SOUTH SYDNEY HIGH SCHOOL

PRELIMINARY
AUGUST ASSESSMENT

YEAR 11

MATHEMATICS

2002

Time Allowed—2Periods

Directions to Candidates

- Attempt ALL questions
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work.
- Board approved calculators maybe used.

Question One

a) Solve

i) $2a = \frac{a+1}{3} - 5(a-2)$

ii) $2x^2 - 5x - 12 = 0$

iii) $|3x+2| = 1$

iv) $2x - y = 6$
$x + 3y = 10$

b) Simplify:

i) $\frac{x^2 - x - 20}{x^2 - 25} \div \frac{x+1}{x^2 + 5x}$

ii) $\frac{125}{(5^n)^3 \times (125)^{1-n}}$

c) Rationalise the denominator:

i) $\frac{\sqrt{2}+1}{\sqrt{3}}$

ii) $\frac{5}{3-\sqrt{2}}$

Question Two

a) Sketch the graph of each of the following and state the domain and range of each.

i) $y = x^2$ ii) $y = \frac{4}{x+1}$ iii) $y = |1+x|$

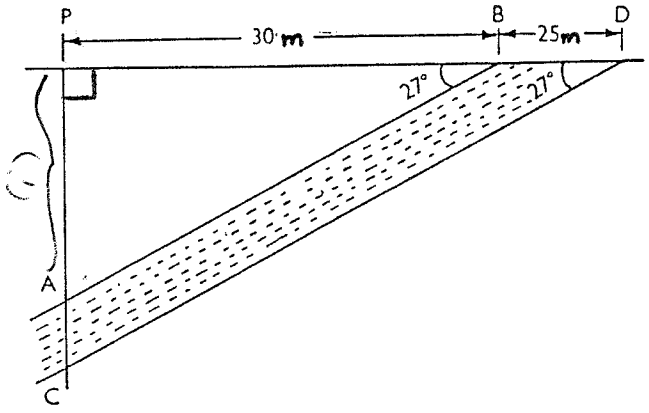
iv) $y = x^2 - 4x + 3$ v) $y = \sqrt{1-x^2}$ vi) $y = 2 - x^3$

b) i) If $g(x) = 2x^3$ find $g(-x)$
ii) Hence prove that $g(x)$ is an odd function

c) Draw a sketch showing where $x^2 + y^2 \leq 4$, $y \leq 1 - x^2$ and $y \geq -1$ hold simultaneously.

Question Three

a) In the diagram, AB and CD are the sides of a straight seam of coal, and BD is the width of coal visible on the horizontal ground surface PBD. PAC represents a vertical bore.



Find:

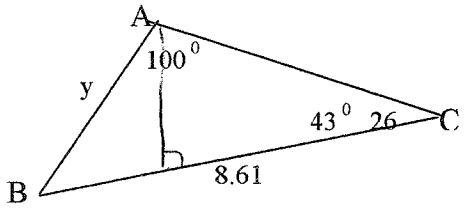
i) the depth PA where the bore meets the seam.

ii) the distance AC through the seam along the bore

iii) the thickness of the seam.

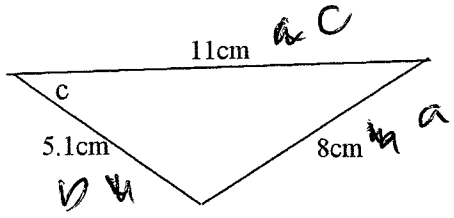
b)

i) Find the value of the pronumeral (to two significant figures). Measurements in metres.



ii) Find the area of triangle ABC. (nearest square metre)

c) Find c (to the nearest minute)



d) A ship sailing due north observes two lighthouses bearing 043^0 and 022^0 respectively. After sailing for a further 18km, the lighthouses appear in line with and due east of the ship.Find the distance between the lighthouses.

e) Simplify:

i) $(\sec^2 x - 1)\cot^2 x$

ii) $\dfrac{\tan x \sec x}{1 + \tan^2 x}$

f) Prove

$$\frac{1}{SinxCosx} - \tan x = Cotx$$

g) Find the exact value for:

i) Sin 120^0 ii) Sec 300^0 iii) Cot (-30^0)

h) Solve the following equations:

i) $Sinx = \dfrac{1}{2}$, $0 \le x \le 360^0$

ii) $Cotx = -\sqrt{3}$, $0 \le x \le 720^0$

Question Four

a) A is the point (-2, -3) and B is the point (4, 3).Find:

i) The distance AB (as a surd).

ii) The coordinates of the midpoint of AB.

iii) The gradient of the line AB.

iv) The equation of the perpendicular bisector of AB.

v) The equation of the line parallel to AB that passes through the point (11, 0).

