SOUTH SYDNEY HIGH SCHOOL

Year 11

Preliminary Course

Assessment Task No. 1

19th November 1997

MATHEMATICS

2/3 UNIT (COMMON)

Time allowed - 2 PERIODS

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DIRECTIONS TO CANDIDATES

- This assessment is worth **10%.**
- Board-approved calculators may be used.
- Write your Name on **EVERY** page .
- Attempt all **SEVEN** questions.
- Start each question on a **NEW** page.

Question 1 (3 marks) **Marks**

$A(-2, 0)$, $B(3, 7)$ and $C(5, -2)$ are three points on the number plane.

(a) Find the distance BC. 1

(b) Find the midpoint of the line joining AB. 1

(c) What is the gradient of the line joining AC ? 1

Question 2 (8 marks)

$X(-1, 4)$, $Y(5, 1)$ and $Z(3, 6)$ are the vertices of a triangle.

Find :

(a) the equation of XY 2

(b) the equation of the line through Z parallel to ~~YZ~~. XY. 2

(c) the shortest distance from Z to XY. 2

(d) the area of ΔXYZ. 2

Question 3 (5 marks)

(a) Find the equation of the line passing through the point $(1, -2)$ and perpendicular to the line $2x - 3y + 7 = 0$. 3

(b) Find the value of a if the line $2ax + 4y - 1 = 0$ is perpendicular to the line $6x + 8y + 7 = 0$. 2

Question 4 (6 marks)

$A(-1, -2)$, $B(7, 2)$, $C(1, 4)$ are the vertices of a triangle.

Show that :

(a) ΔABC is isosceles. 3

(b) the perpendicular bisector of AB passes through C. 3

Question 5 (7 marks) **Marks**

(a) Find x in Figure 1, giving reasons. (Answer to the nearest degree). **2**

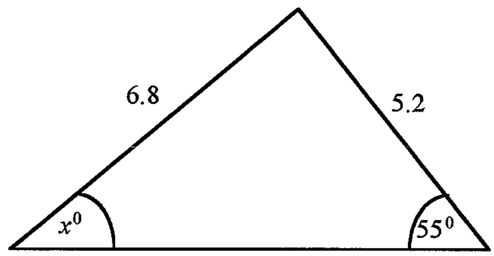

Figure 1

(b) Find (stating the rules used) **5**

(i) the value of x

(ii) the area of the triangle

in Figure 2.

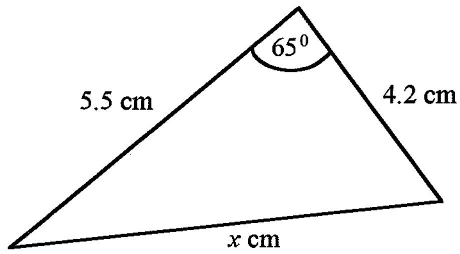

Figure 2

Question 6 (4 marks)

From a port P, a lighthouse L is seen 2.4 kmaway on a bearing of 035^0. A boat leaves port and sails due east to a point B, 4.2 km from the lighthouse.

With the aid of a diagram, find the bearing of the boat from the lighthouse.

Question 7 (17 marks)

(a) What is the exact value of: **4**

(i) $\cos 150^0$ (ii) $\tan 300^0$ (iii) $\sin 150^0 . \cos 330^0$

(b) Solve (i) $2\sin x + \sqrt{3} = 0$ for $0^0 < x < 360^0$. **2**

(ii) $4\sin x\cos x - 2\cos x + 2\sin x - 1 = 0$ for $-180^0 \le x \le 180^0$ **3**

(c) (i) Simplify $\cos^2\theta(1+\tan^2\theta)$ (ii) Show that $\dfrac{1-\tan^2\theta}{1+\tan^2\theta} = 1 - 2\sin^2\theta$ **5**

(d) If $\cos x = -\frac{2}{3}$ and $\tan x > 0$, find the exact value of $\sin x$. **3**

End of paper

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QUESTION 1—2

[redacted]

96%

Q1.

