SOUTH SYDNEY HIGH SCHOOL

SPECTEMUR AGENDO

Year 12

Half-Yearly Examination 1996

MATHEMATICS
3 UNIT

Time allowed - 2 hours

DIRECTIONS TO CANDIDATES

- Board-approved calculators may be used.
- Write your Name on **EVERY** page .
- Attempt all **SEVEN** questions.
- Standard Integrals are provided.
- Start each question on a **NEW** page.

QUESTION 1 **Marks**

(a) Solve the equation $e^{2x} = e^x + 12$ using the substitution $u = e^x$. **3**

(b) Given that $y = ax^b + 2$, and that $y = 7$ when $x = 3$ and $y = 52$ when $x = 9$, **3**

find the values of a and b.

(c) If $A(-1, 2)$ and $B(5, 6)$ are the endpoints of the line AB and **3**

C is the midpoint of AB. Find the coordinates of P which divides CB

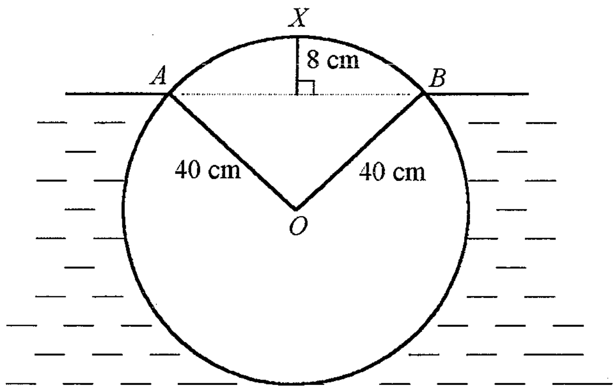
internally in the ratio 3 : 1.

(d) Given that $\log_b(x^3y) = p$ and $\log_b\left(\frac{y}{x^2}\right) = q$. **3**

Express $\log_b(xy)$ in terms of p and q.

QUESTION 2 **Marks**

(a) The figure shows the circular cross-section of a uniform log of radius 40 cm floating in water. The points A and B are on the surface and the highest point X is 8 cm above the surface.



Show that $\angle AOB$ is approximately 1.29 radians. **2**

Calculate :

(i) the length of the arc AXB. **2**

(ii) the area of the cross-section below the surface. **2**

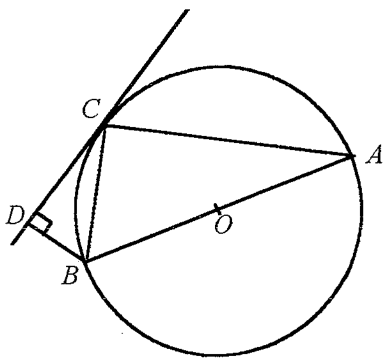
(iii) the percentage of the volume of the log below the surface. **1**

(b) Prove by mathematical induction **5**

$$\sum_{r=0}^{n} 4^r = \frac{4^n - 1}{3}$$

QUESTION 3 **Marks**

(a)

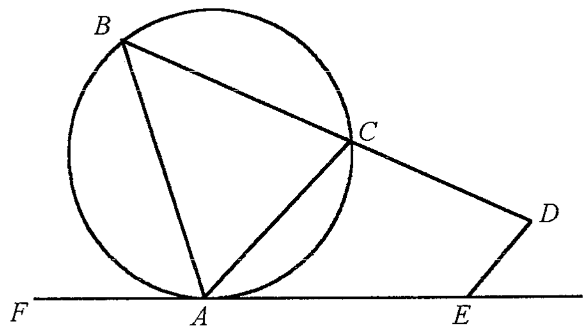


In the above diagram, AB is a diameter. BD is perpendicular to the tangent at C.

(i) Draw this diagram in your examination booklet. **1**

(ii) Prove that : (α) BC bisects $\angle ABD$ (β) $BC^2 = BA \times BD$ **6**

(b)



ABC is a triangle inscribed in a circle. FA is a tangent to the circle.

ED is drawn parallel to AC and meets BC produced at D.

(i) Copy the diagram into your examination booklet **1**

(ii) Prove that $AEDB$ is a cyclic quadrilateral. **4**

QUESTION 4 **Marks**

(a) A function and its inverse is said to be mutually inverse functions if

$$f(f^{-1}(x)) = f^{-1}(f(x)) = x.$$

(i) Show that the function $y = \dfrac{1}{x+1}$ and its inverse **2**

are mutually inverse functions.

