2006

Year 11

HSC Assessment Task 1

Friday 7^{th} December

Mathematics
EXTENSION 1

Weighting: 10%

Working time: 2 periods

Total marks: 48

Topics examined:

Sequences and Series
Mathematical Induction
Polynomials

Outcomes assessed:

Question	Mark
1	
2	
3	
4	
Bonus	
TOTAL	

neral Instructions:

- Write using blue or black pen
- Board-approved calculators and templates may be used
- All necessary working should be shown in every question
- Questions are of equal value
- Full marks may not be awarded for careless or badly arranged work
- Questions are not necessarily arranged in order of difficulty
- Begin each question on a new page
- There is a bonus question at the end of the paper (marks will be awarded for this question)

Question 1: (12 marks)

a) In the arithmetic series $3+8+13+18+..........$, find :

i) the 20^{th} term.

ii) the sum of the first 20 terms. 1

iii) the sum of the 21^{st} to 30^{th} terms 2

b) In an arithmetic series, the 3^{rd} term is -8 and the 6^{th} term is 28. Find the first term and the common ~~ratio~~. difference 2

c) The first three terms of a sequence are $3, 7, 11, ...$ What is the first term to exceed 200? 2

d) Find the sum of the series $4+9+14+............+149$ 2

~~Find the series.~~ 3

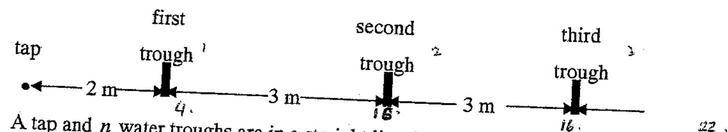
Question 2: (12 marks)

a) Use mathematical induction to prove that:

i) $\sum_{r=1}^{n}(4r+2) = 2n^2+4n$ 4

ii) 5 is a factor of $8^n - 3^n$

b) 3



A tap and n water troughs are in a straight line. The tap is first in line, 2 metres from the first trough, and there is 3 metres between consecutive troughs. A stable hand fills the troughs by carrying a bucket of water from the tap to each trough and then returning to the tap. Thus she walks $2+2=4$ metres to fill the first trough, 10 metres to fill the second trough, and so on.

i) How far does the stable hand walk to fill the k th trough?

ii) How far does the stable hand walk to fill all n troughs? 1

iii) The stable hand walks 1220 metres to fill all the troughs. How many water troughs are there? 2

estion 3: (12 marks) 2

a) A monic – polynomial $P(x)$ has a single root at $x=-2$ and a double root at $x=3$. Write down one possible polynomial which satisfies these conditions. 2

b) The polynomial $P(x)=2x^4-3x^3-4x^2+ax+b$, where a and b are constants, is divisible by both $(x-2)$ and $(x+1)$.

i) Find the values of a and b.

ii) Hence find all real roots of $P(x)=0$ for these values of a and b. 3

4

c) When $P(x)=x^2+2x+3$ is divided by $(x-k)$ the remainder is 2. Find the value of k.

3

Marks

stion 4: (12 marks)

a) i) Using long division, prove that $(2x-3)$ is a factor of $P(x)=6x^3+5x^2-33x+18$. 3

ii) Hence completely factorise $P(x)$. 1

iii) Sketch $P(x)$. 1

b) $(x-k)$ is a factor of $x^2-5x+(2k+2)$. Find the values of k. 2

c) If α,β,γ are the roots of $x^3-2x^2+3x+7=0$, find the values of:

i) $(\alpha+1)(\beta+1)(\gamma+1)$. 2

ii) $\dfrac{2}{\alpha}+\dfrac{2}{\beta}+\dfrac{2}{\gamma}$ 1

iii) $\alpha^2+\beta^2+\gamma^2$ 2

BONUS QUESTION (2 marks)

a) $P(x)$ and $Q(x)$ are two polynomials. $P(x)$ has degree 3 and $Q(x)$ has degree 4. 1

What is the degree of $[P(x)\times Q(x)]$?

b) The year 11 extension 1 class were discussing the possible values of m in this polynomial of degree 4: 1

$$mx^4-5x^3+\frac{2}{3}x^2+x-9$$

Joshua said "m must be 1" — Nelson said "m must be positive"

Fung Fung said "m must be an integer" — Antony said "m must be non-~~zero~~"

Who was correct?

