

SOUTH SYDNEY HIGH SCHOOL
MATHEMATICS DEPARTMENT
YEAR 11 EXTENSION 1 TASK 1

72
74

Question 1:

- (a) Solve the following inequalities

(i) $|2x-1| < 4$

(ii) $|3x-5| \geq 4$

✶ (iii) $\frac{2}{|x-1|} < 1$

(iv) $2x^2 + 5x - 3 \geq 0$

- (b) A function is given by $g(x) = 2x^2 + 3x - 2$.

(i) Evaluate $g(-2)$

✶ (ii) For what values of x is $g(x) < 2x^2 + 1$?

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Question 2:

- (a) A function is defined by:

$$f(x) = \begin{cases} 3 & \text{for } x \leq -2 \\ x^2 - 1 & \text{for } -2 < x \leq 1 \\ x - 1 & \text{for } x > 1 \end{cases}$$

(i) Evaluate $f(2)$

(ii) Evaluate $f(-3)$

(iii) Sketch the above function

- ✶ (b) Determine if the following function is odd or even? (Give reasons)

$$f(x) = x^2 - 3x$$

- (c) Show the region where each inequality holds simultaneously.

✶ - $y > x^2$ and $x + y \leq 2$.

Question 3:

Sketch the following functions, showing all critical points.
State the domain and range in each case.

(a) $y = \sqrt{4 - x^2}$

(b) $y = \frac{3}{x-1}$

(c) $y = 2 - x^3$

(d) $y = 4x - x^2$

(e) $y = |x| - 2$

Question 4:

(a) Sketch $y = x^2 - 6$ and $y = |x|$ on the same set of axes.

(i) Find the x co ordinates of the points of intersection.

(ii) Hence solve $x^2 - 6 \leq |x|$

(b) Solve the following inequalities:

(i) $\frac{1}{x-2} \geq 3$

(ii) $|x-3| < |2x+1|$

(iii) $\frac{1}{x-4} \leq \frac{1}{2x}$

Question 5:

(a) For the following function $y = \frac{2x^2}{x^2 - 9}$

(i) Is it odd or even? (Give reasons)

(ii) Find any other asymptotes.

(iii) Find where it cuts the axes.

(iv) Hence sketch the curve.

(b) (i) Sketch the graph $y = |2x+4| + |x-1| - 5$

(iii) On the same number plane draw the lines $y = -1$ and $y = 2$.

Hence solve the inequation $-1 \leq |2x+4| + |x-1| - 5 \leq 2$

$$-1 \leq 4 + |x-1| - 5 \leq 2$$

$$3 + |x-1| \leq 2$$

$$6 - 2 + 0 - 1 - 5$$

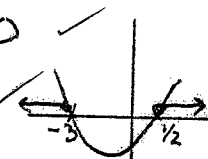
Question 1 (11 marks)

(a) (i) $|2x-1| < 4$
 $-4 < 2x-1 < 4$ ✓
 $-3 < 2x < 5$
 $-\frac{3}{2} < x < \frac{5}{2}$ ✓

(ii) $|3x-5| \geq 4$
 $3x-5 \geq 4$ or $3x-5 \leq -4$ ✓
 $3x \geq 9$ or $3x \leq 1$
 $x \geq 3$ or $x \leq \frac{1}{3}$ ✓

(iii) $\frac{2}{|x-1|} < 1$
 $2 < |x-1|$ ✓
 $x-1 > 2$ or $x-1 < -2$ ✓
 $x > 3$ or $x < -1$

