NAME :

SOUTH SYDNEY HIGH SCHOOL

SPECTEMUR AGENDO

Year 11 Half-yearly Examination
May 2000

MATHEMATICS EXTENSION 1

Instructions : **Time Allowed:** $1\frac{1}{2}$ Hours

1. All questions may be attempted.
2. All necessary working should be shown.
3. Marks may be deducted for poorly arranged or missing working.
4. Approved calculators may be used.
5. Questions are **not** of equal value.

Question 1 (12marks) **Marks**

(a) Simplify, without the calculator, with rational denominators : **4**

(i) $2\sin 45^0 \sin 45^0$

(ii) $\dfrac{\tan 60^0 + \tan 45^0}{1 - \tan 60^0 \tan 45^0}$

(b) Find the exact value of : **5**

(i) $\sin 225^0$

(ii) $\tan(-210^0)$

(iii) $\cos(510^0)$

(c) If $\sin x = -\dfrac{3}{4}$ and $0^0 < x < 270^0$, find the exact value of : **3**

(i) $\cos x$ (ii) $\cot x$

Question 2 (12marks)

(a) Simplify $\dfrac{4\cos 10^0}{3\sec 66^0} \times \dfrac{9\text{cosec}24^0}{16\sin 80^0}$ **2**

(b) Solve for x if $\cot(2x + 15^0) = \tan 15^0$ **2**

(c) Simplify :

(i) $\sec\theta \sin\theta \cot\theta$ **2**

(ii) $(\sec\theta + \tan\theta)(\sec\theta + \tan\theta)$ **2**

Continue ....

Question 2 (Continued) **Marks**

(d) Simplify the following results : **4**

(i) $\dfrac{\sin\theta}{\cos\theta}$ (ii) $\cos^2\theta + \sin^2\theta$

hence prove that

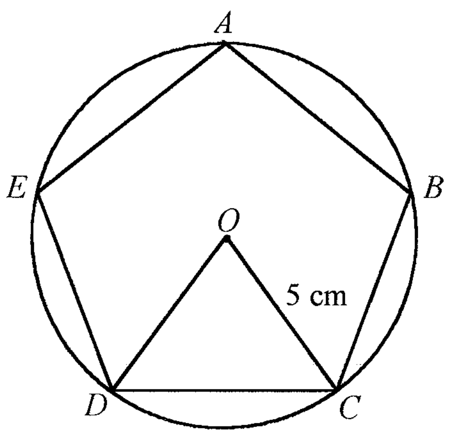
(iii) $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = 1 - 2\sin^2\theta$

Question 3 (10 marks)

(a) (i) A parallelogram has opposite sides that are parallel and equal. Name two other properties of a parallelogram. **2**

(ii) Find the size of each interior angle of a regular hexagon. **1**

(b) A regular pentagon *ABCDE* is inscribed in a circle, with centre *O* and radius 5 cm. **3**



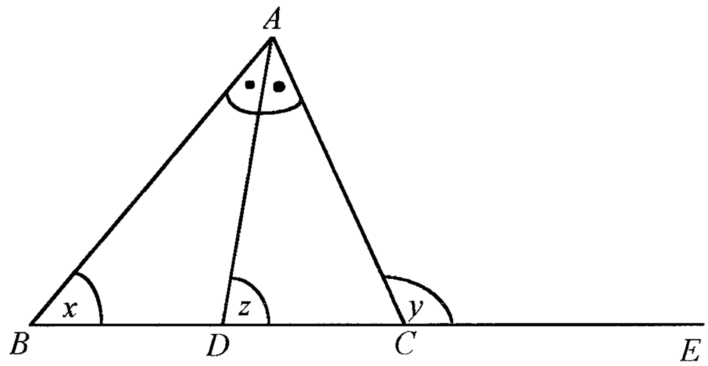
(i) Find the size of $\angle DOC$.

(ii) Hence find the area of the pentagon (to the nearest sq. cm).

Continue ....

