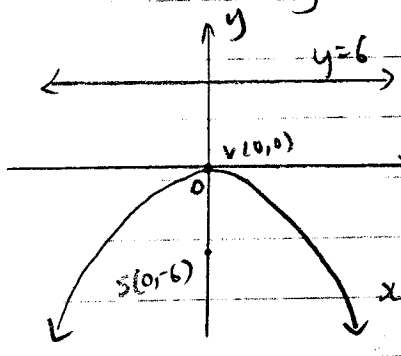
$B = -2$ $C = 6$

∴ $A = -1$, $B = -2$, $C = 6$ ✓

(4) 12

a) i, $x^2 + 24y = 0$

$x^2 = -4(6)y$



vertex (0, 0)

focus (0, -6) ✓

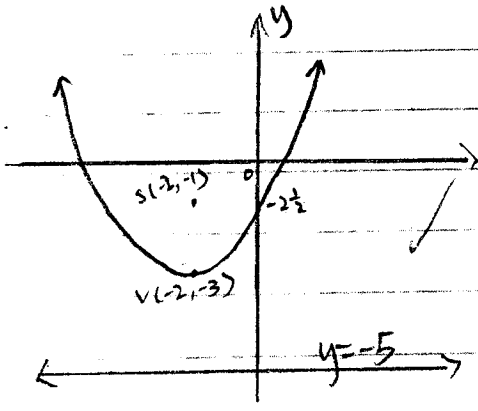
eqn of directrix : $y = 6$

length of latus rectum : 24 units

ii, $(x+2)^2 = 8y + 24$

$(x+2)^2 = 8(y+3)$

$(x+2)^2 = 4(2)(y+3)$



vertex (-2, -3)

focus (-2, -3+2) ⇒ (-2, -1) ✓

eqn of directrix : $y = -3 - 2$

$y = -5$

latus rectum : 8 units ✓

b) i, $\sqrt{(x-1)^2 + (y-5)^2} = \sqrt{(x+2)^2 + (y+3)^2}$

$x^2 - 2x + 1 + y^2 - 10y + 25 = x^2 + 4x + 4 + y^2 + 6y + 9$

$x^2 - 2x + y^2 - 10y + 26 = x^2 + 4x + y^2 + 6y + 13$

$-6x - 16y + 13 = 0$

$6x + 16y - 13 = 0$ ✓

ii, $x = -3$

$x + 3 = 0$

$\therefore \sqrt{(x-1)^2 + (y-2)^2} = \left| \frac{x+3}{\sqrt{1^2+0^2}} \right|$

$\sqrt{(x-1)^2 + (y-2)^2} = x + 3$

$x^2 - 2x + 1 + (y-2)^2 = x^2 + 6x + 9$

$(y-2)^2 = 8x + 8$

$(y-2)^2 = 8(x+1)$ ✓

~~$x^2 - 2x + 1 + y^2 - 4y + 4 = x^2 + 6x + 9$~~

~~$-8x + y^2 - 4y - 4 = 0$~~

~~$y^2 - 4y - 8x - 4 = 0$~~

iii)

$$\frac{y-3}{x+1} \times \frac{y+1}{x-3} = -1$$

$$\frac{(y-3)(y+1)}{(x+1)(x-3)} = -1$$

$$y^2 - 2y - 3 = -(x^2 - 2x - 3)$$

$$y^2 - 2y - 3 = -x^2 + 2x + 3$$

$$x^2 - 2x + y^2 - 2y = 6$$

$$(x^2 - 2x + 1) + (y^2 - 2y + 1) = 6 + 2$$

$$\underline{(x-1)^2 + (y-1)^2 = 8}$$

centre = $\underline{(1,1)}$

radius = $\sqrt{8} = \underline{2\sqrt{2} \text{ units}}$

(5)

12

a) i, years = 12

~~$1000 (1+ 12%/4)^12~~

$\$1000\left(1+\frac{12\%}{4}\right)^{48}$ ✓

$= \$\underline{4132.25}$ ✓ (2 d.p.)