(ii) Sketch the above function and its inverse on the same axes. **3**

(b) Consider the function $f(x) = 2\cos^{-1}x$

(i) Find the exact value of $f\left(\dfrac{1}{\sqrt{2}}\right)$. **1**

(ii) Sketch the graph of $y = f(x)$ showing its domain and range. **2**

(iii) Find the equation of the normal to the curve at the point where **2**

$x = \dfrac{1}{\sqrt{2}}$.

(c) Find $\displaystyle\int \frac{4}{4+9x^2}\,dx$ **2**

QUESTION 5 **Marks**

(a) Using the substitution $u = 1 - x^2$, evaluate **4**

$$\int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx$$

(b) Find $\frac{d}{dx}\left(x\sin^{-1}x\right)$. Hence or otherwise, evaluate $\int_0^1 \sin^{-1}x\,dx$. **4**

(c) Express $2\sin^2 A$ in terms of $\cos 2A$ and hence evaluate **4**

$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2\sin^2\frac{x}{2}\,dx$$

QUESTION 6 **Marks**

(a) During last year's drought the rate of decrease of the number (N) of sheep in a region was given by the equation :

$\frac{dN}{dt} = -\frac{N-1000}{5}$ where time t is measured in months.

Initially there 9000 sheep.

(i) Show that $N = 1000 + Ae^{-\frac{t}{5}}$ satisfies the equation and evaluate the constant A. **3**

(ii) Calculate the number of sheep in the region after 5 months. **1**

(iii) How long will it take for the number of sheep to decrease by 60% ? **2**

(b) The acceleration of a particle, when x metres from the origin on a directed axis, is given by $(4x - 2x^3)$ m/s^2.

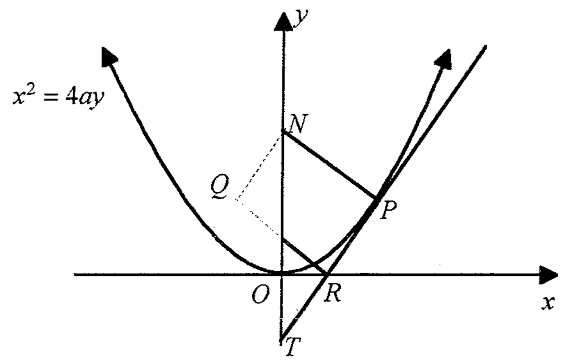
It is released from rest at $x = 2$.

(i) Determine the position at which it next comes to rest. **3**

(ii) Determine the acceleration at this point. **1**

(iii) Hence describe the motion. **2**

QUESTION 7 **Marks**



In the above diagram, P is any point on the parabola $x^2 = 4ay$, whose focus is the point S. The tangent at P cuts the x– and y–axes at R and T. The normal at P cuts the y–axis at N. A line is drawn through N parallel to PT and meets RS produced at Q.

(i) Prove that RS is parallel to NP. **4**

(ii) Prove that S is the mid-point of NT. **4**

(iii) Show that the locus of the point Q is a horizontal line and state its position. **4**

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82/84 V. Good effort

Q1 A.) $e^{2x} - e^{x} - 12 = 0$ $u = e^{x}$

$u^2 = e^{2x}$

$u^2 - u - 12 = 0$

$(u-4)(u+3) = 0$

$u = 4$ or $u = -3$

$e^{x} = 4$ $e^{x} = -3$. $e^{x} > 0$ for all x. $\therefore$ no solution

$x = \ln 4$

$\therefore x = \ln 4 = 2\ln 2$

B) $y = ax^{b} + 2$

when $x = 3$, $y = 7$

$7 = a.3^{b} + 2$; $a.3^{b} = 5$

when $x = 9$, $y = 52$ $3^{b} = \frac{5}{a}$; $9^{b} = \frac{25}{a^2}$ — (1)

$\therefore 52 = a.9^{b} + 2$ — (2)

$\therefore$ (sub in (1))

$52 = a\left(\frac{25}{a^2}\right) + 2$

$52 = \frac{25}{a} + 2$; $\frac{25}{a} = 50$; $a = \frac{1}{2}$ (sub into (1))

