SOUTH SYDNEY HIGH SCHOOL

YEAR 12

MATHEMATICS

2006

HALF YEARLY

Time Allowed—2 hours

Directions to Candidates

- Attempt ALL questions
- All necessary working must be shown. Marks may be deducted for careless or badly arranged work.
- Board approved calculators maybe used.

Answer each question in a SEPARATE writing booklet.

Marks

Question 1 (12 marks) Use a SEPARATE writing booklet.

(a) On average, Abdullah's heart beats approximately 74 times per minute. How many times will Abdullah's heart beat in 65 years (assuming 365 days in a year). Give your answer in scientific notation correct to three significant figures. **2**

(b) Solve for x: $|1 - 2x| = 5$. **2**

(c) The radius, r, of a conical flask of height h and volume V is given by **2**

$$r = \sqrt{\frac{3V}{\pi h}}.$$

A manufacturer is required to produce a conical flask with a volume of 1000 m^3. Find the radius of this flask if the height and radius are to be of equal length. Answer correct to two decimal places.

(d) Rationalise the denominator and express in simplest terms **2**

$$\frac{\sqrt{2} - \sqrt{6}}{\sqrt{6} + \sqrt{2}}.$$

(e) Factorise $2x^2 - 10x - kx + 5k$. **2**

(f) Simplify $\dfrac{125}{(5^n)^6 \times 125^{1-2n}}$. **2**

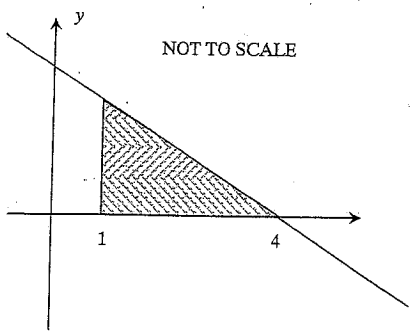
Question 3 (12 Marks) Use a Separate Sheet of paper **Marks**

(a) Differentiate the following with respect to x.

(i) $\sqrt[4]{x^3}$ **2**

(ii) $3x^2(4x-1)$ **2**

(iii) $x\cos x$ **2**



(b) $y = 4 - x$ is shown on the graph. **3**

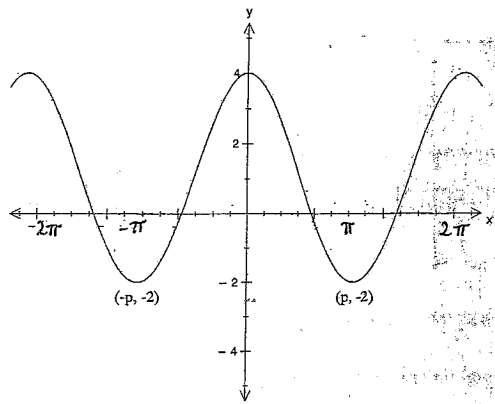
Calculate the volume of the solid formed when the area bounded by the function, x axis and $x = 1$ is rotated around the x axis.

(c) $g'(x) = 3x^2 - 4 + \frac{1}{x^2}$ **3**

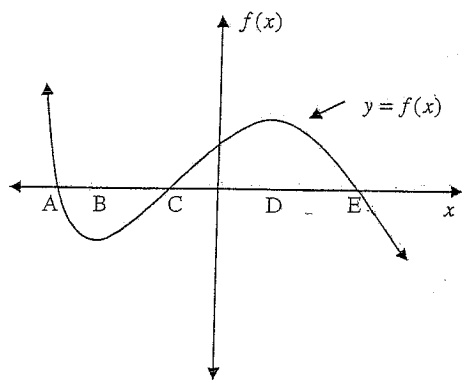
$g(x)$ takes the value 4 when $x = 1$. Find $g(x)$.

Question 5

(c) Write the function represented by the following graph **2**



(d)



The above diagram shows a sketch of the function $y = f(x)$.
Copy this diagram into your writing booklet, and on the same diagram sketch the gradient function $y = f'(x)$. **2**

Question 6 (12 marks) Use a SEPARATE writing booklet: **Marks**

(a) Consider the function $y=\sqrt{4-x^2}$.

(i) State its domain. 1

(ii) Sketch the graph 1

(b) The gradient function of a curve is given by $f'(x)=(3x-4)(x-4)$ and the curve $y=f(x)$ passes through (1,9).