(A) Joshua (B) Nelson (C) Fung Fung (D) Antony

SOLUTIONS TO ASSESSMENT MATHS EXT 1 TASK 1 7th December 2006

Question 1

a)(i) $T_n=a+(n-1)d$

$a=3,\ n=20,\ d=5$ $\therefore T_{20}=3+19\times5$

$=\underline{98}$ 1

(ii) $S_n=\frac{n}{2}[a+l]$

$S_{20}=10[3+98]$

$=\underline{1010}$

or

$S_n=\frac{n}{2}[2a+(n-1)d]$ 2

$S_{20}=10[6+19\times5]$

$=1010$

(iii) $S_{30}=15[2\times3+29\times5]$

$=2265$

$\therefore$ Sum of 21st to 30th term $=S_{30}-S_{20}$ 2

$=2265-1010$

$=\underline{1255}$

b) $T_3=-8 \longrightarrow a+2d=-8$ —(1)

$T_6=27 \longrightarrow a+5d=28$ —(2)

(2)−(1) $3d=36$ 2

$\underline{d=12}$

$\underline{a=-32}$

c) $T_n>200$

$a+(n-1)d>200$

$a=3,\ d=4,\ n=?$ 2

$3+(n-1)4>200$

$3+4n-4>200$

$4n-1>200$

$4n>201$

$n>50\frac{1}{4}$

$\therefore$ ~~51st term~~ $\therefore T_{51}=3+50\times4$ $=203$

d) $S_n=\frac{n}{2}[a+l]$

Need to find n?

$\therefore T_n=a+(n-1)d$

$149=4+(n-1)5$

$149=4+5n-5$

$149=5n-1$

$150=5n$ 3

$n=30$

$\therefore S_{30}=15[4+149]$

$=15\times153$

$=\underline{2295}$

Question 2

i) $6+10+14+\ldots+4n+2=2n^2+4n$

Step 1: Prove for $n=1$

$6=2(1)^2+4$

$=6$

$\therefore$ true for $n=1$

Step 2: Assume true for $n=k$

i.e. $6+10+\ldots+4k+2=2k^2+4k=2k(k+2)$

Step 3. Prove for $n=k+1$

$6+10+\ldots+4k+2+4k+6=2k^2+4k+4k+6$

$=2k^2+8k+6$ 4

$=2(k^2+4k+3)$

$=2(k+1)(k+3)$

$=S_{k+1}$

$\therefore$ true for $n=k+1$

If true for $n=k$ then true for $n=k+1$, but true for $n=1$ then true for $n=2$ etc

ii) Step 1: Prove for $n=1$

ie $8^1 - 3^1 = 5$

$\therefore$ true for $n=1$

Step 2: Assume true for $n=k$

ie $8^k - 3^k = 5m$ where m is an integer

Step 3: Prove for $n=k+1$

$$8^{k+1} - 3^{k+1} = 8.8^k - 3.3^k$$
$$= 8(8^k - 3^k) + 5 3^k$$
$$= 8(5m) + 5.3^k$$
$$= 5[8m + 3^k]$$

$\therefore$ true for $n=k+1$

If true for $n=k$, then true for $n=k+1$, but it is true for $n=1$, then true for $n=2$ etc

3

b) i) $4 + 10 + 16 + 22 + \dots$

$\therefore \; T_n = a + (n-1)d$

$T_k = 4 + (k-1)6$

$= 4 + 6k - 6$

$\underline{T_k = 6k - 2}$

1

(ii) $S_n = \frac{n}{2}[2a + (n-1)d]$

$= \frac{n}{2}[8 + (n-1)6]$

$= \frac{n}{2}[8 + 6n - 6]$

$= \frac{n}{2}[2 + 6n]$

$S_n = \underline{n + 3n^2}$

2

iii) $S_n = n + 3n^2$

$1220 = n + 3n^2$

$3n^2 + n - 1220 = 0$

$n = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

$= \frac{-1 \pm \sqrt{1 + 14640}}{6}$

$= \frac{120}{6}, \; -\frac{122}{6}$

$= 20, \; -20\frac{1}{3}$

$\therefore$ 20 water troughs

2

Question 3

a) $P(x) = (x+2)(x-3)^2$

2

b) i) $P(x) = 2x^4 - 3x^3 - 4x^2 + ax + b$

$P(2) = 0 \Rightarrow 32 - 24 - 16 + 2a + b = 0$

ie $2a + b = 8$ — (1)

