South Sydney High School

Extension I Mathematics

Trigonometry Task 3

Term: Term 3 Week 6	**Date:** Monday 23rd August 2010
Time allowed: 50 minutes	**Assessment:** Extension I Mathematics

Question 1:

(a) I. Prove that $\sin\theta\sec\theta = \tan\theta$ **1**

II. Hence solve $\sin\theta\sec\theta = \sqrt{3}$. $0 \le x \le 360$ **1**

(b) i. Show that $\sin x - \cos 2x = 2\sin^2 x + \sin x - 1$ **2**

II. Hence or otherwise solve $\sin x - \cos 2x = 0$ for for $0 \le x \le 360$ **3**

Question 2:

(a) I. Expand $\sin(\alpha+\beta)$. **1**

II. Hence or otherwise find the exact value of $\sin 105^\circ$ **2**

(b) I. Show that $\dfrac{1+\cos 2A}{\sin 2A} = \cot A$ **2**

II. Hence find the exact value of $\cot 15^\circ$ **2**

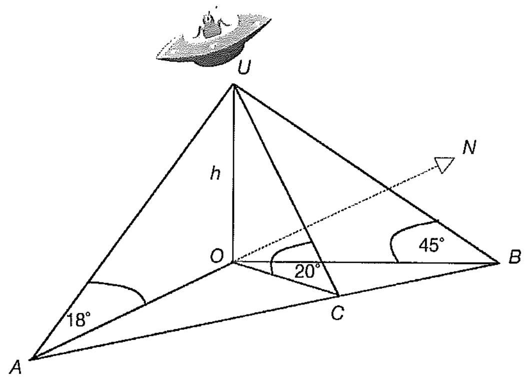
Question 3:

(a) i. Express $8\cos x - 6\sin x$ in the form $R\cos(x+\alpha)$, where R and α are constants. **3**

(Give α correct to the nearest minute.)

ii. Hence find, correct to the nearest degree, the two angles between 0° and 360° that satisfy the equation $8\cos x - 6\sin x = 3$. **2**

(b)



An Undentified Flying Object (UFO) at *U*, due north of an observer *A* is sighted at an angle of 18°. At the same time, another observer *B* due east of the UFO sighted it at an angle of 45°. A third observer *C* in the line of sight between *A* and *B* also sighted the UFO at an angle of 20°.

i. Find *OA*, *OB* and *OC* in terms of h. **3**

ii. Find $\angle OBC$ and $\angle OCB$. (Answer to the nearest minute). **3**

iii. Hence or otherwise, find the bearing of *C* from *O*. **1**

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Question 1:

(a) i. Prove that $\sin\theta\sec\theta=\tan\theta$ **1**

ii. Hence solve $\sin\theta\sec\theta=\sqrt{3}$. $0\le x\le 360$ **1**

i. $\sin\theta\sec\theta=\sin\theta.\frac{1}{\cos\theta}$

$=\tan\theta$ ✓

ii. $\sin\theta\sec\theta=\sqrt{3}$

ie $\tan\theta=\sqrt{3}$

$\therefore\theta=60^\circ,\ 240^\circ$ ✓

(b) i. Show that $\sin x-\cos 2x=2\sin^2 x+\sin x-1$ **2**

ii. Hence or otherwise solve $\sin x-\cos 2x=0$ for for $0\le x\le 360$ **3**

i.

$\sin x-\cos 2x=\sin x-\left(1-2\sin^2\right)$ ✓

$=\sin x-1+2\sin^2 x$

$=2\sin^2 x+\sin x-1$ ✓

ii.