Question 3 (Continued) **Marks**

(c) 4

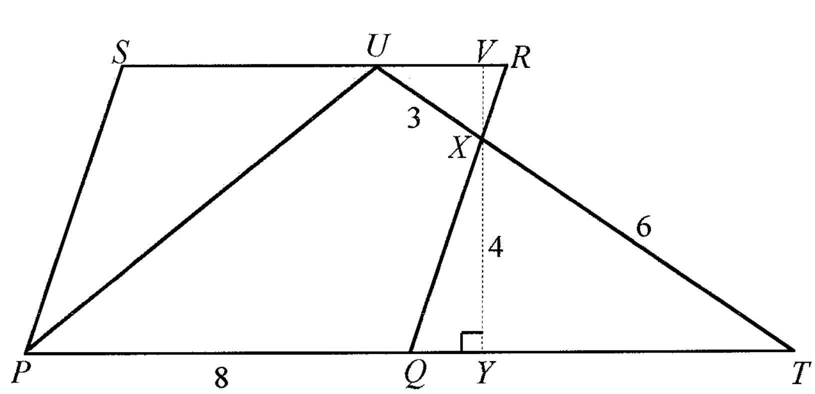


In ΔABC, AD bisects $\angle BAC$. $x = \angle ABC$, $y = \angle AEC$, $z = \angle ADC$

Prove that $x + y = 2z$.

Question 4 (12 marks)

(a) 6



$PQRS$ is a parallelogram and $VY \perp PT$. UT intersects QR and VY at X.
$PQ = QT = 8$ cm, $UX = 3$cm, $XT = 6$ cm, $XY = 4$cm.

(i) Prove that $\Delta UVX \;|||\; \Delta TYX$.

(ii) Find the length of VX.

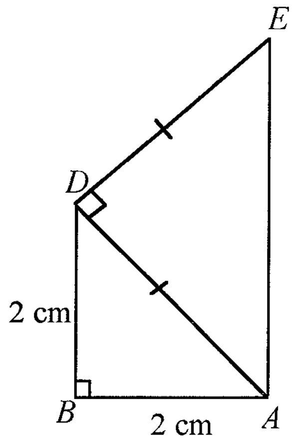
(iii) Find the area of parallelogram $PQRS$.

(iv) Find the area of ΔPUT.

Continue ....

Question 4 (Continued) **Marks**

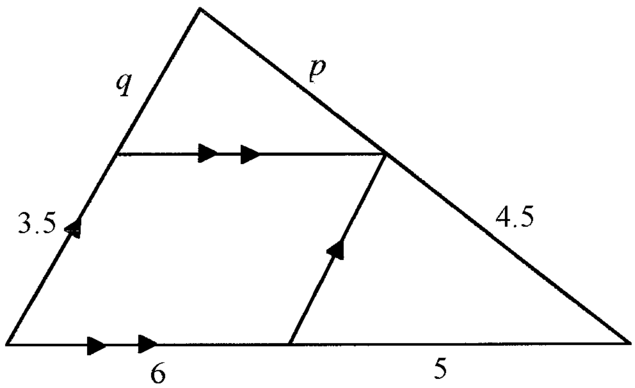
(b) **4**



ΔABD and ΔADE are isosceles right triangles. $AB = BD = 2$ cm and $AD = DE$.

Find the exact length of AE.

(c) **2**



Find the values of p and q

Continue ...

Question 5 (12 marks) **Marks**

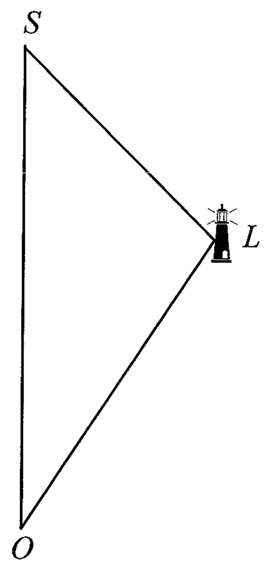
(a) Solve these inequalities : **6**

(i) $\dfrac{1-x^2}{2x+1} < 0$

(ii) $\frac{1}{x} < \frac{4}{5}$

(iii) $x \geq \dfrac{3}{x+2}$

(b) From a ship, S, that is sailing due North the lighthouse, L, is seen at a bearing 030^0T After a further 5 nautical miles, the bearing of the lighthouse from the ship is 140^0T. **3**
Copy the diagram below into your exam pad and fill in the relevant information.



Find the distance, SL, the ship is from the lighthouse to the nearest nautical mile.

(c) The lengths of the three sides of a triangle are 8 cm, 6 cm and 9 cm. **3**
Find :

(i) the size of the largest angle in degrees and minutes.

(ii) the area of the triangle to the the nearest cm^2.

End of paper.