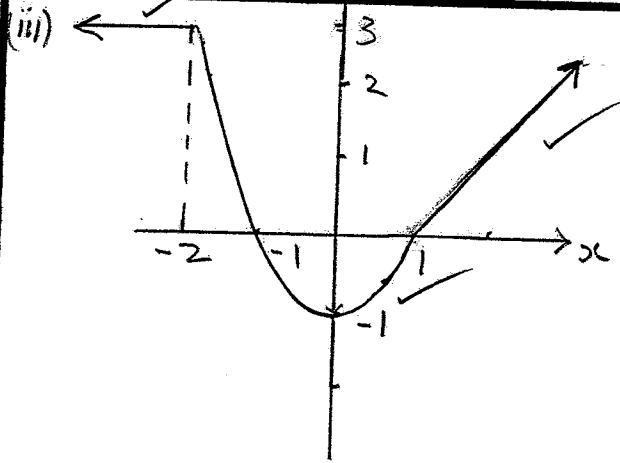
(iv) $2x^2 + 5x - 3 \geq 0$
 $(2x-1)(x+3) \geq 0$ ✓
 $x \geq \frac{1}{2}$ or $x \leq -3$ ✓



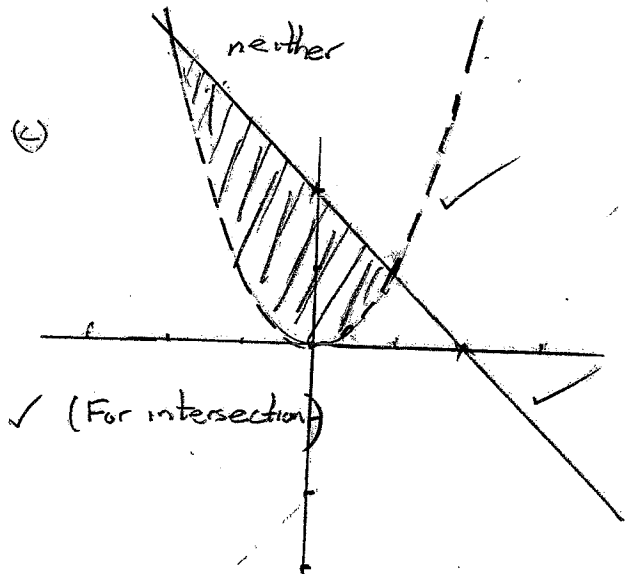
(b) $g(x) = 2x^2 + 3x - 2$
 (i) $g(-2) = 2(-2)^2 + 3(-2) - 2$ ✓
 $= 8 - 6 - 2$
 $= 0$
 (ii) $g(x) < 2x^2 + 1$
 $2x^2 + 3x - 2 < 2x^2 + 1$ ✓
 $3x - 3 < 0$ ✓
 $3x < 3$
 $x < 1$

Question 2 (9 marks)

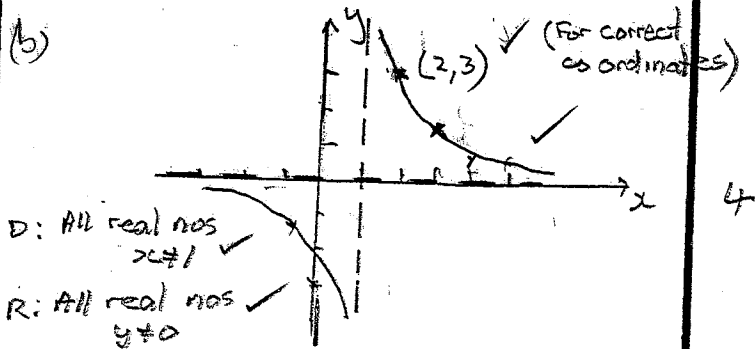
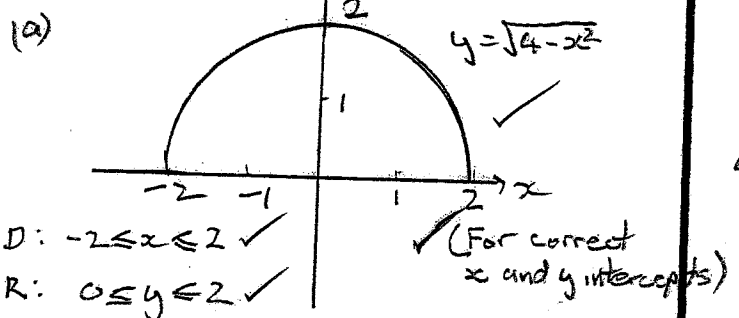
(a) (i) $f(2) = 2 - 1$
 $= 1$ ✓
 (ii) $f(-3) = 3$ ✓