$9^{b} = \frac{25}{\frac{1}{4}} = 100$

$b \ln 9 = \ln 100$

$b = \frac{2\ln 10}{\ln 9}$

$\therefore a = \frac{1}{2}$, $b = \frac{2\ln 10}{\ln 9}$

c) $C = \left(\frac{-1+5}{2}, \frac{2+6}{2}\right)$

$= (2, 4)$ ✓

$P = \left(\frac{3(2)+1(5)}{4}, \frac{3(4)+1(6)}{4}\right)$

$P = \left(\frac{6+5}{4}, \frac{12+6}{4}\right)$

$P = \left(\frac{11}{4}, \frac{9}{2}\right)$ ✓

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D) $\log_b (x^3 y) = p$ and $\log_b \left(\frac{y}{x^2}\right) = q$.

or solve simultaneously
$3 \log_b x + \log_b y = p \cdots (i)$
$-2 \log_b x + \log_b y = q \cdots (ii)$
$(i) - (ii)$
$5 \log_b x = p - q$
$\log_b x = \frac{p-q}{5}$

$\log_b x^3 + \log_b y = p$. $\log_b y - \log_b x^2 = q$

$\log_b y = p - \log_b x^3$ — ① $\log_b y = \log_b x^2 + q$ — ②

∴ Solve ① and ② simult. to find x. ✓

$p - 3\log_b x = 2\log_b x + q$.

$p - q = 5 \log_b x$ $x = b^{\frac{p-q}{5}}$ (sub into ① to find y)

$\log_b y = p - 3\log_b \left(b^{\frac{p-q}{5}}\right)$

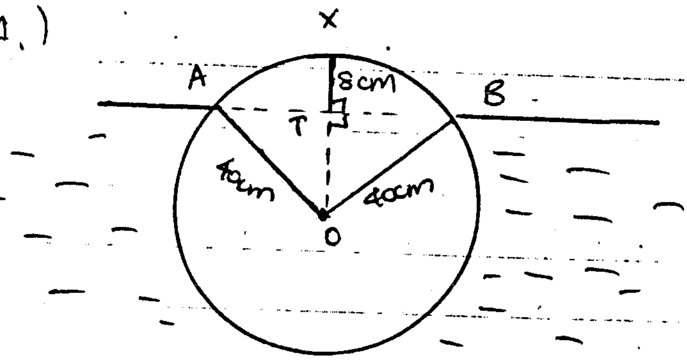
$= p - 3\left(\frac{p-q}{5}\right) = \frac{5p - 3p + 3q}{5} = \frac{2p+3q}{5}$ ✓

$y = b^{\frac{2p+3q}{5}}$

now $\log_b (xy) = \log_b \left(b^{\frac{p-q+2p+3q}{5}}\right)$ ✓

$\boxed{= \frac{3p+2q}{5}}$

2) A.)



Show that $\angle AOB$ is approx 1.29^c.

In ΔTOB, $OT = 40cm - 8cm$ (radius $OX = 40cm$)

$= 32cm$

$\cos \angle TOB = \frac{32}{40}$

$\angle TOB = 0.644^c$

Similarly, $\cos \angle AOT = \frac{32}{40}$; $\angle AOT = 0.644^c$

$\therefore \angle AOB = 0.644^c \times 2$

$= 1.29^c$ (2dp)

i.) Arc $AXB = r\theta$

$= 40 \times 1.29 =$ 51.48 cm (2dp)

ii.) Area of minor segment AXB = Area of sector AOB − Area of ΔAOB

Using pythag. theorem, $TB^2 = 40^2 - 32^2$

$TB = 24cm$

Since $\Delta ATO \equiv \Delta TOB$ (SAS), $TB = AT$ (corresp. sides of $\equiv \Delta$ equal)

$\therefore AB = 2(24) = 48cm$.