(i) Find the equation of the curve $y=f(x)$. 2

(ii) Find any stationary points and their nature 2

(iii) Sketch the curve $y=f(x)$ clearly labelling turning points. 2

(c) Consider the geometric series: $1+(5-\sqrt{a})+(5-\sqrt{a})^2+(5-\sqrt{a})^3+\cdots$

(i) Find the values of a for which this geometric series has a limiting sum. 2

(ii) Find the limiting sum of the series given that a is 20. Write your answer with a rational denominator. $\frac{a}{1-r}$ 2

Year 12 Half Yearly Examination 11/4/06 – Solutions

Question 1

(a) Number of times $= 74\times60\times24\times365\times65$ ✓
$= 2\,528\,136\,600$
$= 2.53\times10^9$ ✓

(b) $|1-2x|=5$
$1-2x=5$ or $1-2x=-5$ ✓
$-2x=4$ or $-2x=-6$
$x=-2$ $x=3$ ✓

(c)
$r=\sqrt{\frac{3V}{\pi h}}$
$r=\sqrt{\frac{3V}{\pi r}}$ since $r=h$ ✓
$r^2=\frac{3V}{\pi r}$
$\pi r^3=3V$
$r^3=\frac{3V}{\pi}$
$r^3=\frac{3\times1000}{\pi}$ ✓½
$r=9.85$ ✓½

(d) $\frac{\sqrt2-\sqrt6}{\sqrt6+\sqrt2}=\frac{\sqrt2-\sqrt6}{\sqrt6+\sqrt2}\times\frac{\sqrt6-\sqrt2}{\sqrt6-\sqrt2}$ ✓
$=\frac{\sqrt{12}-2-6+\sqrt{12}}{6-2}$
$=\frac{-8}{4}$
$=-2$ ✓

(e) $2x^2-10x-kx+5k=2x(x-5)-k(x-5)$ ✓
$=(x-5)(2x-k)$ ✓

(f) $\frac{125}{(5^m)^2\times125^{1-2m}}=\frac{5^3}{5^{2m}\times(5^3)^{1-2m}}$ ✓
$=\frac{5^3}{5^{2m}\times5^{3-6m}}$ ✓½
$=\frac{5^3}{5^3}=1$ ✓½

Question 2

(a) $\cos210^\circ+\tan480^\circ=-\cos30-\tan60$ ✓
$=-\frac{\sqrt3}{2}-\sqrt3$
$=\frac{-\sqrt3-2\sqrt3}{2}$ ✓
$=\frac{-3\sqrt3}{2}$

(b) $\sin\theta=\frac{-\sqrt3}{2}$
$\theta=180+60$ or $360-60$ (½ for each answer) ✓
$=240$ or 300 ✓

(c) (i) $\frac{y-y_1}{x-x_1}=\frac{y_2-y_1}{x_2-x_1}$ ✓
$\frac{y-4}{x-5}=\frac{1}{3}$ ✓
$3y-12=x-5$
$x-3y+7=0$

(ii) $M=\left\{\frac{x_1+x_2}{2},\frac{y_1+y_2}{2}\right\}$
$=\left\{\frac{7}{2},\frac{7}{2}\right\}$ ✓

(iii) $3x+y=14$ ✓✓

(iv) $3x+y=14$
when $y=0$: $3x=14$
$x=4\frac{2}{3}$
$\therefore C(4\frac{2}{3},0)$ ✓

(v) $d_{OC}=4\frac{2}{3}$ [from diagram]
$h=4$ [from diagram]
$A=\frac{1}{2}bh$
$=\frac{1}{2}\times4\frac{2}{3}\times4$
$=9\frac{1}{3}$

Question 3

(a) (i) $\frac{d}{dx}\left(4\sqrt{x^3}\right) = \frac{d}{dx}\left(x^{3/4}\right)$ ✓