(a) $BC = \sqrt{(3-5)^2 + (7-(-2))^2} = \sqrt{4+81} = \sqrt{85}$. ✓

(b) midpoint AB $\left(\frac{3-2}{2}, \frac{0+7}{2}\right) = \left(\frac{1}{2}, \frac{7}{2}\right)$ ✓

(c) Gradient $AC = \frac{0-(-2)}{-2-5} = \frac{2}{-7}$ ✓

3

Q2. $x(-1,4)$, $y(5,1)$, $Z(3,6)$

(a) EQ. XY : $(y-1) = \frac{4-1}{-1-5}(x-5)$

$y = \frac{3}{-6}(x-5)+1 = -\frac{1}{2}x + \frac{5}{2} + 1 = -\frac{1}{2}x + \frac{7}{2}$. ✓

$\therefore 2y + x - 7 = 0$. ✓

(b) Eq : parallel to XY $\therefore$ Gradient $= -\frac{1}{2}$.

through $Z(3,6)$.

$y - 6 = -\frac{1}{2}(x-3)$ ✓ $\therefore 2y - 12 = -x + 3$.

$\therefore 2y + x - 15 = 0$ ✓

7

(c) $Z(3,6)$ XY : $2y + x - 7 = 0$.

$\therefore$ Shortest distance $= \frac{|2\times(3) + (6) - 7|}{\sqrt{2^2+1^2}} = \frac{|6+6-7|}{\sqrt{5}}$ ✓

$= \frac{8\sqrt{5}}{5}$

$= \frac{5}{\sqrt{5}} = \sqrt{5}$.

[redacted]

(d) Area of $\triangle XYZ$.

height $= \sqrt{5}$.

Distance of $XY = \sqrt{(-1-5)^2 + (4-1)^2} = \sqrt{36+9}$

$= \sqrt{45}$ ✓

$\therefore$ Area of $\triangle XYZ = \frac{1}{2} \times \sqrt{5} \times \sqrt{45} = \frac{1}{2} \times \sqrt{225}$

$= \frac{1}{2} \times \frac{8\sqrt{5}}{5} \times 3\sqrt{5} \qquad = \frac{15}{2}$. ✓

$=$ **12 sq. units**

Q3

(a) Eq through $(1, -2)$ perpendicular : $y = \frac{2}{3}x + \frac{7}{3}$

$\therefore$ Gradient of Eq $\times \frac{2}{3} = -1$

$\therefore$ Gradient $= -\frac{3}{2}$.

$\therefore$ Eq : $(y - (-2)) = -\frac{3}{2}(x-1)$

$\therefore 2y + 4 = -3x + 3$. $\qquad \therefore 2y + 3x + 1 = 0$.

(b) Gradient of Eq $= -\frac{2a}{4} \quad \therefore -\frac{a}{2}$

Gradient of perpendicular Eq $= -\frac{6}{8} = -\frac{3}{4}$.

$\therefore -\frac{a}{2} \times -\frac{3}{4} = -1 \qquad \therefore \frac{3a}{8} = -1 \qquad 3a = -8 \qquad \therefore a = -\frac{8}{3}$

Q4. $A(-1, -2)$, $B(7, 2)$, $C(1, 4)$.

(a) $AB = \sqrt{(-1-7)^2 + (-2-2)^2} = \sqrt{64+16} = \sqrt{80}$.

$BC = \sqrt{(7-1)^2 + (2-4)^2} = \sqrt{36+4} = \sqrt{40}$.

$CA = \sqrt{(-1-1)^2 + (-2-4)^2} = \sqrt{4+36} = \sqrt{40}$

$\therefore BC = CA \qquad \therefore$ isosceles.

QUESTION 3.