Area of minor segment $AXB = \frac{1.29^c}{2\pi} \times \pi(40^2) - \frac{1}{2} \times 48 \times 32$

$= 261.6 \; cm^2$

$\therefore$ area of cross section below surface $= \pi(40^2) - 26$

length

iii.) Volume of log below surface = $(4764.95 \times l)\ cm^3$.
Volume of entire log = $\pi . 40^2 \times l$

% of volume of log below surface

$$= \frac{4764.95 \times \cancel{l}}{\pi \times 40^2 \cancel{l}} \times 100$$

$$= 94.8\%$$

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B) R.T.P $4^0 + 4^1 + 4^2 + \ldots 4^n = \frac{4^n - 1}{3}$

Step 1
Let $n = 1$
LHS $4^n = 4^0 = 1$
RHS $\frac{4^n - 1}{3} = \frac{4^1 - 1}{3} = 1 =$ LHS $\therefore$ true for $n=1$

Step 2
Assume true for $n=k$.
$4^0 + 4^1 + \ldots 4^{k-1} = \frac{4^k - 1}{3}$

R.T.P. also true for $n = k+1$
$4^0 + 4^1 + \ldots 4^{k-1} + 4^k = \frac{4^{k+1} - 1}{3}$

LHS $4^0 + 4^1 + 4^2 + \ldots 4^{k-1} + 4^k = \frac{4^k - 1}{3} + 4^k$ (from assumption

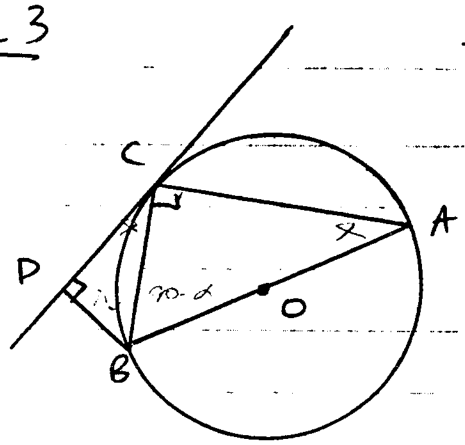
$$= \frac{4^k - 1 + 3.4^k}{3} = \frac{4^k(4) - 1}{3}$$

$$= \frac{4^{k+1} - 1}{3} = \text{RHS}$$

Step 3
If true for $n=k$ and $n=k+1$ and also true for $n=1$, then it is true for $n=1+1=2$, $n=2+1=3$ and so on. $\therefore$ by the POMI, it is true for all positive integers n.

Question 3

A.) i.)



ii.) α) Let $\angle DCB = \alpha$.

$\angle DCB = \angle CAB = \alpha$ ($\angle$ made by tgt and chord equals $\angle$ in alt segment)

In Δ CDB, $\angle CBD = 180° - 90° - \alpha$ ($\angle$ sum $\Delta = 180°$)

$= 90 - \alpha°$

$\angle BCA = 90°$ ($\angle$ in semi-circle $= 90°$)

In ΔBCA, $\angle CBA = 180° - 90° - \alpha$ ($\angle$ sum $\Delta = 180°$)

$= 90 - \alpha = \angle CBD$

$\therefore \angle CBA = \angle CBD$

$\therefore$ BC bisects $\angle ABD$

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β) In ΔCDB and ΔACB,

① $CBD = \angle CBA$ (BC bisects $\angle DBA$)

② $\angle CDB = \angle BCA = 90°$

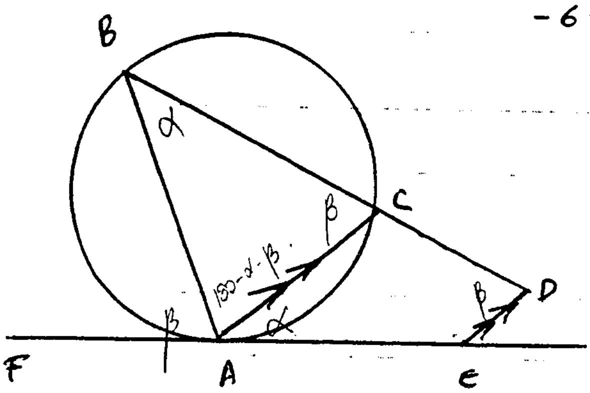
③ $\angle DCB = \angle CAB$ ($\angle$ made by tgt and chord equals $\angle$ in alt segment)

$\therefore$ $\Delta CDB \parallel\!\mid \Delta ACB$ (equiangular)

$\frac{BC}{BD} = \frac{BA}{BC}$ (corresponding sides of similar Δ are in same ratio)

$\therefore BC^2 = BA \times BD$

3)



i.) done

ii.) ~~I don't think~~

AEDB is a cyclic quad.

Let $\angle ABC = \alpha = \angle CAE$ (Angle in the alternate segment)

and $\angle BCA = \beta = \angle CDE$ (Corr. $\angle$s $AC \parallel ED$)

~~[illegible]~~

Also $\angle BCA = \angle BAF$ (Angle in the alternate segment.