Nalan Ektas

89% V. Good effort 6

Question 1.

a) i) $2\sin 45^\circ \times \sin 45^\circ$

$= 2 \times \frac{1}{\sqrt{2}} \times \frac{1}{\sqrt{2}}$ ✓

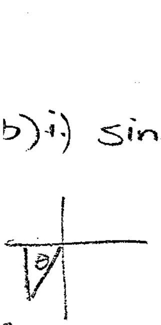
$= \frac{2}{2} = 1$ ✓

ii) $\frac{\cancel{\tan 60 + \tan 45}}{1 - \tan 60^\circ \tan 45} = \frac{\sqrt{3} + 1}{1 - \sqrt{3} \times 1}$ ✓

$= \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}} = \frac{(\sqrt{3}+1)(1+\sqrt{3})}{1-3}$

$= \frac{3 + 2\sqrt{3} + 1}{-2}$ ✓ ✗ $= \frac{\overset{2}{\cancel{4}} + \cancel{2}\sqrt{3}}{-\cancel{2}}$

b) i) $\sin 225$



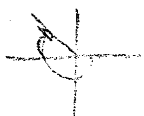
$225 - 180 = 45$

Acute ∠ is 45.

$\therefore$ exact value of $\sin 225 = -\sin 45$ ✓

$= -\frac{1}{\sqrt{2}}$

ii) $\tan(-210)^\circ = \tan 150$



$= -\tan 30$ = Acute ∠

$\therefore$ exact value of $\tan(-210)^\circ$ ✓

$= -\tan 30$

$= -\frac{1}{\sqrt{3}}$ ✓

iii) $\cos(510)^\circ$ $(510 - 360 = 150)$

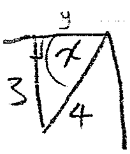
$= -\cos 30$

$\therefore$ exact value of $\cos(510)^\circ$ ✓

$= -\cos 30$

$= -\frac{\sqrt{3}}{2}$ ✓

c) $\sin x = -\frac{3}{4}$



$y^2 = 4^2 - 3^2$

$= 16 - 9$

$y = \sqrt{7}$ ✓ ✗

i). $\cos x = -\frac{\sqrt{7}}{4}$ ✓ ✗

ii). $\tan x = \frac{3}{\sqrt{7}}$ $\therefore \cot x = \frac{\sqrt{7}}{3}$ ✓

$10\frac{1}{2}$

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Question 2.

a) $\cancel{\frac{{}^{1}\cancel{4}\cos 10}{1\cancel{8}\sec 66} \times \frac{\cancel{9}^{3}\operatorname{cosec} 24}{4\cancel{16}\sin 80}}$

$\cancel{= \left(\cos 10 \div \frac{1}{\cos 66}\right) \times \left(\frac{3}{\sin 24} \div 4\sin 80\right)}$?

b) $\cot(2x+15) = \tan 15$

$\tan 15 = \cot(90-15)$

$= \cot(75)$

$\cot 75 = \cot 2x+15$

$x = 30$

c) i) $\sec\theta \sin\theta \cot\theta$

$= \frac{1}{\cos\theta} \times \sin\theta \times \frac{1}{\tan\theta}$

$= \frac{\sin\theta}{\cos\theta} \times \frac{1}{\tan\theta}$

$= \tan\theta \times \frac{1}{\tan\theta}$

$= 1$

ii) $(\sec\theta + \tan\theta)(\sec\theta + \tan\theta)$

$= \sec^2\theta + 2\sec\theta\tan\theta + \tan^2\theta$

$= 1 + \tan^2\theta + \tan^2\theta + 2\sec\theta\tan\theta$

$= 1 + 2\tan^2\theta + 2\sec\theta\tan\theta$

d) i) $\frac{\sin\theta}{\cos\theta} = \tan\theta$

ii) $\cos^2\theta + \sin^2\theta = 1.$

iii) L.H.S $= \frac{1-\tan^2\theta}{1+\tan^2\theta}$

$= 1 - \frac{\sin^2\theta}{\cos^2\theta} \div 1 + \frac{\sin^2\theta}{\cos^2\theta}$

$= \frac{\cos^2\theta - \sin^2\theta}{\cancel{\cos^2\theta}} \times \frac{\cancel{\cos^2\theta}}{\cos^2\theta + \sin^2\theta}$?

$= \frac{\cos^2\theta - \sin^2\theta}{\cos^2\theta + \sin^2\theta}$

$= \cos^2\theta - \sin^2\theta$

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Question 3

a) i). opposite ∠s are equal. in a parallelogram.