$P(-1) = 0 \Rightarrow 2 + 3 - 4 - a + b = 0$

$-a + b = -1$ — (2)

(1) − (2) $3a = 9$

$a = 3$

$b = 2$

3

(ii) $P(x) = 0$

$2x^4 - 3x^3 - 4x^2 - 53x - 98 = 0$

$$\begin{array}{r} 2x^2 - x - 1 \\ x^2 - x - 2 \,\big)\, 2x^4 - 3x^3 - 4x^2 + 3x + 2 \\ 2x^4 - 2x^3 - 4x^2 \\ \hline -x^3 \quad\quad + 3x \\ -x^3 + x^2 + 2x \\ \hline -x^2 + x + 2 \\ -x^2 + x + 2 \end{array}$$

$P(x) = (x-2)(x+1)(2x^2 - x - 1)$

$= (x-2)(x+1)(2x+1)(x-1)$

$\therefore x = 2, -1, -\frac{1}{2}, 1$

4

c) $P(x) = x^2 + 2x + 3$

$P(k) = 2$

$k^2 + 2k + 3 = 2$

$k^2 + 2k + 1 = 0$

$(k+1)^2 = 0$

$\underline{k = -1}$

3

Question 4

a) (i)

$$\begin{array}{r} 3x^2 + 7x - 6 \\ 2x - 3 \,\big)\, 6x^3 + 5x^2 - 33x + 18 \\ 6x^3 - 9x^2 \\ \hline 14x^2 - 33x \\ 14x^2 - 21x \\ \hline -12x + 18 \\ -12x + 18 \\ \hline 0 \end{array}$$

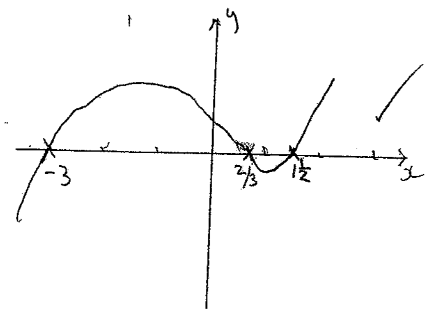
3

(ii) $P(x) = (2x-3)(3x^2 + 7x - 6)$

$= \underline{(2x-3)(3x-2)(x+3)}$

1

(iii)



1

b) $P(x) = x^2 - 5x + (2k+2)$

$P(k) = 0$

$k^2 - 5k + 2k + 2 = 0$

$k^2 - 3k + 2 = 0$

$(k-1)(k-2) = 0$

$\underline{k = 1, 2}$

2

c) $x^3 - 2x^2 + 3x + 7 = 0$

$\alpha + \beta + \gamma = -\frac{b}{a}$

$= 2$

$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a}$

$= 3$

$\alpha\beta\gamma = -7$

(i) $(\alpha+1)(\beta+1)(\gamma+1)$

$= (\alpha+1)(\beta\gamma + \beta + \gamma + 1)$

$= \alpha\beta\gamma + \alpha\beta + \alpha\gamma + \alpha + \beta\gamma + \beta + \gamma + 1$

$= \alpha\beta\gamma + \alpha\beta + \alpha\gamma + \beta\gamma + \alpha + \beta + \gamma + 1$

$= -7 + 3 + 2 + 1$

$= \underline{-1}$

2

(ii) $\frac{2}{\alpha} + \frac{2}{\beta} + \frac{2}{\gamma} = \frac{2(\alpha\beta + \alpha\gamma + \beta\gamma)}{\alpha\beta\gamma}$

$= \frac{2(3)}{-7}$

$= \underline{-\frac{6}{7}}$

1

(iii) $\alpha^2 + \beta^2 + \gamma^2 = (\alpha+\beta+\gamma)^2 - 2(\alpha\beta + \alpha\gamma + \beta\gamma)$

$= 4 - 2(3)$

$= \underline{-2}$

2

Bonus Question

a) $\underline{7}$

b) Antony

2