$\therefore \angle BAF = \angle EDB = \beta$ (Exterior angle of cyclic quad)

$\therefore$ AEDB is a cyclic quadrilateral.

4) A.) i.) $y = f(x) = \frac{1}{x+1}$

For $f^{-1}(x)$, swap x and y values

$$x = \frac{1}{y+1} \qquad y+1 = \frac{1}{x}$$

$$y = \frac{1}{x} - 1$$

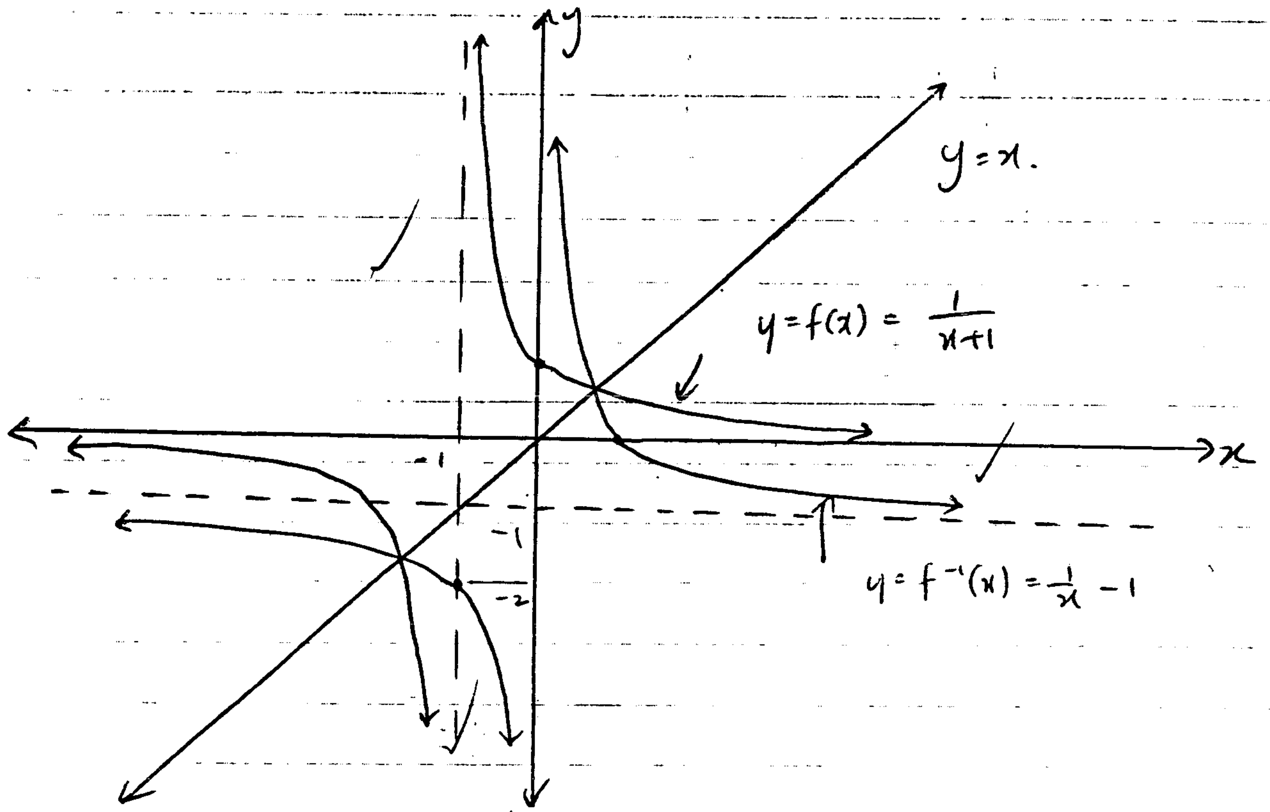
$$\therefore f^{-1}(x) = \frac{1}{x} - 1$$

$$f(f^{-1}(x)) = f\left(\frac{1}{x} - 1\right) = \frac{1}{\frac{1}{x} - 1 + 1} = \frac{1}{\frac{1}{x}} = x$$

$$f^{-1}(f(x)) = f^{-1}\left(\frac{1}{x+1}\right) = \frac{1}{\frac{1}{x+1}} - 1 = x+1-1 = x$$

$$\therefore f(f^{-1}(x)) = f^{-1}(f(x)) = x$$

ii.)

