SOUTH SYDNEY HIGH SCHOOL

Name: ____________________

2006

Year 11

HSC Assessment Task 1

Friday 2nd December

Mathematics

Weighting: 10%

Working time: 2 periods

Total marks: 48

Topics examined:

Tangent and the derivative
Quadratic polynomial

Outcomes assessed:

Question	Mark
1	
2	
3	
4	
Bonus	
TOTAL	

General Instructions:

- Write using blue or black pen
- Board-approved calculators and templates may be used
- All necessary working should be shown in every question
- Questions are of equal value
- Full marks may not be awarded for careless or badly arranged work
- Questions are not necessarily arranged in order of difficulty
- Begin each question on a new page
- There is a bonus question at the end of the paper (marks will be awarded for this question)

Question 1. (12 marks) Marks

Differentiate and simplify the following:

a) $3x^2-7x+8$ 1

b) $(5-7x)^4$ 1

c) $(x^3-7x)(3x^2+12)$ 2

d) $\dfrac{3x-5}{2x+3}$ 2

e) $\left(\sqrt[3]{x}\right)^5$ 2

f) $\dfrac{1}{x\sqrt{x}}$ 2

g) $\dfrac{5x^3+3x^2-4}{x}$ 2

Question 2. (12 marks)

a) If $f(x)=\dfrac{3}{x}+x^4$, find $f'(2)$. 2

b) For $f(x)=x^3+2$, find the values of x for which $f'(x)=1$. 2

c) Find the equation, in general form, of the tangent to the parabola $y=\dfrac{1}{3}x^2$ at the point $\left(2,\dfrac{4}{3}\right)$. 3

d) A tangent to the curve $y=2x^3-2x+3$, is parallel to the line $4x-y-3=0$. Find the point of contact. i.e where the curve and tangent intersect. Hence find the equation of this tangent. 4

e) Draw a possible graph of an **increasing function** indicating what you know about its derivative. 1

Question 3. (12 marks)

a) Solve the quadratic equation $8+2x-x^2=0$. 2

b) Form a quadratic equation in x in expanded form whose roots are: 2

(i) 3 and −5.

(ii) $3+\sqrt{5}$ and $3-\sqrt{5}$.

c) Draw on **separate** axes, a possible graph of the quadratic function $y=ax^2+bx+c$ 3

(i) if $a<0$ and $\Delta>0$

(ii) if $a>0$ and $\Delta<0$

(iii) if $a>0$ and $\Delta=0$.

d) Find the value of m if the equation $2x^2-5x+m=0$ has one root twice the other root. 3

e) Solve for x, $\left(2^x\right)^2-9\left(2^x\right)+8=0$. 2

Question 4. (12 marks)

a) If α and β are the roots of the quadratic equation $2x^2-5x-4=0$, find the value of: 4

(i) $\alpha+\beta$

(ii) $\alpha\beta$

(iii) $\frac{2}{\alpha}+\frac{2}{\beta}$

b) Find the derivative from first principles for the function $f(x)=3x^2+4x$. 3

c) Express $15-8x-x^2$ in the form $a-(x+b)^2$, by completion of squares method or otherwise. 3

Hence find the maximum value of $15-8x-x^2$, and the value of x for which this occurs.

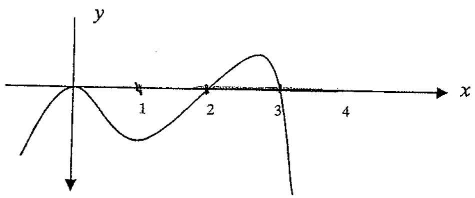
d) Which equation best suits the graph drawn? 2

A] $y=x(x-2)(x-3)$

B] $y=x^2(2-x)(3-x)$

C] $y=x(2-x)(3-x)$

D] $y=x^2(x-2)(3-x)$



Hint: As this is a cubic equation, there are 3 roots. Find the roots and hence determine the appropriate equation.

Please give reasons for your answer. (no marks will be given if there is no reason)

BONUS QUESTION (2 marks)

The tangent to the curve $y=ax^2+bx+1$ passes through the point $(1,5)$ and is parallel to the line $y-6x-2=0$. Find the values of a and b.