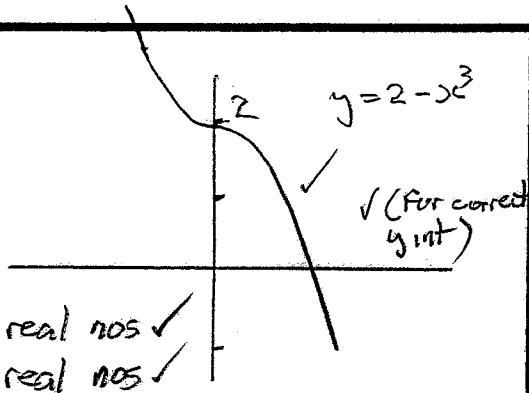
(b) $f(x) = x^2 - 3x$
 $f(-x) = (-x)^2 - 3(-x)$
 $= x^2 + 3x$ ✓



Question 3



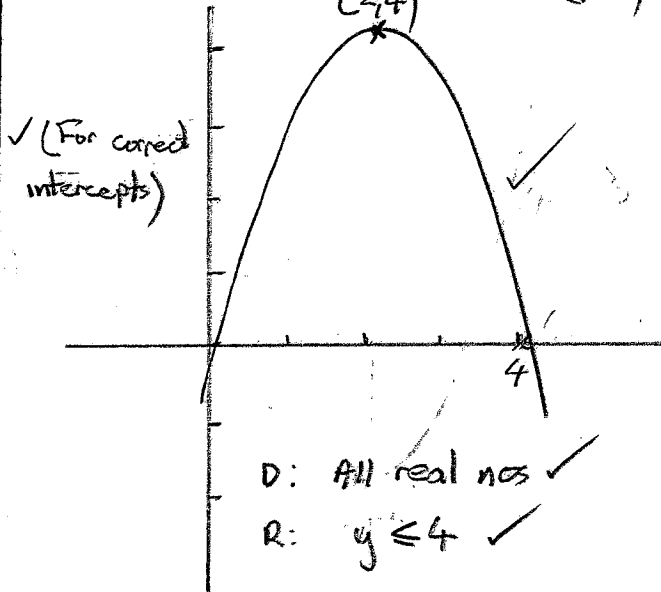
(c)



D: All real nos ✓
R: All real nos ✓

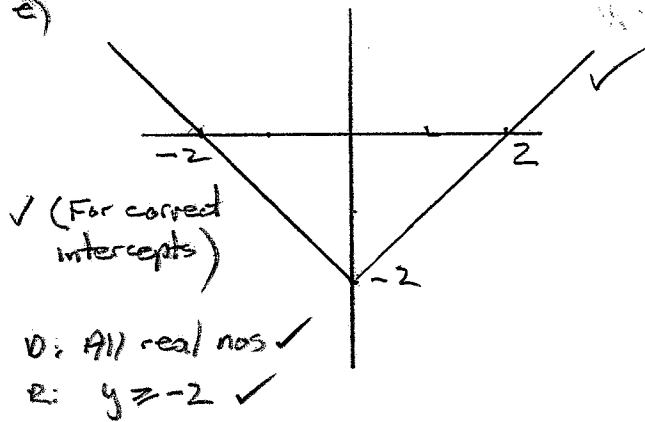
d)

$$y = 4x - x^2 = x(4-x)$$



D: All real nos ✓
R: $y \leq 4$ ✓

e)