ii, $\$1000\left(1+\frac{12\%}{4}\right)^{48} + 500\left(1+\frac{12\%}{4}\right)^{44} + 500\left(1+\frac{12\%}{4}\right)^{40} + \cdots + 500\left(1+\frac{12\%}{4}\right)^{4}$

$= \$1000\left(1+\frac{12\%}{4}\right)^{48} + 500\left(1+\frac{12\%}{4}\right)^{4} + 500\left(1+\frac{12\%}{4}\right)^{8} + \cdots + 500\left(1+\frac{12\%}{4}\right)^{44}$

$= \$1000\left(1+\frac{12\%}{4}\right)^{48} + 500\left(1+\frac{12\%}{4}\right)^{4}\left[1 + \left(1+\frac{12\%}{4}\right)^{4} + \left(1+\frac{12\%}{4}\right)^{8} + \cdots 500\left(1+\frac{12\%}{4}\right)^{40}\right]$

$\left[\begin{array}{l} a = 1 \\ r = \left(1+\frac{12\%}{4}\right)^{4} \\ n = 11 \end{array}\right]$ ✓

$\therefore = \$1000\left(1+\frac{12\%}{4}\right)^{48} + 500\left(1+\frac{12\%}{4}\right)^{4}\frac{\left\{1-\left[\left(1+\frac{12\%}{4}\right)^{4}\right]^{11}\right\}}{1-\left(1+\frac{12\%}{4}\right)^{4}}$ ✓

$= \underline{\$16110.47}$ ✓ (2 d.p.)

b) i, $\$120000\ (1+1.5\%)$

$= \underline{\$121800}$ ✓

ii, $A_6 = \underline{120000\ (1+1.5\%)^6 - Q} = \underline{120000\ (1.015)^6 - Q}$ ✓

iii, ~~At the end of 2nd year~~

~~$A = \{[120000(1+1.5\%)^6 - Q](1+1.5\%)^6 - Q$~~

$A_{12} = A_6(1+1.5\%)^6 - Q$

$= [120000(1+1.5\%)^6 - Q](1+1.5\%)^6 - Q$

(After 12 months) $= 120000(1+1.5\%)^{12} - Q(1+1.5\%)^6 - Q$ ✓

$= 120000(1+1.5\%)^{12} - Q[1 + (1+1.5\%)^6]$

$A_{18} = A_{12}(1+1.5\%)^6 - Q$

(After 18 months) $= 120000(1+1.5\%)^{18} - Q(1+1.5\%)^{12} - Q(1+1.5\%)^6 - Q$

$= 120000(1+1.5\%)^{18} - Q[1 + (1+1.5\%)^6 + (1+1.5\%)^{12}]$ ✓

∴ At the end of 2nd year,

$A_{24} = 120000(1+1.5\%)^{24} - Q[1+(1+1.5\%)^6+(1+1.5\%)^{12}+(1+1.5\%)^{18}]$

$a = 1$
$r = (1.015)^6$
$n = 4$

$$A_{24} = 120000(1.015)^{24} - \frac{Q\{[(1.015)^6]^4 - 1\}}{(1.015)^6 - 1}$$ ✓

~~$A_{24} = 0$ (owe nothing at the end of 2nd year)~~

$$\therefore A_{24} = 120000(1.015)^{24} - \frac{Q[(1.015)^{24} - 1]}{(1.015)^6 - 1}$$

$A_{24} = 0$ ✓

$$\frac{Q[(1.015)^{24} - 1]}{(1.015)^6 - 1} = 120000(1.015)^{24}$$

$$\therefore Q = \frac{120000(1.015)^{24}((1.015)^6 - 1)}{(1.015)^{24} - 1}$$ ✓

iv) Total interest pay = $4Q - 120000$ ✓

$= \$29282.273$ ✓ (3 d.p.)