SOLUTIONS TO ASSESSMENT MATHEMATICS TASK 1-4th December 2006

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Question 1

a) $y=3x^2-7x+8$

$y'=6x-7$ ✓ 1

b) $y=(5-7x)^4$

$y'=4(5-7x)^3(-7)$ ✓ 1

$=-28(5-7x)^3$

c) $y=(x^3-7x)(3x^2+12)$

$y'=uv'+vu'$ ✓

$=(x^3-7x)6x+(3x^2+12)(3x^2-7)$ ✓ 2

$=6x^4-42x^2+9x^4+15x^2-84$

$=15x^4-27x^2-84$

d) $y=\frac{3x-5}{2x+3}$ $\begin{matrix}u\\v\end{matrix}$

$y'=\frac{vu'-uv'}{v^2}$ ✓

$=\frac{(2x+3)3-(3x-5)2}{(2x+3)^2}$ ✓ 2

$=\frac{6x+9-6x+10}{(2x+3)^2}$

$=\frac{19}{(2x+3)^2}$

e) $y=\sqrt[3]{x^5}$

$y=x^{5/3}$ ✓

$y'=\frac{5}{3}x^{2/3}$ ✓ 2

f) $y=\frac{1}{x\sqrt{x}}$

$y=\frac{1}{x^{3/2}}$

$y=x^{-3/2}$ ✓

$y'=-\frac{3}{2}x^{-5/2}$ ✓ 1

$=\frac{-3}{2\sqrt{x^5}}$

g) $y=\frac{5x^3+3x^2-4}{x}$

$y=\frac{5x^3}{x}+\frac{3x^2}{x}-\frac{4}{x}$ ✓

$y=5x^2+3x-4x^{-1}$ ✓ 2

$y'=10x+3+4x^{-2}$

$=10x+3+\frac{4}{x^2}$

Question 2

a) $f(x)=\frac{3}{x}+x^4$

$=3x^{-1}+x^4$

$f'(x)=-3x^{-2}+4x^3$ ✓

$=\frac{-3}{x^2}+4x^3$ 2

$\therefore f'(2)=\frac{-3}{4}+32$ ✓

$=31\frac{1}{4}$

b) $f(x)=x^3+2$

$f'(x)=1$

$3x^2=1$ ✓

$x^2=\frac{1}{3}$ 2

$x=\pm\frac{1}{\sqrt{3}}$ ✓

c) $y=\frac{1}{3}x^2$

$y'=\frac{2}{3}x$ ✓

when $x=2$, $y'=\frac{4}{3}$

$\therefore m_T=\frac{4}{3}$ ✓

Reqd eqn: $y-y_1=m(x-x_1)$

$y-\frac{4}{3}=\frac{4}{3}(x-2)$ ✓ 3

$3y-4=4(x-2)$

$3y-4=4x-8$

$4x-3y-4=0$

d) $y = 2x^3 - 2x + 3$

$y' = 6x^2 - 2$

Since parallel to $4x - y - 3 = 0$

$y = 4x - 3$

$m_T = 4$ ✓

i.e. $6x^2 - 2 = 4$

$6x^2 = 6$

$x^2 = 1$

$x = \pm 1$ ✓

Points of contact $(1,3)$ and $(-1,3)$

Reqd tangents:

$y - y_1 = m(x - x_1)$ $\quad$ $y - y_1 = m(x - x_1)$

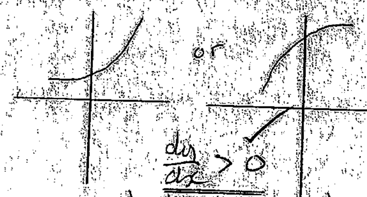
$y - 3 = 4(x-1)$ $\quad$ $y - 3 = 4(x+1)$

$y - 3 = 4x - 4$ ✓ $\quad$ $y - 3 = 4x + 4$ ✓

$\underline{4x - y - 1 = 0}$ $\quad$ $\underline{4x - y + 7 = 0}$

3

e)