Area = b×h

ii. $(2\times6-4)\times90 = 6\times x$ $\quad$ (x = interior ∠)

$8\times 90 = 6x$

$720 = 6x$

$x = 120°$

b) i) $(2\times5-4)\times90 = 5\times x$

$540 = 5x$

$x = 108$

= interior ∠s

∠ODC is half interior ∠

$= 54°$.

∴ in △ ODC, $54 + 54 + \angle DOC = 180$

(∠ sum of △)

$\angle DOC = 180 - 108$

$= \underline{72°}$

ii) $A = 5\times\frac{1}{2}ab\sin C$

$= 2.5\times25\times\sin 72$

$= 59.44\ cm^2$

$= 59\ cm^2$ (nearest cm^2).

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c). $\underbrace{LHS}$ $= x + y$.

$= z - \angle BAD + z + \angle BAD$

$= 2z$.

= RHS

as required

V. Good!

in △ ABD

$x = 180 - \angle BAD - (180 - z)$

$= 180 - \angle BAD - 180 + z$

$x = z - \angle BAD$.

$y = x + 2\angle BAD$.

(exterior ∠).

$= z - \angle BAD + 2\angle BAD$

$= z + \angle BAD$.

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Question 4.

a) i). $\angle UXV = \angle YXT$ (vertically opp).

$\angle VUX = \angle XTY$ (alternate $\angle$s, $SR \parallel PT$).

$\therefore$ remaining $\angle$s are equal also ($\angle$ sum of Δ)

$\angle XVU = \angle XYT$.

$\therefore \Delta UVX \;|||\; \Delta TYX$ (equiangular).

ii). $\frac{VX}{XY} = \frac{UX}{TX} \Rightarrow \frac{VX}{4} = \frac{3}{6} \quad \therefore VX = \frac{3 \times 4}{6} = 2.$

iii). $A = \text{base} \times \text{height}$

$= 8 \times 6$

$= 48\,cm^2$

iv) $\text{Area} = \frac{1}{2} \times \text{base} \times \text{height}$

$= \frac{1}{2} \times (8 * 5) \times 6$

$= 48\,cm^2$

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b). $DA^2 = 2^2 + 2^2$

$= 4 + 4$

$DA = \sqrt{8} = DE$

Let $x = EA$.

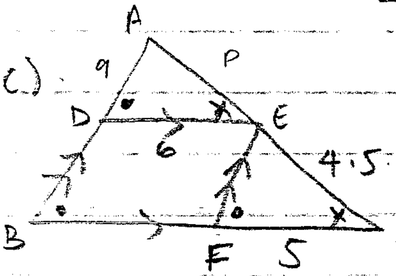
$x^2 = DA^2 + DE^2$

$= \sqrt{8}^2 + \sqrt{8}^2$

$x^2 = 16$

$x = 4$ $(EA = 4)$.

c).



Not asked for

$\angle AED = \angle ECF$ (corresponding $\angle$s, $DE \parallel BC$)

(Note $\angle DBF = \angle EFC$, corresponding $\angle$s, $BD \parallel EF$).

$\therefore \angle ADE = \angle DBF = \angle EFC$ (corresponding $\angle$s, $DE \parallel BF$)

$\therefore \angle DAE = \angle FEC$ (remaining $\angle$s must be equal)

Since $\therefore \Delta ADE \;|||\; \Delta EFC$.

$\frac{P}{4.5} = \frac{6}{5}$

$P = 5.4.$

$\frac{9}{3.5} = \frac{6}{5} = \frac{P}{4.5} = \frac{5}{6}$

$9 = \frac{3.5 \times 6}{5}$ X

$= 4.2$

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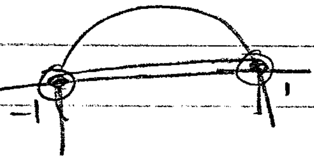
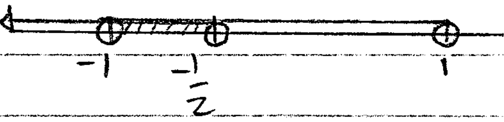
Question 5.

a) i) $\dfrac{1-x^2}{2x+1} < 0$ $\quad \therefore 1-x^2 > 0 \;\cap\; 2x+1 < 0$

$(1-x)(1+x) > 0 \qquad 2x < -1$

$x = 1, -1 \qquad x < -\frac{1}{2}$





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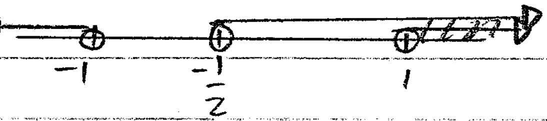