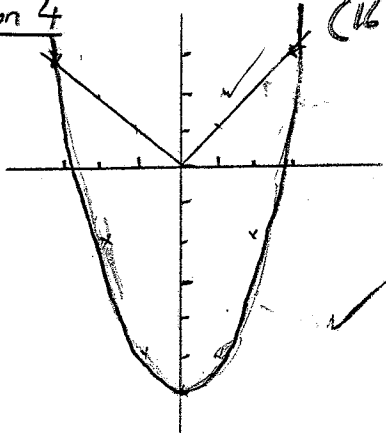


D: All real nos ✓
R: $y \geq -2$ ✓

Question 4

(16 marks)

(4)



(i)

$$y = x^2 - 6 \quad \text{--- (1)}$$

$$y = x \quad \text{--- (2)}$$

$$x^2 - 6 = x$$

1st QUAD

$$x^2 - x - 6 = 0$$

$$(x-3)(x+2) = 0$$

$$x = 3 \text{ or } x = -2 \quad \checkmark$$

$$y = x^2 - 6 \quad \text{--- (1)}$$

$$y = -x \quad \text{--- (2)}$$

2nd QUAD

$$x^2 + x - 6 = 0$$

$$(x+3)(x-2) = 0$$

$$x = -3 \text{ or } x = 2 \quad \checkmark$$

(ii)

$$x^2 - 6 \leq |x|$$

$$-3 \leq x \leq 3 \quad \checkmark \checkmark$$

(b) (i) $\frac{1}{x-2} \geq 3$

$$(x-2) \geq 3(x-2)^2 \quad \checkmark \left\{ \begin{array}{l} \text{mult both} \\ \text{sides by } (x-2)^2 \end{array} \right.$$

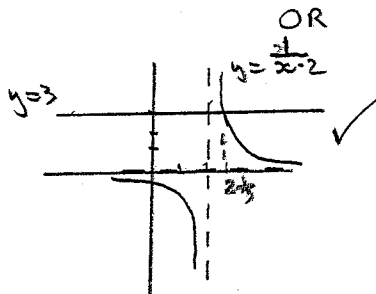
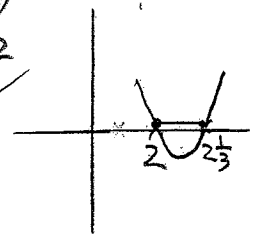
$$x-2 \geq 3(x^2 - 4x + 4)$$

$$x-2 \geq 3x^2 - 12x + 12$$

$$3x^2 - 13x + 14 \leq 0 \quad \checkmark$$

$$(3x-7)(x-2) \leq 0$$

$$2 < x \leq 2\frac{1}{3} \quad \checkmark$$



OR

$$y = \frac{1}{x-2}$$

$$y = 3$$

$$\frac{1}{x-2} = 3$$

$$3x - 6 = 1$$

$$3x = 7$$

$$x = 7/3 \quad \checkmark$$

From graph:
 $2 < x \leq 2\frac{1}{3} \quad \checkmark$

(ii) $|x-3| < |2x+1|$

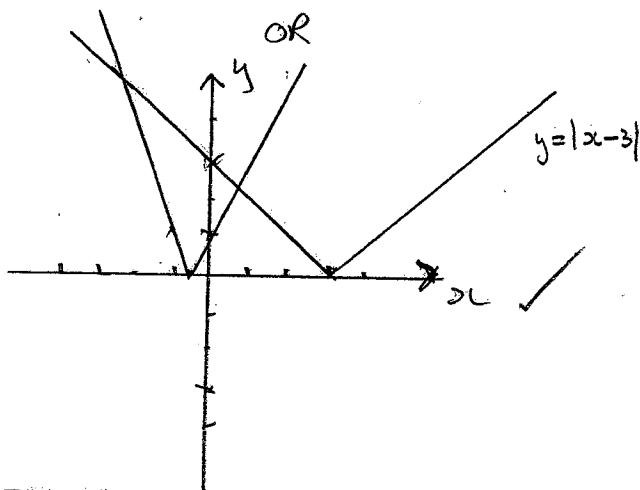
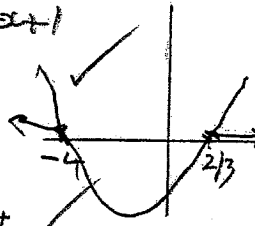
$(x-3)^2 < (2x+1)^2$ ✓
 {square both sides}