$= \frac{3}{4}x^{-1/4}$ ✓

$= \frac{3}{4\sqrt[4]{x}}$ 2

(ii) $\frac{d}{dx}\left(3x^2(4x-1)\right) = uv' + vu'$ ✓

$= 3x^2 \cdot 4 + (4x-1)6x$ ✓

$= 12x^2 + 6x(4x-1)$ 2

$= 12x^2 + 24x^2 - 6x$

$= 36x^2 - 6x$

$= 6x(6x-1)$

(iii) $\frac{d}{dx}(x\cos x) = uv' + vu'$ ✓

$= x(-\sin x) + \cos x$ ✓

$= -x\sin x + \cos x$ 2

(b) $V = \pi\int_1^4 y^2\,dx$ ✓

$= \pi\int_1^4 (4-x)^2\,dx$ ✓

$= \pi\left[\frac{(4-x)^3}{-3}\right]_1^4$ ✓ 3

$= \pi(0+9)$

$= 9\pi$

(c) $g'(x) = 3x^2 - 4 + \frac{1}{x^2}$

$g(x) = x^3 - 4x - \frac{1}{x} + C$ ✓

when $x=1$, $g(x)=4$: $4 = 1-4-1+C$

$C = 8$ ✓

$\therefore g(x) = x^3 - 4x - \frac{1}{x} + 8$ ✓

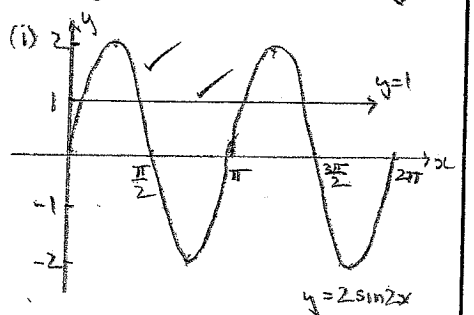
Question 4

(a) $3x+2y-6 \geqslant 0$ ✓ deduct ½ for not having equality sign

$x \leqslant 4$ ✓½

$y \leqslant 0$ ✓½ 2

(b) (i)



2

(ii) $y = 2\sin 2x$ — (1)

$y = 1$ — (2)

$2\sin 2x = 1$

$\sin 2x = \frac{1}{2}$ ✓

$2x = \frac{\pi}{6}, \frac{5\pi}{6}, \frac{13\pi}{6}, \frac{17\pi}{6}$ ✓½

$x = \frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$ ✓½ 2

(c) (i) $A_{AOB} = \frac{1}{2}r^2\theta$ ✓ (Sector)

$= \frac{1}{2} \times 100 \times \frac{\pi}{3}$ ✓

$= \frac{100\pi}{6}$

$= \frac{50\pi}{3}$ 2

(ii) $A_{AOB} = \frac{1}{2}ab\sin C$ ✓ (Triangle)

$= \frac{1}{2} \times 100 \times \sin\frac{\pi}{3}$ ✓

$= \frac{100\sqrt{3}}{4}$

$= 25\sqrt{3}$ 2

(iii) $A_{segment} = \frac{50\pi}{3} - 25\sqrt{3}$

$= \frac{50\pi - 75\sqrt{3}}{3}$ ✓ 1

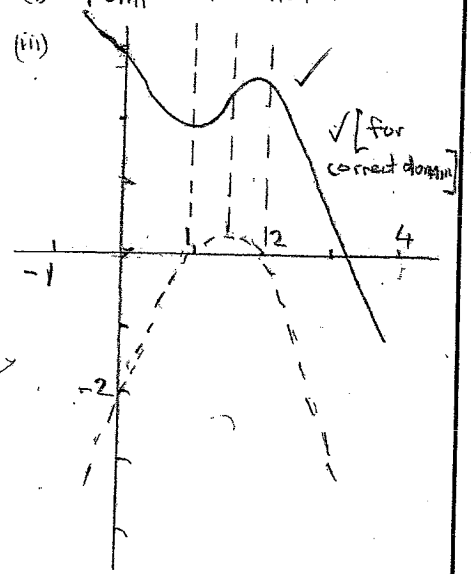
(d) $\int \sin 2x\,dx = -\frac{1}{2}\cos 2x + C$ ✓✓ 2

Question 5

(a) (i) $x=1$ and $x=2$ ✓½ ✓½ 1

(ii) Point of inflexion ✓ 1

(iii)



✓ for correct domain 2

(iv) The graph of derivative indicates slope $\geqslant 2$ between $x=0$ and $x=1$. The slope would have to be less than -3 to reach from $y=3$ to $y=0$ in the space of 1 unit.