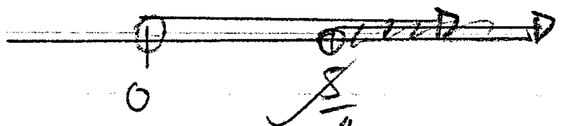
$\therefore -1 < x < -\frac{1}{2},\ x > 1.$

ii. $\dfrac{1}{x} - \dfrac{4}{5} < 0$ $\quad \therefore 5-4x < 0 \;\cap\; 5x > 0$

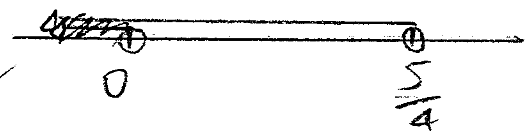
$5 < 4x \qquad x > 0.$

$\dfrac{5-4x}{5x} < 0 \qquad 5/4 < x$

$x > 5/4$



OR

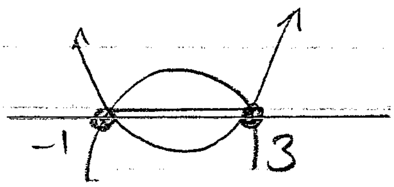


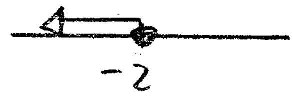
$\therefore x > 5/4,\ x < 0.$

iii). $x - \dfrac{3}{x+2} \geq 0$ $\quad x \neq -2 \therefore (x-3)(x+1) \geq 0 \;\cap\; x+2 \leq 0$

$x = 3, -1 . \qquad x \leq -2$

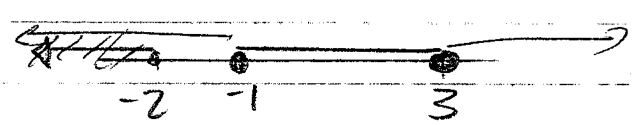
$\dfrac{x(x+2)-3}{x+2} \geq 0$



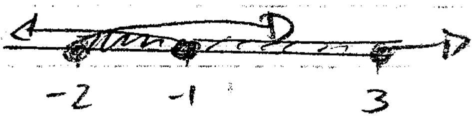


$x < -2$

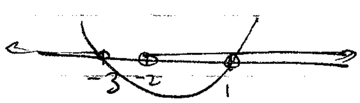
$\dfrac{x^2+2x-3}{x+2} \geq 0.$



$\dfrac{(x+3)(x-1)}{x+2} \geq 0$ OR



$-1 \leq x \leq 3$



$x \geq 1$

$-3 \leq x < -2$

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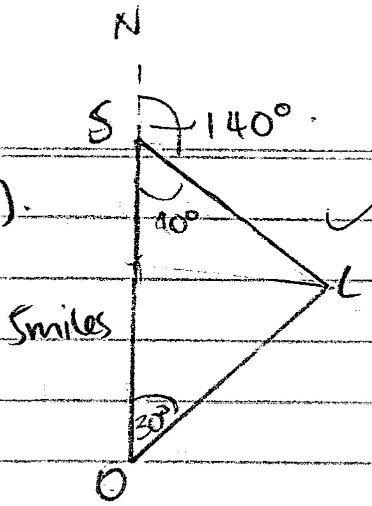
b).



$\angle L = 180 - 40 - 30$
$= 110°$

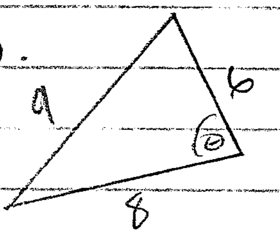
$\dfrac{5\text{miles}}{\sin 110} = \dfrac{SL}{\sin 30}$

$SL = \dfrac{5 \times \sin 30}{\sin 110}$

$= 2.66\ldots$ miles

$= 3$ nautical miles.

c).



i.) $\cos\theta = \dfrac{6^2 + 8^2 - 9^2}{2 \times 6 \times 8}$

$= 0.1979\ldots$

$\theta = 78°35'$

ii). $\left(A = \frac{1}{2} ab \sin C\right)$

$A = \frac{1}{2}\, 6 \times 8 \times \sin 78°35'$

$= 23.52\ldots \quad = 24 \text{ cm}^2$ (nearest cm^2)