$x^2 - 6x + 9 < 4x^2 + 4x + 1$

$3x^2 + 10x - 8 > 0$

$(3x-2)(x+4) > 0$

$x > \frac{2}{3}$ or $x < -4$



1st QUAD

$y = 2x+1$ — (1)

$y = -x+3$ — (2)

$2x+1 = -x+3$

$3x = 2$

$x = \frac{2}{3}$

✓ $\frac{1}{2}$

2ND QUAD

$y = -2x-1$ — (1)

$y = -x+3$ — (2)

$-2x-1 = -x+3$

$-x = 4$

$x = -4$

✓ $\frac{1}{2}$

From graph: $x > \frac{2}{3}$ or $x < -4$

(iii) $\frac{1}{x-4} \leq \frac{1}{2x}$

$4x^2(x-4) \leq 2x(x-4)$ ✓

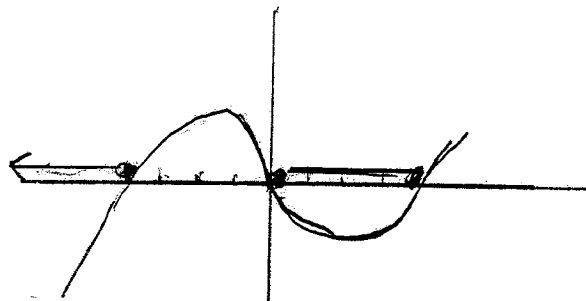
$4x^2(x-4) - 2x(x-4) \leq 0$ ✓

$2x(x-4)\{2x - (x-4)\} \leq 0$ ✓

$2x(x-4)(2x-x+4) \leq 0$ ✓

$2x(x-4)(x+4) \leq 0$ ✓

✓ mult both sides by $4x^2(x-4)$



From graph: $x < -4$ or $0 < x < 4$

Question 5: (18 marks)

(a) (i) $f(x) = \frac{2x^2}{x^2-9}$

$f(-x) = \frac{2(-x)^2}{(-x)^2-9}$ ✓

$= \frac{2x^2}{x^2-9}$

$= f(x)$ ✓

∴ even function ✓

(ii) $y = \frac{2x^2}{(x-3)(x+3)}$

∴ vertical asymptotes $x=3$ and $x=-3$ ✓

horizontal asymptotes:

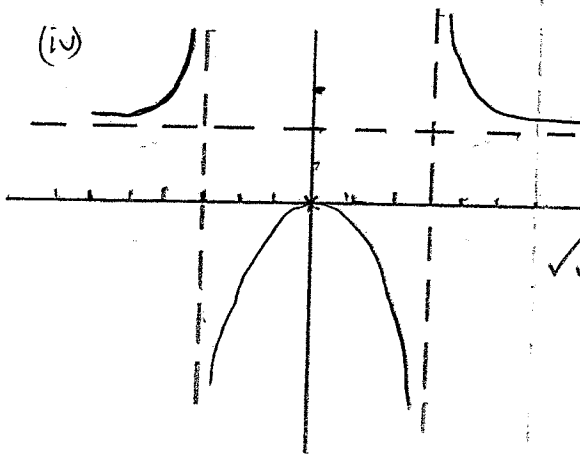
$\lim_{x \rightarrow \infty} \frac{2x^2}{x^2-9} = \lim_{x \rightarrow \infty} \frac{2x^2/x^2}{x^2/x^2 - 9/x^2}$ ✓