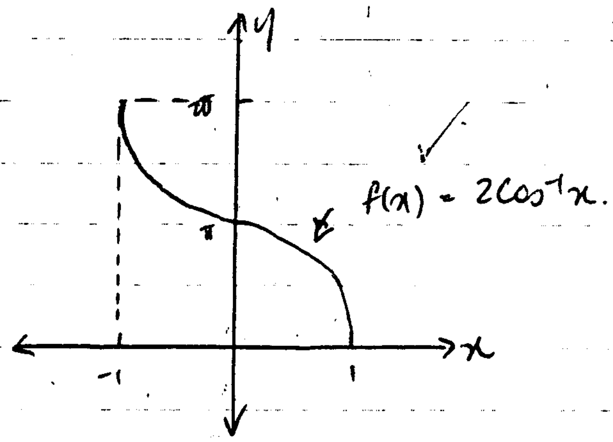


B) $f(x) = 2\cos^{-1} x$

i.) $f\left(\frac{1}{\sqrt{2}}\right) = 2\cos^{-1}\frac{1}{\sqrt{2}} = 2\left(\frac{\pi}{4}\right) = \frac{\pi}{2}$

ii.) D: $-1 \leq x \leq 1$

R: $0 \leq f(x) \leq 2\pi$



iii.) $y = 2\cos^{-1} x$

$$\frac{dy}{dx} = \frac{-2}{\sqrt{1-x^2}}$$

At $x = \frac{1}{\sqrt{2}}$, $y = \frac{\pi}{2}$ and $\frac{dy}{dx} = \frac{-2}{\sqrt{1-\frac{1}{2}}} = \frac{-2}{\sqrt{\frac{1}{2}}} = -2 \times \sqrt{2} = -2\sqrt{2}$

$\therefore$ grad. of normal $= \frac{1}{2\sqrt{2}}$. $(m_1 m_2 = -1)$

Eqt of normal at $x = \frac{1}{\sqrt{2}}$ is:

$$\left(y - \frac{\pi}{2}\right) = \frac{1}{2\sqrt{2}}\left(x - \frac{1}{\sqrt{2}}\right)$$

$$y - \frac{\pi}{2} = \frac{x}{2\sqrt{2}} - \frac{1}{4}$$

$$2\sqrt{2}y - \sqrt{2}\pi = x - \frac{\sqrt{2}}{2}.$$

$$4\sqrt{2}y - 2\sqrt{2}\pi = 2x - \sqrt{2}.$$

$$\therefore \underline{2x - 4\sqrt{2}y + 2\sqrt{2}\pi - \sqrt{2} = 0} \text{ or.}$$

$$\underline{\sqrt{2}x - 4y + 2\pi - 1 = 0}$$

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c) $\displaystyle\int \frac{4}{4+9x^2}\,dx = 4\int \frac{1}{9(\frac{4}{9}+x^2)}\,dx$

$$= \frac{4}{9}\int \frac{1}{\frac{4}{9}+x^2}\,dx$$

$$= \frac{4}{9}\left(\frac{1}{\frac{2}{3}}\tan^{-1}\frac{x}{\frac{2}{3}}\right) + c$$

$$= \frac{4^2}{9_3}\left(\frac{3}{2}\tan^{-1}\frac{3x}{2}\right) + c = \underline{\frac{2}{3}\tan^{-1}\frac{3x}{2} + c}$$

b) i.) $N = 1000 + Ae^{-\frac{t}{5}}$; $Ae^{-\frac{t}{5}} = N - 1000$ — ①

$$\frac{dN}{dt} = \frac{-1}{5}Ae^{-\frac{t}{5}} \quad \text{(sub in ①)}$$

$$= \frac{-1}{5}(N-1000) = \underline{\underline{\frac{-(N-1000)}{5}}}$$

$\therefore$ when $t = 0$, $N = 9000$

$9000 = 1000 + A$; $\underline{A = 8000}$

ii.) when $t = 5$, $N = 1000 + 8000e^{-1}$

$= \underline{3943 \text{ sheep}}.$

iii.) Find t when there are only 40% × 9000 sheep left.