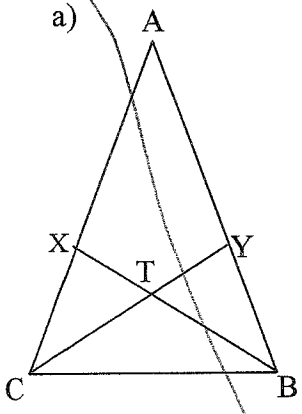
vi) The equation of the line AB.

vii) The perpendicular distance from the point (-1, 3) to the line AB.

b) Find the equation of the line passing through the point of intersection of $5x-2y+3=0$ and $2x+6y-7=0$ and which is perpendicular to the line $2y-x+4=0$

Question Five

a)

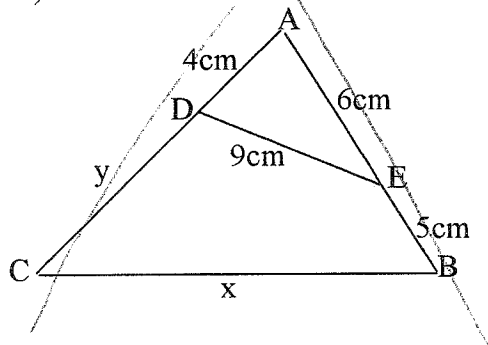


Triangle ABC is isosceles with AB=AC. BX is perpendicular to AC and CY is perpendicular to AB.Prove

i) $\Delta BXC \equiv \Delta CYB$

ii) Hence prove that XT = YT.

b)



$\angle ADE = \angle ABC$.

i) Prove $\Delta ADE \;|||\; \Delta ABC$

ii) Find x and y.

Year 11 Solns – AUG 2002

a)

$6a = a + 1 - 15a + 30$

$\therefore 20a = 31 \Rightarrow a = 31/20$ (2m)

$(2x+3)(x-4) = 0$

$\Rightarrow x = -3/2, 4$ (2m)

$3x + 2 = 1$ or $3x + 2 = -1$

$x = -1/3$ or $x = -1$

~~CHECK!~~ (2m)

$2x - y = 6$ —①

$2x + 6y = 20$ —②

} (–)

$-7y = -14 \Rightarrow y = 2$

$\therefore 2x - 2 = 6 \Rightarrow x = 4$

$x = 4, y = 2$ (2m)

$\frac{(x-5)(x+4)}{(x-5)(x+5)} \times \frac{x(x+5)}{x+1}$

$\frac{x(x+4)}{x+1}$ (3m)

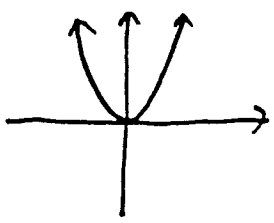
$\frac{5^3}{3^n \times (5^3)^{1-n}} \Rightarrow \frac{5^3}{5^3}$

$\Rightarrow 1$ // (2m)

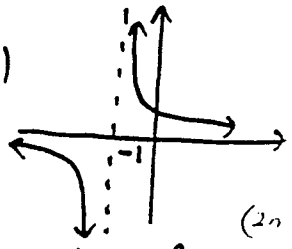
$\frac{\sqrt{2}+1}{\sqrt{3}} \times \frac{\sqrt{3}}{\sqrt{3}} \Rightarrow \frac{\sqrt{6}+\sqrt{3}}{3}$ (2m)

$\frac{\quad}{\sqrt{2}} \times \frac{3+\sqrt{2}}{3+\sqrt{2}} \Rightarrow \frac{5(3+\sqrt{2})}{7}$ (2m)