[redacted]

Q3. (a) Eq through $(1,-2)$ perpendicular to : $y=\frac{2}{3}x+\frac{7}{3}$

$\therefore$ Gradient of Eq $\times \frac{2}{3} = -1$ ✓

$\therefore$ Gradient of Eq $= -\frac{3}{2}$. ✓

$\therefore$ Eq : $y-(-2) = -\frac{3}{2}(x-1)$

$\therefore 2y+4 = -3x+3$.   3

$\therefore 2y+3x+1=0$. ✓

(b) Gradient of Eq $= -\frac{2a}{4} = -\frac{a}{2}$ ✓

Gradient of perpendicular Eq $= -\frac{6}{8} = -\frac{3}{4}$. ✓

$\therefore -\frac{a}{2} \times -\frac{3}{4} = -1$ $\therefore$

$\therefore \frac{3a}{8} = -1$ $\therefore 3a=-8$

$\therefore a = -\frac{8}{3}$. ✓ 2

5

QUESTION 4 — 7

[redacted]

Q4. $A(-1,-2)$, $B(7,2)$, $C(1,4)$

(a) $AB = \sqrt{(-1-7)^2+(-2-2)^2} = \sqrt{64+16} = \sqrt{80}$

$BC = \sqrt{(7-1)^2+(2-4)^2} = \sqrt{36+4} = \sqrt{40}$

$CA = \sqrt{(-1-1)^2+(-2-4)^2} = \sqrt{4+36} = \sqrt{40}$

3.

$\therefore BC = CA$ $\therefore$ isosceles

(b) Gradient $AB = \dfrac{-2-2}{-1-7} = \dfrac{-4}{-8} = \dfrac{1}{2}$

Midpoint AB $\left(\dfrac{-1+7}{2}, \dfrac{-2+2}{2}\right) \equiv (3, 0)$

$\therefore$ Gradient perpendicular Eq $\times \frac{1}{2} = -1$

$\therefore$ Gradient $= -2$

5

This Eq through $(3,0)$

$\therefore$ Eq : $y - 0 = -2(x-3)$ $\therefore y = -2x + 6$ 2

You have not shown that this passes thru' C

Q5.

(By sine rule.)

(a) $\dfrac{6.8}{\sin 55^\circ} = \dfrac{5.2}{\sin \alpha}$ $\therefore \sin\alpha = \dfrac{5.2 \times \sin 55}{6.8} = 0.6264$

$\therefore \alpha = 38^\circ 48' = 39^\circ$ (nearest degree)

+2

(By cosine rule.)

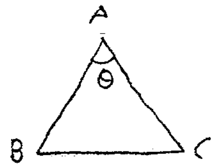
(b) $x^2 = (5.5)^2 + (4.2)^2 - 2\cdot(5.5)\cdot(4.2)\cdot\cos 65^\circ$

$= 28.365$

$\therefore x = 5.326$ cm

(ii) Area of triangle

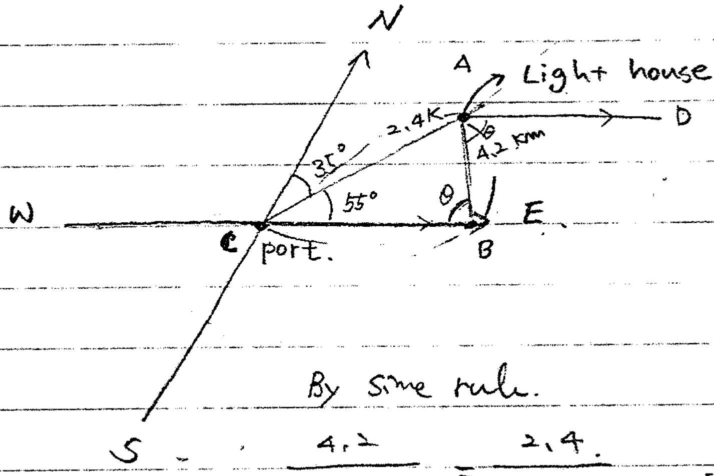
$= \frac{1}{2} \times \sin\theta \times AB \times AC.$



$\therefore$ Area $= \frac{1}{2} \times \sin 65^\circ \times 5.5 \times 4.2$

$= 10.468 \text{ cm}^2$ ✓ 5

Q6.



By sine rule.

$\frac{4.2}{\sin 55} = \frac{2.4}{\sin\theta}$ $\therefore \sin\theta = \frac{2.4 \times \sin 55}{4.2}$

$= 0.468$

$\therefore \theta = 27^\circ 54'$

Let's put AD which is parallel to CB.