$\sin x-\cos 2x=0$

$2\sin^2 x+\sin x-1=0$

$2\sin^2 x+2\sin x-\sin x-1=0$ ✓

$2\sin x(\sin x+1)-(\sin x+1)=0$

$(2\sin x-1)(\sin x+1)=0$

So $\sin x=\frac{1}{2}$ or $\sin x=-1$ ✓

$x=30^\circ,\ 150^\circ,\ 270^\circ$ ✓

Question 2:

(a) i. Expand $\sin(\alpha+\beta)$. **1**

ii. Hence or otherwise find the exact value of $\sin 105^\circ$ **2**

i. $\sin(\alpha+\beta)=\sin\alpha\cos\beta+\cos\alpha\sin\beta$ ✓

ii. $\sin 105^\circ=\sin(60+45)$

$=\sin 60\cos 45+\cos 60\sin 45$ ✓

$=\frac{\sqrt{3}}{2}\times\frac{1}{\sqrt{2}}+\frac{1}{2}\times\frac{1}{\sqrt{2}}$

$=\frac{\sqrt{3}+1}{2\sqrt{2}}$ ✓

(b) i. Show that $\frac{1+\cos 2A}{\sin 2A}=\cot A$ **2**

ii. Hence find the exact value of $\cot 15^\circ$ **2**

i LHS=$\frac{1+2\cos^2A-1}{2\sin A\cos A}$ ✓

$=\frac{2\cos^2A}{2\sin A\cos A}$

$=\cot A=\text{RHS}$ ✓

ii sub $A=15^0$

$\cot 15^0=\frac{1+\cos 30^0}{\sin 30^0}$ ✓

$=\frac{1+\frac{\sqrt{3}}{2}}{\frac{1}{2}}$

$=\frac{2+\sqrt{3}}{2}\times 2$

$=2+\sqrt{3}$ ✓

Question 3:

(a) i. Express $8\cos x - 6\sin x$ in the form $R\cos(x+\alpha)$, where R and α are constants. **3**

(Give α correct to the nearest minute.)

ii. Hence find, correct to the nearest degree, the two angles between 0^o and 360^o that satisfy the equation $8\cos x - 6\sin x = 3$. **2**

i. $R=\sqrt{8^2+6^2}=10$ ✓

Let $8\cos x - 6\sin x = 10\cos(x+\alpha)$

$\therefore 8\cos x - 6\sin x = 10\cos\alpha\cos x - 10\sin\alpha\sin x$

$\therefore 10\cos\alpha = 8$ and $10\sin\alpha = 6$

$\therefore \cos\alpha = \frac{4}{5}$ and $\sin\alpha = \frac{3}{5}$ ✓

$\therefore \alpha = 36^o52'$

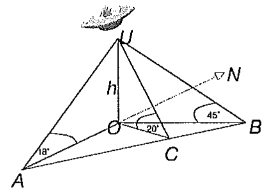
$\therefore 8\cos x - 6\sin x = 10\cos(x+36^o52')$ ✓

ii. $\cos(x+36^o52') = \frac{3}{10}$ ✓

$\therefore x+36^o52' = 72^o33', 360^o-72^o33'$ ✓

$\therefore x \approx 36^o, 251^o$

(b)



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i. Find *OA, OB* and *OC* in terms of *h*. **3**

ii. Find $\angle OBC$ and $\angle OCB$. (Answer to the nearest minute). **3**

iii. Hence or otherwise, find the bearing of *C* from *O*. **1**

i. In ΔAOU, $\tan 18^0 = \frac{h}{OA}$

$\therefore OA = h\tan 18^0$ ✓

In ΔCOU, $\tan 20^0 = \frac{h}{OC}$

$\therefore OC = h\tan 20^0$ ✓

Since ΔBOU is isosceles, $\therefore OB = h$ ✓

ii $\tan\angle OBC = \frac{OA}{OB}$ (since $\angle AOB = 90^0$)

$= \frac{\not{h}\cot 18^0}{\not{h}} = \cot 18^0$

$\therefore \angle OBC = 72^o$ ✓

Using sine rule in ΔOBC,

$\frac{\sin\angle OCB}{h} = \frac{\sin\angle OBC}{h\cot 20^0}$ ✓

$\sin\angle OCB = \frac{\sin\angle OBC}{\not{h}\cot 20^0}\times\not{h}$

$= \frac{\sin 72^o}{\cot 20^o} = 0.3462$

$\therefore \angle OCB = 20^015'$ ✓

iii. $\angle COB = 180^o - 20^o15' - 72^o$

$= 87^o45'$

$\therefore$ bearing is $90^o + 87^o45'$

$= 177^045'$ T ✓