S.S.H.S. – YEAR 11 ASSESSMENT TASK 3 – 2006

EXTENSION 1

Question 1: (12 marks) **Marks**

(a) The points $A(-3,2)$ and $B(5,8)$ lie on a number plane.

(i) Find the equation of AB in general form. 2

(ii) Find the midpoint of AB. 1

(iii) Find the length of AB. 1

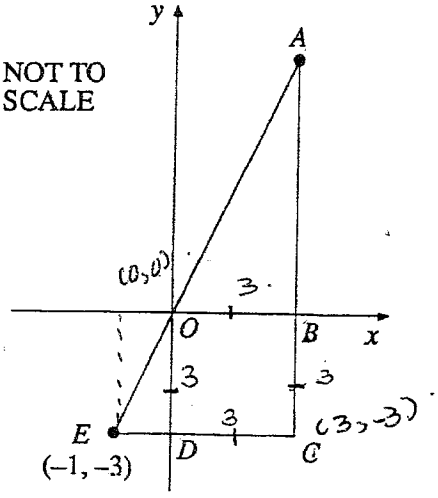
(iv) Show that the equation of the circle with diameter AB is $x^2 + y^2 - 2x - 10y + 1 = 0$. 1

(v) Prove that the point $C(1,10)$ lies on this circle. 1

(vi) Prove that $AC \perp CB$. 2

(vii) Find the coordinates of the point which divides the join of $(5,3)$ and $(1,-3)$ externally in the ratio $3:2$. 2

(b) 2



The line AE passes through the origin.

AC and EC cut the axes at B and D respectively.

$OBCD$ is a square.

E has coordinates $(-1,-3)$.

Find the coordinates of A?

Question 2: (12 marks)

(a) Prove that the lines $12x - 5y + 3 = 0$ and $24x - 10y - 7 = 0$ are parallel. 2

(b) The point $A(1,k)$ lie on the line $12x - 5y + 3 = 0$. Find the value of k. 1

(c) Hence find the perpendicular distance between the lines $12x - 5y + 3 = 0$ and $24x - 10y - 7 = 0$. 2

(d) Find the equation of the line that passes through the point of intersection of the lines $x - 2y + 3 = 0$ and $3x + y - 7 = 0$, and the point $(5,-2)$. Express your answer in the general form. 3

(e) In what ratio does the line $3x + y - 17 = 0$ intersect the interval joining $(-4,2)$ and $(6,8)$? 4

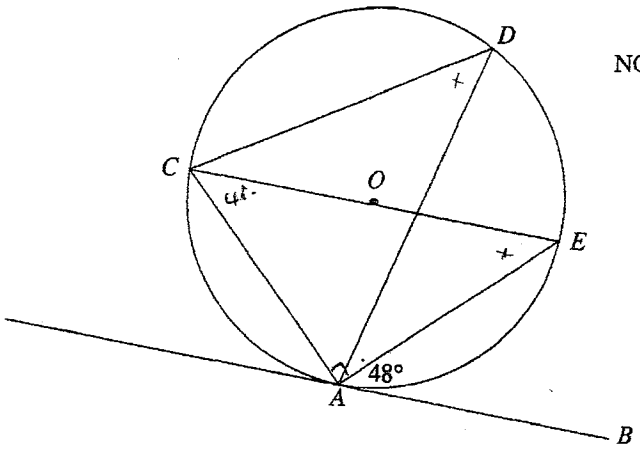
$y = -3x + 17$

Question 3: (12 marks)

(a) Find the acute angle between the lines $3x - y + 4 = 0$ and $3x - 11y + 44 = 0$. 4

(b) What are the two possible gradients of lines that make an angle of 45° with the line $2x - y + 6 = 0$? 4

(c)



NOT TO SCALE

AB is a tangent and CE is a diameter to a circle of centre O. $\angle BAE = 48$ and D lies on the circumference as shown in the diagram.