(b)

x	1	1.25	1.5	1.75	2
y	1.00	0.64	0.44	0.33	0.25
	y_0	y_1	y_2	y_3	y_4

✓½ each

$h = \frac{b-a}{n} = 0.25$

$\int_1^2 \frac{1}{x^2}\,dx = \frac{h}{3}\left[(y_0+y_4) + 4(y_1+y_3) + 2(y_2)\right]$

$= \frac{0.25}{3}\left[(1+0.25) + 4(0.64+0.33) + 2(0.44)\right]$

$= 0.501$ ✓

c) period $= 2p$ (from graph) ✓½

$\therefore$ period $= \frac{2\pi}{n}$

$2p = \frac{2\pi}{n}$

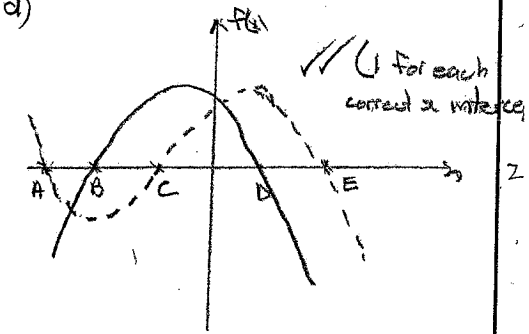
$n = \frac{\pi}{p}$ ✓½ 2

amplitude $= 3$ ✓½

Graph shifted upwards by 1 ✓½

$\therefore y = 1 + 3\cos\frac{\pi}{p}x$

d)



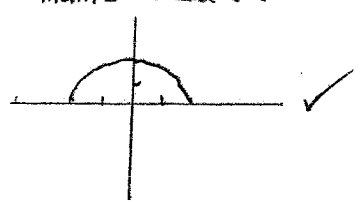
✓✓ ½ for each correct x intercept 2

Question 6

(a) $y = \sqrt{4-x^2}$

(i) domain $= -2 \leqslant x \leqslant 2$ ✓ 1

(ii)



✓ 1

(b) $f'(x) = (3x-4)(x-4)$

$= 3x^2 - 12x - 4x + 16$

$= 3x^2 - 16x + 16$

(i) $f(x) = x^3 - 8x^2 + 16x + C$ ✓

when $x=1$, $f(x)=9$: $9 = 1 - 8 + 16 + C$

$C = 0$ ✓

$\therefore f(x) = x^3 - 8x^2 + 16x$

(ii) For stationary points

$f'(x) = 0$

$(3x-4)(x-4) = 0$

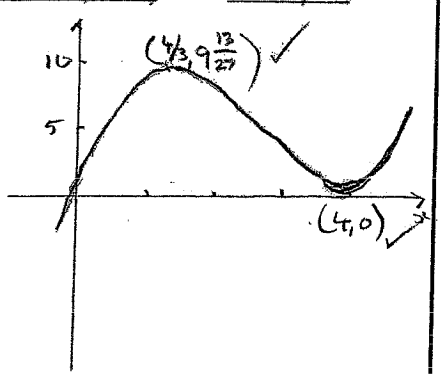
$x = 4/3$ or $x = 4$ ✓ ½ each

x	$4/3^-$	$4/3$	$4/3^+$
y'	+	0	−

x	4^-	4	4^+
y'	−	0	+

$\therefore$ Max turning pt at $(4/3, 9\frac{13}{27})$ ✓½ Min turning pt at $(4,0)$ ✓½

(iii)



✓

(c) $1 + (5-\sqrt{a}) + (5-\sqrt{a})^2 + (5-\sqrt{a})^3 + \ldots$

(i) $|r| < 1$ ✓

$\therefore 5 - \sqrt{a} < 1$ and $5 - \sqrt{a} > -1$

$\sqrt{a} > 4$ $\qquad$ $\sqrt{a} < -6$

$a > 16$ $\qquad$ $a < 36$

$\therefore \underline{16 < a < 36}$ ✓ 2

(ii) $S_\infty = \frac{a}{1-r}$ ✓

$= \frac{20}{1-(5-\sqrt{a})}$ ✓½

$= \frac{20}{\sqrt{a}-4} \times \frac{\sqrt{a}+4}{\sqrt{a}+4}$

$= \frac{20(\sqrt{a}+4)}{a-16}$ ✓½ 2

$= \frac{20(\sqrt{20}+4)}{4}$

$= \underline{5(2\sqrt{5}+4)}$