$= \lim_{x \rightarrow \infty} \frac{2}{1 - 9/x^2}$

$= 2$

∴ $y=2$ horizontal asymptote ✓

(iii) when $x=0$ $y=0$ ✓



2
2
4
1
3

$$b) y = |2x+4| + |x-1| - 5$$

$$|2x+4| = 2x+4 \quad \text{if } 2x+4 \geq 0$$

$$|2x+4| = -(2x+4) \quad \text{if } x \leq -2$$

$$|x-1| = x-1 \quad \text{if } x \geq 1$$

$$|x-1| = -(x-1) \quad \text{if } x < 1$$

∴ For $x \geq -2$

$$y = (2x+4) + x-1 - 5$$

$$y = 3x-2 \quad \checkmark$$

For $-2 \leq x < 1$

$$y = 2x+4 - (x-1) - 5$$

$$= 2x+4 - x+1 - 5$$

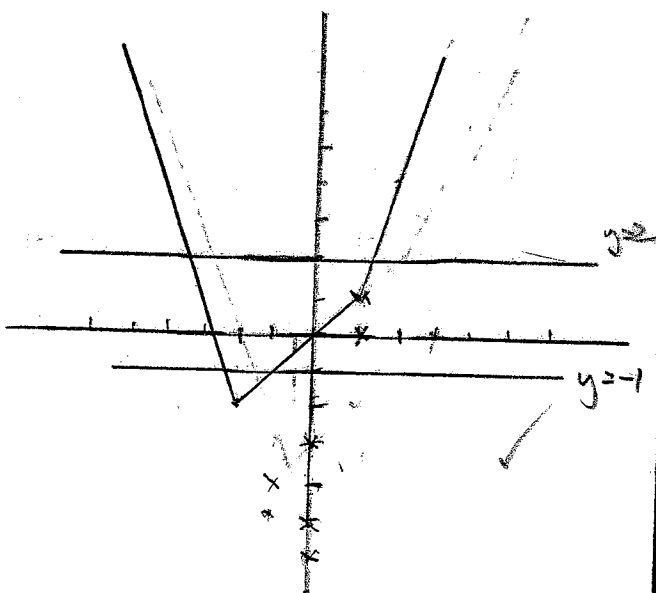
$$y = x \quad \checkmark$$

For $x < -2$

$$y = -(2x+4) - (x-1) - 5$$

$$= -2x-4 - x+1 - 5$$

$$y = -3x-8 \quad \checkmark$$



1st Quad:

$$y = 3x-2 \quad \text{--- (1)}$$

$$y = 2 \quad \text{--- (2)}$$

$$3x-2 = 2$$

$$3x = 4$$

$$x = \frac{4}{3} \quad \checkmark \frac{1}{2}$$

2nd Quad:

$$y = -3x-8 \quad \text{--- (1)}$$

$$y = 2$$

$$-3x-8 = 2$$

$$-3x = 10$$

$$x = -\frac{10}{3} \quad \checkmark \frac{1}{2}$$

3rd Quad:

$$y = -3x-8 \quad \text{--- (1)}$$

$$y = -1 \quad \text{--- (2)}$$

$$-3x-8 = -1$$

$$-3x = 7$$

$$x = -\frac{7}{3} \quad \checkmark \frac{1}{2}$$

3rd Quad:

$$y = x \quad \text{--- (1)}$$

$$y = -1 \quad \text{--- (2)}$$

$$\therefore x = -1 \quad \checkmark \frac{1}{2}$$

From graph:

$$-1 \leq |2x+4| + |x-1| - 5 \leq 2$$

$$-\frac{10}{3} \leq x \leq -\frac{7}{3} \quad \text{and}$$

$$-1 \leq x \leq \frac{4}{3} \quad \checkmark \checkmark$$