(i) Copy the diagram and find the size of angle ACE, giving reasons. 1

(ii) Find the size of angle ADC. Justify your answer. 3

Question 4: (12 marks)

Differentiate and simplify the following:

(a) $3x^2 - 7x + 8$. 1

(b) $\left(5 - 7x^3\right)^4$. 1

(c) $(x^3 - 7x)(3x^2 + 12)$. 2

(d) $\dfrac{3x-5}{2x+3}$. 2

(e) $\left(\sqrt[3]{x}\right)^5$ (Leave answer in surd form) 2

(f) $\dfrac{1}{x\sqrt{x}}$ (Leave answer in surd form) 2

(g) $\dfrac{5x^3 + 3x^2 - 4}{x}$. (Do not use quotient rule) 2

Question 5: (12 marks)

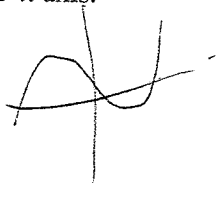
Find:

(a) $\lim\limits_{x \to 2} \dfrac{x^2 + x - 6}{x - 2}$ 2

(b) $\lim\limits_{x \to \infty} \dfrac{3x^2 - 5x + 1}{2x^2 + 3x - 5}$ 2

(c) Find the equation in general form, of the tangent to the parabola $y = \dfrac{1}{3}x^2$ at the point $\left(2, \dfrac{4}{3}\right)$ 4

(d) Find the coordinates of the points of contact of the tangents to the curve $y = x^3 - 2x + 3$, Which have an angle of inclination of 45° to the positive direction of the x axis. 4



ASSESSMENT TASK 3 SOLUTIONS 24/8/06

Question 1

(a) (i) $\frac{y-y_1}{x-x_1} = \frac{y_2-y_1}{x_2-x_1}$ $A(-3,2)$ $B(5,8)$

$\frac{y-2}{x+3} = \frac{6}{8}$ ✓

$8(y-2) = 6(x+3)$

$8y - 16 = 6x + 18$

$6x - 8y + 34 = 0$ ✓

$3x - 4y + 17 = 0$ 2

(ii) $M = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$= \left(\frac{2}{2}, \frac{10}{2}\right)$

$= (1,5)$ ✓ 1

(iii) $d = \sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$

$= \sqrt{64+36}$

$= 10$ ✓ 1

(iv) Centre $= (1,5)$

Radius $= 5$

∴ Required eqn is:

$(x-1)^2 + (y-5)^2 = 25$ ✓

$x^2 + y^2 - 2x - 10y + 1 = 0$ 1

(v) $(x-1)^2 + (y-5)^2 = 25$

Subst $(1,10)$: $0^2 + 5^2 = 25$ ✓

$25 = 25$

Satisfies eqn

∴ $(1,10)$ lies on circle 1

(vi) $m_{AC} = \frac{y_2-y_1}{x_2-x_1}$ $C = (1,10)$

$= \frac{8}{4}$ ✓

$= 2$ ½ $\therefore m_{AC} \times m_{CB} = -1$

$\therefore$ perp

$m_{CB} = \frac{y_2-y_1}{x_2-x_1}$

$= -\frac{2}{4} = -\frac{1}{2}$ ✓ ½ 2

(vii) $\frac{mx_2+nx_1}{m+n}$ $\frac{my_2+ny_1}{m+n}$ ✓ ✓

$(x_1,y_1) = (5,3)$

$(x_2,y_2) = (1,-3)$

$m = 3$

$n = -2$

$\left[\frac{3(1)+(-2)(5)}{1}, \frac{3(-3)+(-2)(3)}{1}\right]$

$[-7, -15]$

(b) $m_{AC} = \frac{\text{rise}}{\text{run}}$

$= 3$

∴ Eqn of AC: $y = 3x$ ✓

C has x coordinate of 3

∴ A has x coordinate of 3

when $x=3$: $y = 3 \times 3 = 9$ ✓

∴ Coords of A are $(3,9)$

Question 2

(a) $12x - 5y + 3 = 0$

$5y = 12x + 3$

$y = \frac{12}{5}x + \frac{3}{5}$

$\therefore m_1 = \frac{12}{5}$ ½

$24x - 10y - 7 = 0$

$10y = 24x - 7$

$y = \frac{24}{10}x - \frac{7}{10}$

$m_2 = \frac{24}{10} = \frac{12}{5}$ ½

$m_1 = m_2 \therefore$ parallel ✓

(b) $12x - 5y + 3 = 0$

Sub $(1,k)$: $12 - 5k + 3 = 0$ ✓

$5k = 15$

$k = 3$

c) $d = \dfrac{|ax_1 + by_1 + c|}{\sqrt{a^2 + b^2}}$

$= \dfrac{|24(1) + (-10)(3) - 7|}{\sqrt{24^2 + 10^2}}$ $\quad a \quad b \quad c$ $\quad 24x - 10y - 7 = 0$ $\quad (x_1, y_1)$ $\quad A(1,3)$