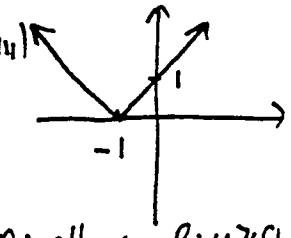
Q2.

i) 

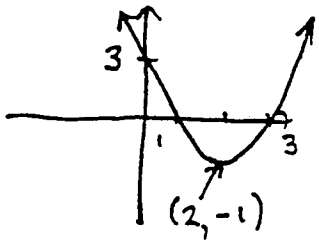
D: all x, R: $y \geq 0$ (2m)

ii) 

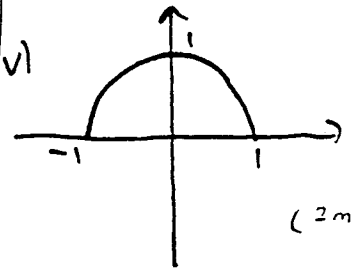
D: $x \neq -1$, R: $y \neq 0$ (2m)

iii) 

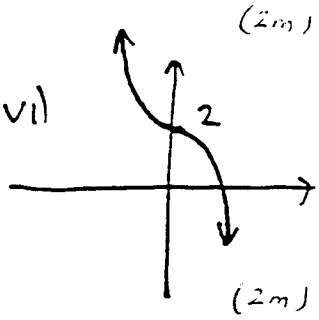
D: all x, R: $y \geq 0$ (2m)

iv) 

D: All x, R: $y \geq -1$ (2m)

v) 

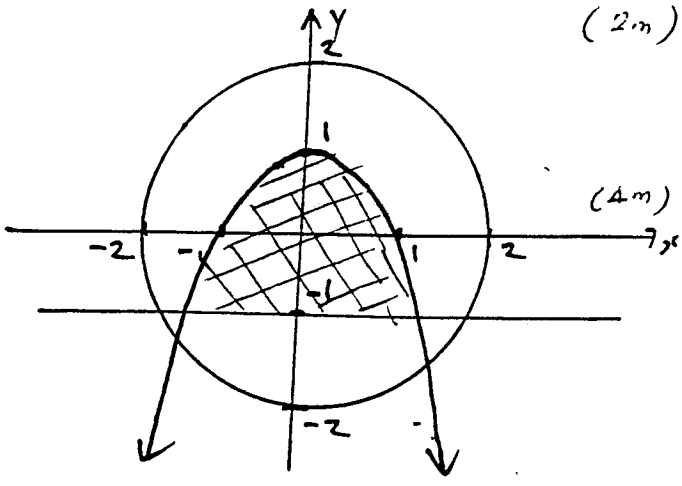
D: $-1 \leq x \leq 1$, R: $0 \leq y \leq 1$ (2m)

vi) 

D: all x, R: all y (2m)

b) i) $g(-x) = 2(-x)^3 = -2x^3$ (1m)

ii) Since $-g(x) = g(-x)$, function is odd (or graph has point symmetry about origin) (2m)

c) 

(4m)

$\tan 27 = \frac{PA}{30} \Rightarrow PA = 15.29\,m$ (2dp)

$\tan 27 = \frac{PC}{55} \Rightarrow PC = 28.02\,m$ (2dp)

$\therefore AC = PC - PA = 12.73\,m$

$\sin 27 = \frac{x}{25} \Rightarrow x$ (thickness) $= 11.35\,m$

$\frac{y}{\sin 43^\circ 26'} = \frac{8.61}{\sin 100} \Rightarrow y = 6.0\,m$

$A = \frac{1}{2} \times 8.61 \times 6.01 \times \sin 36^\circ 34' = 15\,m^2$

$\cos C = \frac{11^2 + 5.1^2 - 8^2}{2 \times 11 \times 5.1} \Rightarrow C = 42^\circ 17'$