$\frac{dy}{dx} > 0$ must also have graph

Question 3

a) $8 + 2x - x^2 = 0$ ✓

$(4 - x)(2 + x) = 0$ ✓

$\underline{x = 4 \text{ or } x = -2}$ ✓

b) (i) $x^2 - (3 - 5)x + 3(-5) = 0$

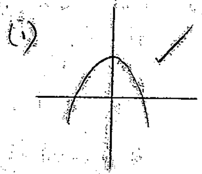
$\underline{x^2 + 2x - 15 = 0}$ ✓

(ii) $x^2 - (3+\sqrt{5}+3-\sqrt{5})x + (3+\sqrt{5})(3-\sqrt{5}) = 0$

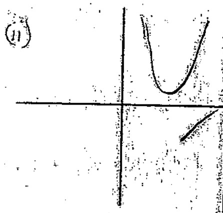
$\underline{x^2 - 6x + 4 = 0}$ ✓

c) $y = ax^2 + bx + c$

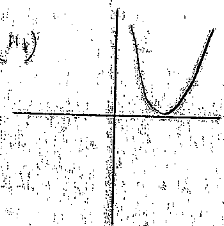
(i)



(ii)



(iii)



3

d) $2x^2 - 5x + m = 0$

Let the roots be $\alpha, 2\alpha$

$\alpha + 2\alpha = -\frac{b}{a}$

$3\alpha = \frac{5}{2}$ ✓

$\alpha = \frac{5}{6}$

Also $2\alpha^2 = \frac{m}{2}$ ✓

$m = 4\alpha^2$

$= 4\left(\frac{5}{6}\right)^2$ ✓

$m = \underline{\frac{25}{9}}$

e) $(2^x)^2 - 9(2^x) + 8 = 0$

let $u = 2^x$

$u^2 - 9u + 8 = 0$ ✓

$(u-1)(u-8) = 0$

$u = 1$ or $u = 8$

$2^x = 1$ or $2^x = 8$

$\underline{x = 0 \text{ or } x = 3}$ ✓

Question 4

(a)(i) $\alpha + \beta = -\frac{b}{a}$

$= \frac{3}{2}$ ✓

(ii) $\alpha\beta = \frac{c}{a}$

$= \frac{-4}{2}$

$= \underline{-2}$ ✓

(iii) $\frac{2}{\alpha} + \frac{2}{\beta} = \frac{2\beta + 2\alpha}{\alpha\beta}$

$= \frac{2(\alpha+\beta)}{\alpha\beta}$ ✓

$= \frac{2(3/2)}{-2}$ ✓

$= \underline{-\frac{3}{2}}$

4

b) $f(x) = 3x^2 + 4x$

$f'(x) = \lim_{h\to 0} \frac{f(x+h) - f(x)}{h}$ ✓

$= \lim_{h\to 0} \frac{3(x+h)^2 + 4(x+h) - (3x^2+4x)}{h}$

$= \lim_{h\to 0} \frac{3(x^2+2xh+h^2) + 4x + 4h - 3x^2 - 4x}{h}$

$= \lim_{h\to 0} \frac{3x^2 + 6xh + 3h^2 + 4x + 4h - 3x^2 - 4x}{h}$

$= \lim_{h\to 0} \frac{6xh + 3h^2 + 4h}{h}$ ✓

$= \lim_{h\to 0} \frac{h(6x + 3h + 4)}{h}$

$= \lim_{h\to 0} 6x + 3h + 4$

$= \underline{6x + 4}$ ✓

c) $15 - 8x - x^2 = a - (x+b)^2$

$= a - (x^2 + 2bx + b^2)$

$= a - x^2 - 2bx - b^2$

$= a - b^2 - 2bx - x^2$ ✓

$\therefore$ $2b = 8$ $\quad$ $a - b^2 = 15$

$b = 4$ ✓ $\quad$ $a - 16 = 15$

$a = 31$ ✓

$\therefore 15 - 8x - x^2 = \underline{31 - (x+4)^2}$

OR

$15 - 8x - x^2 = -(x^2 + 8x) + 15$ ✓

$= -(x^2 + 8x + 16) + 15 + 16$ ✓

$= -(x+4)^2 + 31$ ✓

$= \underline{31 - (x+4)^2}$

3

d) D Roots are $-1, 2, 3$ ✓

when $x = 1$ y is negative ✓

$\therefore$ D is correct graph

2

Bonus Question

$y = ax^2 + bx + 1$

$y' = 2ax + b$

Since // to $y - 6x - 2 = 0$

$y = 6x + 2$

$m_T = 6$ ✓

when $x = 1$: $y' = 6$

$2a + b = 6$ — (1)

when $x = 1$: $y = 5$ ✓

$a + b + 1 = 5$

$a + b = 4$ — (2)

Solving simultaneously $\underline{a = 2,\ b = 2}$