$= \dfrac{|24 - 30 - 7|}{26}$

$= \dfrac{13}{26}$

$= \underline{\underline{\dfrac{1}{2}}}$

2

(d) Reqd eqn is:

$x - 2y + 3 + k(3x + y - 7) = 0$ ✓

Subst $(5,-2)$: $5 + 4 + 3 + k(15 - 2 - 7) = 0$

$12 + 6k = 0$

$6k = -12$

$k = -2$ ✓

$\therefore x - 2y + 3 - 2(3x + y - 7) = 0$

$x - 2y + 3 - 6x - 2y + 14 = 0$ ✓

$-5x - 4y + 17 = 0$

$\underline{\underline{5x + 4y - 17 = 0}}$

(e) Substitute the point which divides the join of $(-4,2)$ and $(6,8)$ in the ratio $m_1 : m_2$ into the eqn $3x + y - 17 = 0$

$3\left(\dfrac{m_1 \times 6 + m_2 \times -4}{m_1 + m_2}\right) + 4\left(\dfrac{m_1 \times 8 + m_2 \times 2}{m_1 + m_2}\right) - 17 = 0$ ✓

$3(6m_1 - 4m_2) + 4(8m_1 + 2m_2) - 17(m_1 + m_2) = 0$

$18m_1 - 12m_2 + 8m_1 + 2m_2 - 17m_1 - 17m_2 = 0$ ✓

$9m_1 = 27m_2$

$\dfrac{m_1}{m_2} = \dfrac{3}{1}$ ✓

Ratio is $3:1$ ✓

Alternative Solution:

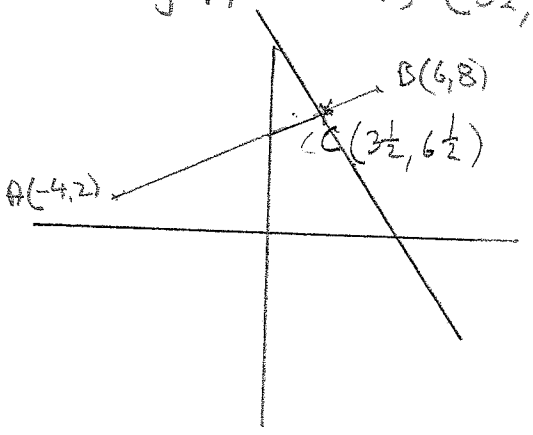
Equation of join of $(-4,2)$ and $(6,8)$ is:

$3x - 5y + 22 = 0$ ✓

Point of intersection of

$3x - 5y + 22 = 0$ and

$3x + y - 17 = 0$ is $(3\frac{1}{2}, 6\frac{1}{2})$ ✓



4

AC : CB = ratio of difference in x values

$= 7\frac{1}{2} : 2\frac{1}{2}$ ✓

$= \underline{3:1}$ ✓

Question 3

(a) $3x - y + 4 = 0$

$y = 3x + 4$

$m_1 = 3$ ✓

$3x - 11y + 44 = 0$

$11y = 3x + 44$

$y = \dfrac{3}{11}x + 4$

$m_2 = \dfrac{3}{11}$ ✓

$\tan\theta = \dfrac{m_1 - m_2}{1 + m_1 m_2}$

$= \dfrac{3 - 3/11}{1 + 9/11} = \dfrac{3}{2}$ ✓

$\underline{\theta = 56^\circ 19'}$ ✓

4

(b) Let the gradient be m

$$\tan\theta = \left|\frac{m_1 - m_2}{1 + m_1 m_2}\right|$$

$$1 = \left|\frac{m-2}{1+2m}\right| \checkmark$$

$$1 = \frac{|m-2|}{|1+2m|} \checkmark$$

$$|1+2m| = |m-2|$$

$$1 + 2m = m - 2 \quad \text{or} \quad 1+2m = -(m-2)$$

$m = -3$ ✓ or $1+2m = -m+2$

$3m = 1$

$m = \frac{1}{3}$ ✓

c) $\frac{d}{dx}(x^3 - 7x)(3x^2 + 12) = uv' + vu'$

$= (x^3 - 7x)6x + (3x^2+12)(3x^2-7)$ ✓ 2

$= 6x^4 - 42x^2 + 9x^4 - 21x^2 + 36x^2 - 84$ ✓

$= \underline{15x^4 - 27x^2 - 84}$

(d) $\frac{d}{dx}\left(\frac{3x-5}{2x+3}\right)_v^u = \frac{vu' - uv'}{v^2}$

$= \frac{(2x+3)3 - (3x-5)2}{(2x+3)^2}$ ✓

$= \frac{6x + 9 - 6x + 10}{(2x+3)^2}$ ✓ 2

$= \underline{\frac{19}{(2x+3)^2}}$

(e) $\frac{d}{dx}(\sqrt[3]{x})^5 = \frac{d}{dx}(x^{5/3})$ ✓

$= \frac{5}{3}x^{2/3}$ ✓½ 2

$= \underline{\frac{5}{3}\sqrt[3]{x^2}}$ ✓½

(f) $\frac{d}{dx}\left(\frac{1}{x\sqrt{x}}\right) = \frac{d}{dx}(x^{-3/2})$ ✓

$= -\frac{3}{2}x^{-5/2}$ ✓½ 2

$= \underline{\frac{-3}{2\sqrt{x^5}}}$ ✓½

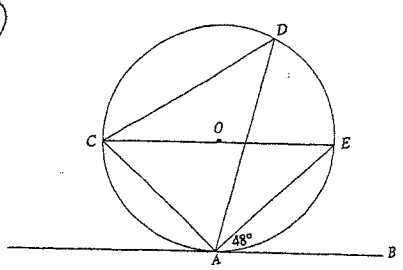
(g) $\frac{d}{dx}\left(\frac{5x^3 + 3x^2 - 4}{x}\right) = \frac{d}{dx}\left(5x^2 + 3x - \frac{4}{x}\right)$ ✓

$= \frac{d}{dx}(5x^2 + 3x - 4x^{-1})$ ½

$= 10x + 3 + 4x^{-2}$ ½

$= \underline{10x + 3 + \frac{4}{x^2}}$ 2

4)



i) $\angle ACE = 48$ (Angle between tangent & chord equal to angle in alt segment) ✓

(ii) $\angle CBE = 90$ (Angle in semi circle) ✓

$\angle AEC = 42$ (Angle sum of $\triangle$) ✓

$\therefore \angle ADC = 42$ (Angles in same segment) ✓

Question 4

(a) $\frac{d}{dx}(3x^2 - 7x + 8) = \underline{6x - 7}$ ✓

(b) $\frac{d}{dx}(5 - 7x^3)^4 = 4(5-7x^3)^3 \cdot (-21x^2)$

$= \underline{-84x^2(5-7x^3)^3}$ ✓

Question 5

(a) $\lim_{x\to 2} \dfrac{x^2+x-6}{x-2} = \lim_{x\to 2} \dfrac{(x+3)(x-2)}{(x-2)}$ ✓

$= \lim_{x\to 2} x+3$ ✓

$= \underline{5}$

(b) $\lim_{x\to\infty} \dfrac{3x^2-5x+1}{2x^2+3x-5} = \lim_{x\to\infty} \dfrac{3x^2/x^2 - 5x/x^2 + 1/x^2}{2x^2/x^2 + 3x/x^2 - 5/x^2}$ ✓

$= \lim_{x\to\infty} \dfrac{3 - 5/x + 1/x^2}{2 + 3/x + 5/x^2}$ ✓

$= \underline{\dfrac{3}{2}}$

(c) $y = \frac{1}{3}x^2$

$y' = \frac{2}{3}x$ ✓

when $x=2$ $\quad y' = \frac{4}{3}$

$\therefore m_T = \frac{4}{3}$ ✓

Reqd eqn is:

$y - y_1 = m(x - x_1)$

$y - \frac{4}{3} = \frac{4}{3}(x-2)$ ✓

$3y - 4 = 4x - 8$

$\underline{4x - 3y - 4 = 0}$ ✓

(d) $y = x^3 - 2x + 3$ ✓

$y' = 3x^2 - 2$ ✓

but $m_T = 1$ ✓ given tan45° = 1

$3x^2 - 2 = 1$ ✓

$3x^2 = 3$

$x^2 = 1$

$x = \pm 1$

$\therefore$ Points of contact are

$\underline{(1,2) \text{ and } (-1,4)}$ ✓ 4