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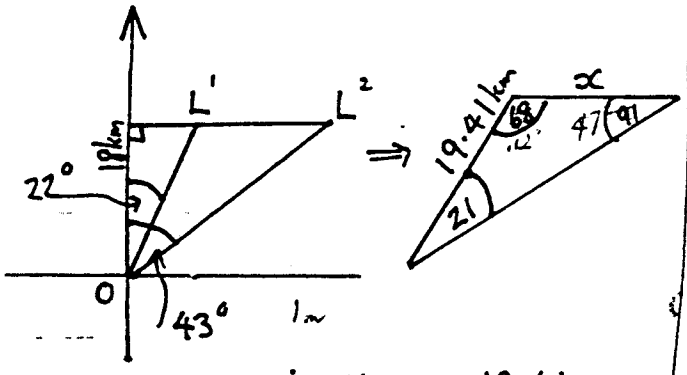

$\therefore \frac{x}{\sin 21} = \frac{19.41}{\sin 47}$

9.51

$x = 6.96$ km 1m.

$\tan^2 x \cdot \frac{1}{\tan^2 x} = 1$ //

$\frac{\tan x \sec x}{\sec^2 x}$

$\frac{\tan x}{\sec x} \Rightarrow \frac{\sin x}{\cos x} \times \frac{\cos x}{1}$

$\frac{\sin x}{1} - \frac{\sin x}{\cos x} \Rightarrow \frac{1 - \sin^2 x}{\sin x \cos x}$

$\frac{\cos^2 x}{x \cos x} = \frac{\cos x}{\sin x} = \cot x$

iii) $\frac{x}{9} = \frac{45}{11\frac{1}{2}} = \frac{6}{4+y} \Rightarrow x = 24,\ y = 12\frac{1}{2}$

g) i) $\frac{\sqrt{3}}{2}$ ii) $+\frac{2}{\sqrt{3}}$ iii) $-\sqrt{3}$

h) i) $x = 30^\circ, 150^\circ$ ii) $150^\circ, 330^\circ, 510^\circ, 690^\circ$

Q4 a) i) $AB = \sqrt{(4+2)^2 + (3+3)^2} = \sqrt{72} = 6\sqrt{2}$

ii) $x = 1,\ y = 0 \Rightarrow (1,0)$ is m.p.

iii) $m = \frac{6}{6} = 1$

iv) $y - 0 = -1(x-1) \Rightarrow y = -x + 1 \Rightarrow x + y - 1 = 0$

v) $y - 0 = 1(x-11) \Rightarrow y = x - 11 \Rightarrow x - y - 11 = 0$

vi) $y - 3 = 1(x-4) \Rightarrow y = x - 1 \Rightarrow x - y - 1 = 0$

vii) $\perp D = \left|\frac{1(-1) + (-1)(3) + (-1)}{\sqrt{1^2 + (-1)^2}}\right| = \frac{5}{\sqrt{2}} = \frac{5\sqrt{2}}{2}$

b) $5x - 2y = -3$ —① $\times 3 \Rightarrow 15x - 6y = -9$

$2x + 6y = 7$ —② $2x + 6y = 7$

$\therefore 17x = -2 \Rightarrow x = -\frac{2}{17},\ y = \frac{41}{34}$

$m_1 = \frac{1}{2} \Rightarrow \therefore m_2 = -2$

$\therefore y - \frac{41}{34} = -2\left\{x + \frac{2}{17}\right\}$

$\Rightarrow y = -2x - \frac{4}{17} + \frac{41}{34} \Rightarrow y = -2x + \frac{33}{34}$

$\Rightarrow 68x + 34y - 33 = 0$

Q5. a) i) $\angle BXC = 90^\circ$ (BX ⊥ to AC) = $\angle CYB$

$\angle ACB = \angle ABC$ (Base $\angle$'s of Isos Δ)

BC is comman

$\therefore \Delta BXC \equiv \Delta CYB$ (S.A.A) 3m

ii) In Δ's XTC & YTB,

$XC = BY$ (Cong Δ's from i)

$\angle TXC = \angle TYC$ (90°)

$\angle XTC = \angle YTB$ (V.O.A.)

$\therefore \Delta XTC \equiv \Delta YTB$ (S.A.A) 3m

$\therefore XT = YT$ (Corr. sides of cong Δ's)