$\frac{40}{100} \times 9000 = \underline{3600 \text{ sheep}}$

$$3600 = 1000 + 8000e^{-\frac{t}{5}}$$

$$8000e^{-\frac{t}{5}} = 2600 \; ; \; e^{-\frac{t}{5}} = \frac{13}{40}$$

$-\frac{t}{5} = \ln \frac{13}{40}$

$t = -5 \ln \frac{13}{40}$

$= 5.6$ months

= 6 months (to nearest month)

B) $\ddot{x} = 4x - 2x^3 \ m/s^2$.

i.) $\ddot{x} = \frac{d}{dx}\left(\frac{1}{2}v^2\right)$

$\frac{1}{2}v^2 = \int 4x - 2x^3 \, dx$

$\frac{1}{2}v^2 = 2x^2 - \frac{x^4}{2} + c$

when $x = 2$, $v = 0$

$0 = 8 - 8 + c$; $c = 0$ $\quad \therefore \frac{1}{2}v^2 = 2x^2 - \frac{x^4}{2}$

$v^2 = 4x^2 - x^4$

Find x when $v = 0$.

$4x^2 - x^4 = 0$

$x^4 - 4x^2 = 0$; $x^2(x^2 - 4) = 0$

$x^2 = 0$ or $x^2 = 4$

$x = 0$ $\quad x = \pm 2$

To find out what direction particle travels, let $x = 3$.

$v^2 = -45$. not possible b/c $v^2 \geq 0$.

Check $x = 1$

$v^2 = 4 - 1 = 3 > 0$ $\therefore$ possible

$\therefore$ particle travels to the left.

$\therefore$ the particle next comes to rest at $x = 0$.

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ii.) when $x=0$, acceleration = $0m/s^2$

iii.) The particle begins at rest at $x=2$. It then travels to the left, slowing until it stops at $x=0$.
(b/c $\ddot{x}>0$ whilst $v<0$, ie the force of acceleration is acting against velocity)

It then changes direction and heads back to the right.

Question 5

A.) $\int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx$

$u = 1-x^2$

$\frac{du}{dx} = -2x.$

$du = -2x\,dx.$

$= \int_1^0 -\frac{1}{2}\frac{du}{\sqrt{u}}$

$= -\frac{1}{2}\int_1^0 u^{-\frac{1}{2}}\,du = -\frac{1}{2}\left(\frac{u^{\frac{1}{2}}}{\frac{1}{2}}\right)_1^0$

$= -\frac{1}{2}\left(2\sqrt{u}\right)_1^0$

$= -\frac{1}{2}(0-2) = -\frac{1}{2}\times -2 = \underline{\underline{1}}$

B) $\frac{d}{dx}(x\sin^{-1}x) = \sin^{-1}x + x\cdot\frac{1}{\sqrt{1-x^2}}$

$= \sin^{-1}x + \frac{x}{\sqrt{1-x^2}}$

$\int \sin^{-1}x + \frac{x}{\sqrt{1-x^2}}\,dx = x\sin^{-1}x + C.$

$\int \sin^{-1}x\,dx = x\sin^{-1}x + C - \int\frac{x}{\sqrt{1-x^2}}\,dx.$

$$\therefore \int_0^1 \sin^{-1}x\,dx = (x\sin^{-1}x)_0^1 - \int_0^1 \frac{x}{\sqrt{1-x^2}}\,dx$$

$$= (\sin^{-1}1) - 1 \quad (\text{from part}(A))$$

$$= \frac{\pi}{2} - 1$$

$$= \boxed{\frac{\pi-2}{2}}$$

c) $\cos 2A = 1 - 2\sin^2 A$

$2\sin^2 A = 1 - \cos 2A$

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$$\int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 2\sin^2\frac{x}{2}\,dx = \int_{\frac{\pi}{2}}^{\frac{3\pi}{2}} 1 - \cos x\,dx$$

$$= (x - \sin x)_{\frac{\pi}{2}}^{\frac{3\pi}{2}}$$

$$= \frac{3\pi}{2} - \sin\frac{3\pi}{2} - \frac{\pi}{2} + \sin\frac{\pi}{2}.$$

$$= \frac{3\pi}{2} + 1 - \frac{\pi}{2} + 1 = \frac{2\pi}{2} + 2 = \boxed{\pi + 2}$$

7) i.) $x^2 = 4ay$, $y = \frac{x^2}{4a}$; $\frac{dy}{dx} = \frac{x}{2a}$.