~~$\therefore \angle BAD = \theta = 27^\circ 54'$~~

$\therefore$ Bearing $= 90^\circ + 27^\circ 54' = 117^\circ 54'$ ✓ 4

24

16

Q7.

(a)

(i) $\cos 150^\circ = \cos(90+60) = -\sin 60 = -\frac{\sqrt{3}}{2}$.

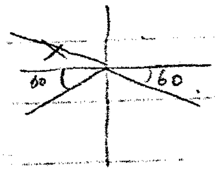
(ii) $\tan 300^\circ = \tan(360-60) = -\tan 60 = -\sqrt{3}$.

(iii) $\sin 150^\circ = \cos 60^\circ = \frac{1}{2}$, $\cos 330 = \cos(360-30) = \cos 30 = \frac{\sqrt{3}}{2}$.

$\therefore \sin 150^\circ \times \cos 330^\circ = \frac{1}{2} \times \frac{\sqrt{3}}{2} = \frac{\sqrt{3}}{4}$. 4

(b) (i) $2\sin\alpha = -\sqrt{3}$ $\therefore \sin\alpha = -\frac{\sqrt{3}}{2}$.

$\therefore \alpha = 240^\circ, 300^\circ$



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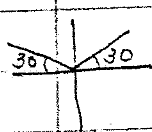
(ii) $4\sin\theta\cos\theta - 2\cos\theta + 2\sin\alpha - 1 = 0$.

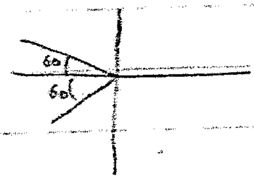
$2\cos\theta(2\sin\theta - 1) + 2\sin\alpha - 1 = 0$.

$\therefore (2\sin\theta - 1)(2\cos\theta + 1) = 0$.

$\therefore \sin\theta = +\frac{1}{2}$, $\cos\theta = -\frac{1}{2}$.



$\theta = 30^\circ, 150^\circ$ $\theta = 120^\circ, \cancel{-120}$ 240°



$\therefore \theta = \cancel{-120^\circ}, 30^\circ, 120^\circ, 150^\circ$ & 240°. 2 +1

$0 < x < 360$!!

$c^2 + s^2 = 1 \qquad 1 + \tan^2\theta = \frac{1}{c^2}$

(c) (i) $\cos^2\theta\,(1 + \tan^2\theta)$

$= \cos^2\theta \times (\sec^2\theta) \qquad (\because\ 1 + \tan^2\theta = \sec^2\theta)$

$= 1.$ ✓ 2

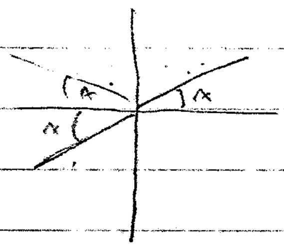
(ii) $\dfrac{1 - \tan^2\theta}{1 + \tan^2\theta} = \dfrac{1 - \tan^2\theta}{\sec^2\theta} = (1 - \tan^2\theta) \times \cos^2\theta.$

$= \left(1 - \dfrac{\sin^2\theta}{\cos^2\theta}\right) \times \cos^2\theta$

$= \cos^2\theta - \sin^2\theta. = (1 - \sin^2\theta) - \sin^2\theta$

$= 1 - 2\sin^2\theta$ ✓ 3

(d) $\cos\alpha = -\frac{2}{3}, \quad \tan\alpha > 0,$



$\tan\alpha > 0, \quad \cos\alpha < 0$

$\therefore\ 180^\circ \leq \alpha \leq 270;$

$\therefore\ \cos\alpha = \sqrt{1 - \sin^2\alpha} = -\frac{2}{3}$

$\therefore\ 1 - \sin^2\alpha = \frac{4}{9}. \quad \therefore\ \sin^2\alpha = 1 - \frac{4}{9} = \frac{5}{9}.$

$\therefore\ \sin\alpha = \pm\frac{\sqrt{5}}{3}.$ (but $180 \leq \alpha \leq 270 \ \therefore\ \sin\alpha < 0$)

$\therefore\ \sin\alpha = -\frac{\sqrt{5}}{3}.$ ✓ 3