

Question 1 (10 marks)**Marks**

- (a) The velocity of a particle after t sec is $\left(64 - \frac{t^2}{3}\right)$ m/s.

Find : (i) its initial velocity. 1

(ii) the time at which the particle reverses its direction of motion. 1

(iii) its distance from the starting point at that instant. 2

(iv) the distance travelled in the first 10 seconds. 2

- (b) A tank is to be emptied by means of a control valve. The valve operates so that V litres, the volume of fluid remaining in the tank, varies with time t , measured in minutes, according to the relation 4

$$\frac{dV}{dt} = -kt, \text{ where } k \text{ is a constant.}$$

- (i) Initially the tank contains 5000 litres of fluid. Show that after t minutes

$$V = 5000 - \frac{1}{2}kt^2$$

- (ii) If $k = 1.44$, at what rate will the tank be emptying when $V = 2000$?

- (iii) Find the time it takes to completely empty the tank ?
(Give your answer to the nearest minute).

Continue next page

Question 2 (10 marks)**Marks**

A particle P moves along a horizontal straight line so that its displacement, x m from a fixed point O , t seconds after motion has begun, is given by

$$x = 28 + 4t - 5t^2 - t^3.$$

- (a) Obtain expressions, in terms of t , for the velocity and acceleration of P , and 2
- (b) state the initial velocity and the initial acceleration of P . 2

A second particle Q moves along the same horizontal straight line as P and starts from O at the same instant that P begins to move. The initial velocity of Q is 2 ms^{-1} and its acceleration, $a \text{ ms}^{-2}$, t seconds after motion has begun, is given by

$$a = 2 - 6t.$$

Find : 6

- (c) the value of t at the instant when P and Q collide and
- (d) determine whether or not P and Q are travelling in the same direction.

Question 3 (11 marks)

- (a) The population of a town over t years is given by the formula 2

$$P = P_0 e^{0.0124t} \quad \text{where } P_0 \text{ is its initial population}$$

How long will it take to **triple** its initial population (to the nearest year).

- (b) The letters of the word **AROUND** are written at random on the circumference of a circle.

(i) How many different permutations are possible? 1

(ii) What is the probability that the three vowels are together? 2

- (c) The rate of change of temperature of an object T° is given by

$$\frac{dT}{dt} = k(T - 16) \text{ degrees/min where } k \text{ is a constant.}$$

(i) Prove that the function $T = 16 + Pe^{kt}$, where P is a constant and t the time in minutes, satisfies this condition. 1

(ii) If, initially $T = 0$, and after 10 minutes, $T = 12$, find the values of P and e^{10k} . 2

(iii) Find the temperature of the object after a further 5 minutes. 1

(iv) Sketch a graph of T as function of t and describe its behaviour as t continues to increase. 2

Continue next page

Question 4 (9 marks)**Marks**

- (a) How many ways are there of arranging all the letters of the word
PARRAMATTA ? **1**
- (b) A sphere is increasing in volume at a rate of 20π cm³/sec. **3**
Given that the volume of a sphere is given by $\frac{4}{3}\pi r^3$, calculate
the radius of the sphere at the instant when the radius is increasing
at the rate of 0.2 cm/s .
- (c) A class of twenty pupils consists of 12 girls and 8 boys. **5**
For a discussion session four "officers" are to be chosen at random
as "Chairman", "Recorder", "Proposer" and "Opposer".
Find, giving your answers correct to three significant figures.
- (i) the probability that all four officers are girls
 - (ii) the probability that two officers are girls and two are boys.
 - (iii) the probability that the Proposer and Opposer are both girls
 - (iv) the probability that the Proposer and Opposer are of opposite sex
given that the Chairman and Recorder are both girls.

End of assessment task

Question 1 (10 Marks)

- (a) (i) $V = 64 \text{ m/s}$ ✓
 (ii) $t = 8\sqrt{3} \text{ s.}$ ✓
 (iii) $x = 591 \text{ m (to the nearest metre).}$ ✓✓
 (iv) $x = 529 \text{ m}$ ✓✓

- (b) (i) Proof ✓

(ii) $\frac{dV}{dt} = -92.95 \text{ L/min}$ ✓✓

- (iii) $t = 83 \text{ min (to the nearest min)}$ ✓

Question 2 (10 Marks)

- (a) (i) $v = 4 - 10t - 3t^2 \text{ m/s}$ ✓

(ii) $a = -10 - 6t \text{ m/s}^2$ ✓

- (b) (i) $v = 4 \text{ m/s}$ ✓

(ii) $a = -10 \text{ m/s}^2$ ✓

- (c) Particles collide by equating x when

$t = \frac{7}{3} \text{ s}$ ✓✓✓✓

- (d) Show that v for P and Q are both negative, ✓
 hence travelling the same direction. ✓

Question 3 (11 Marks)

(a) $3 = \ln e^{0.0124t}$ ✓

$t = 89 \text{ years (nearest yr).}$ ✓

- (b) (i) 120 ✓

(ii) No. of ways 3 vowels together
 $= 3!3! = 36$ ✓

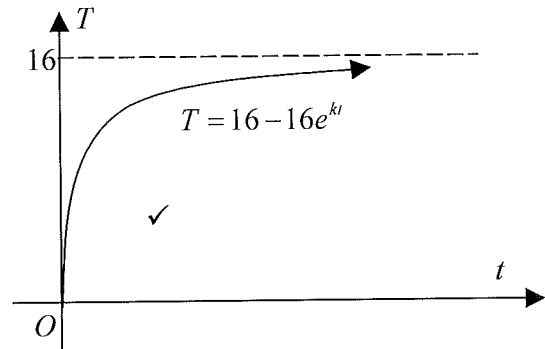
$P(\text{vowels together}) = \frac{36}{120} = \frac{3}{10}$ ✓

- (c) (i) Proof ✓

(ii) $P = -16$ ✓ $e^{10k} = 0.25$ ✓

(iii) 14^0 ✓

(iv) As $t \rightarrow \infty, Pe^{kt} \rightarrow 0, \therefore T \rightarrow 16$ ✓



Question 4 (9 Marks)

(a) $\frac{10!}{4!2!2!} = 37800$ ✓

(b) $T_1 \times T_2 = 32$

(c) (i) $P(\text{all girls}) = \frac{{}^{12}C_4}{{}^{20}C_4} = \frac{33}{323}$ ✓

(ii) $P(2G,2B) = \frac{{}^{12}C_2 \times {}^8C_2}{{}^{20}C_4} = \frac{616}{1615}$ ✓

(iii) $P(\text{P \& O are both girls})$
 $= \frac{12}{20} \times \frac{11}{19} \times 1 \times 1 = 0.347$ ✓

(iv) No. of combinations of $C_G R_G$ & two others
 $= {}^{12}C_2 \times {}^{18}C_2 = 10\,098$ ✓
 No. of combinations of P & O are of different sex
 $= {}^{12}C_2 ({}^{10}C_1 \times {}^8C_1) = 5280$ ✓
 $\therefore P(\text{P\&O opp.sex/both } C_G R_G) = \frac{5280}{10098}$
 $= 0.523$