Grad. of tgt at $P(2ap, ap^2) = \frac{2\cancel{a}p}{\cancel{2a}} = p$

Eqt of tgt at P is:

$y - ap^2 = p(x - 2ap)$

$y - ap^2 - px + 2ap^2 = 0$

$\underline{px - y - ap^2 = 0}$

At R, $y = 0$.

$px - ap^2 = 0$; $px = ap^2$; $x = ap$ $\quad \therefore \underline{R = (ap, 0)}$

At T, $x = 0$

$-y - ap^2 = 0$ $\quad -y = ap^2$; $y = -ap^2$ $\quad \therefore \underline{T = (0, -ap^2)}$

Grad of normal at $p = -\frac{1}{p}$ $(m_1 m_2 = -1)$

$\therefore$ Equation of normal at p is:

$(y - ap^2) = -\frac{1}{p}(x - 2ap)$

$py - ap^3 = 2ap - x.$

$\underline{py + x = ap^3 + 2ap}$

At N, $x = 0$. $py = ap^3 + 2ap$

$y = ap^2 + 2a$

$\therefore$ $\underline{N = (0, a(p^2+2))}$

Grad. of RS $= \frac{a - 0}{0 - ap} = \frac{a}{-ap} = -\frac{1}{p} - m_1$

Grad of NP $= \frac{a(p^2+2) - ap^2}{0 - 2ap} = \frac{ap^2 + 2a - ap^2}{0 - 2ap} = \frac{2\cancel{a}}{-2\cancel{a}p} = -\frac{1}{p} =$

$m_1 = m_2$. $\therefore$ $\underline{RS \parallel NP}$

ii.) $N = (0, a(p^2+2))$

$T = (0, -ap^2)$. Also S(focus) $= (0, a)$

Mdpt of NT $= \left(\frac{0+0}{2}, \frac{a(p^2+2) - ap^2}{2}\right)$

$= \left(0, \frac{ap^2 + 2a - ap^2}{2}\right)$

$= \left(0, \frac{2a}{2}\right) = (0, a) = S$

$\therefore$ S is mdpt of NT.

iii.) Grad. of QN = grad of tgt at P = p (QN ∥ PR)

Eqt of QN is :

$$(y - ap^2 - 2a) = px.$$

$$px - y + ap^2 + 2a = 0.$$

$$\underline{y = px + ap^2 + 2a} \quad -(1)$$

2.

Eqt of SR is

$$y - a = -\frac{1}{p}(x - 0)$$

$$\cancel{py - ap = -x}$$

$$py = a - x$$

$$y = \frac{a-x}{p} \quad -(2)$$

$\Rightarrow \quad x = ap - py$ sub into (1)

$$y = p(ap - py) + ap^2 + 2a$$

$$= ap^2 - p^2y + ap^2 + 2a$$

To find Q, solve (1) and (2) simult.

$$y + p^2y = 2ap^2 + 2a$$

$$y(1+\cancel{p^2}) = 2a(\cancel{p^2+1})$$

$\therefore y = 2a$ is a straight line parallel to the x-axis

$$px + ap^2 + 2a = \frac{a-x}{p}.$$

$$p^2x + ap^3 + 2ap = a - x.$$

$$x = \frac{a - ap^3 - 2ap}{p^2+1}$$

(Sub into (2) to find y)

$$y = \frac{\cancel{a} - \left(\frac{a - ap^3 - 2ap}{p^2+1}\right)}{p}$$

$$= \frac{a(p^2+1) - a + ap^3 + 2ap}{p(p^2+1)}$$

$$= \frac{ap^2 + ap^3 + 2ap}{p(p^2+1)} = \frac{ap + ap^2 + 2a}{p^2+1}$$

$$\therefore Q = \left(\frac{a - ap^3 - 2ap}{p^2+1}, \frac{ap + ap^2 + 2a}{p^2+1}\right)$$

~~Locus of Q~~

$$\cancel{x = a - p^2x - ap^3 - 2ap}$$

$$\cancel{x = a - (p^2x + ap^3 + 2ap)}$$

~~(sub in (A))~~

$$\cancel{x = a - py}$$

$$\cancel{y = px + ap^2 + 2a.}$$

$$\cancel{py = p^2x + ap^3 + 2ap} \quad -(A)$$