# South Sydney High School

# March Assessment Task 1999

# Year 12 Mathematics

# 2/3 Unit Common Paper

**Instructions :** **Time Allowed:** 2 Periods

1. All questions may be attempted.
2. Start each question on a new sheet of paper.
3. All necessary working should be shown.
4. Marks may be deducted for poorly arranged or missing working.
5. Approved calculators may be used.

**Examiners :** Mr Kazzi, Mr Moore & Mr Ooi

**QUESTION 1 (7 marks)** **MARKS**

Find the derivative of (a) $y = 3x^2 - 6x + 5$ (b) $y = (6x+5)^4$ **3**

(c) $y = \dfrac{8x}{5x-2}$ (d) $y = \dfrac{1}{\sqrt{3x-4}}$ **4**

**QUESTION 2 (10 marks)**

Consider the curve given by $y = x^3 - 3x + 4$.

(a) Find the coordinates of the stationary points and determine their nature. **4**

(b) Find the coordinates of any points of inflexion. **2**

(c) Sketch the curve for the domain $-3 \le x \le 3$. **3**

(d) What is the maximum value of $x^3 - 3x + 4$ in the domain $-3 \le x \le 3$. **1**

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**QUESTION 3 (10 marks)** **MARKS**

(a) Mr Kazzi invests \$2 000 in a managed fund, where it earns interest at the rate of 12% p.a. paid quarterly. How much will this accumulate to after three years ? 3

(b) The second term of a geometric series is 52 and the fourth term is 13. Find the two series that satisfy these requirements. 4

(c) Find the equation of the tangent to the curve $y = 3x^2 - x^3$ at $x = -1$. 3

**QUESTION 4 (10 marks)**

(a) The perimeter of a rectangle is 60 metres and its length is $x$ metres. 4

(i) Show that the area of the rectangle, $A$, is given by the equation

$$A = 30x - x^2.$$

(ii) Hence find the maximum area of the rectangle.

(b) Find the sum to infinity of the geometric progression 3

$$1 + \left(\sqrt{2} - 1\right) + \left(\sqrt{2} - 1\right)^2 + \ldots$$

leaving your answer as a surd in rational form.

(c) Using the limiting sum method or otherwise, find the rational equivalent for $1.2\dot{5}$. 3

**QUESTION 5 (10 marks)**

(a) The tenth term of an arithmetic sequence is 29 and the fifteenth term is 44. 4

(i) Find the value of the common difference and the value of the first term.

(ii) Find the sum of the first 75 terms.

(b) Given that $f(x) = \sqrt{9 - x^2}$, simplify $f(2) - f'(2)$, writing your answer with a rational denominator. 3

(c) Find the equation of the normal to the curve $y = \dfrac{x^2}{4 - x}$ at the point $x = 2$. 3

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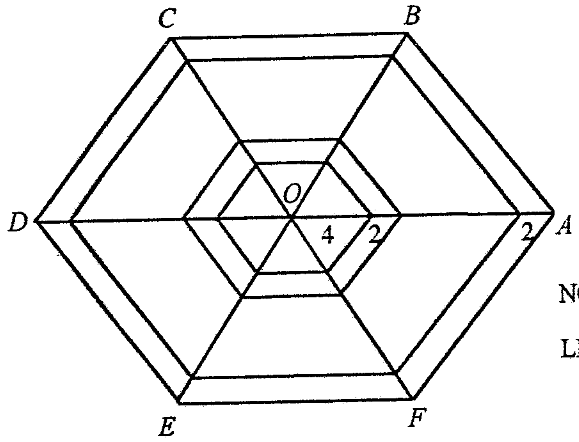
**QUESTION 6 (7 marks)** **MARKS**

A particular spider's web consists of a series of regular hexagon with a common centre $O$, held together by rays through $O$, as shown in the figure, where only some of the hexagons are shown.




NOT TO SCALE

LENGTHS IN CM

The vertices of the smallest hexagon are 4 cm from $O$, the vertices of the next are 2 cm further away and they continue at 2 cm intervals along the rays until the vertices of the last hexagon $ABCDEF$ are 60 cm from $O$.

(a) How many hexagons are there ? **2**

(b) What is the length, in cm, of the perimeter of the smallest hexagon ? **2**

(c) What is the total length of thread used by the spider in making this web (including the six rays from $O$ ) ? **3**

*End of Assessment Task !!!*

SSH '99 2/3 Unit March Assessment Solutions

1] a) $y = 3x^2 - 6x + 5$

$\frac{dy}{dx} = 6x - 6.$

b) $y = (6x+5)^4$

$\frac{dy}{dx} = 4(6x+5)^3 \cdot 6$

$= 24(6x+5)^3$

c) $y = \frac{8x}{5x-2}$

$\frac{dy}{dx} = \frac{(5x-2).8 - 8x(5)}{(5x-2)^2}$

$= \frac{\cancel{40x} - 16 - \cancel{40x}}{(5x-2)^2}$

$= \frac{-16}{(5x-2)^2}$

d) $y = \frac{1}{\sqrt{3x-4}}$

$y = (3x-4)^{-\frac{1}{2}}$

$\frac{dy}{dx} = -\frac{1}{2}(3x-4)^{-\frac{3}{2}} \cdot 3$

$= \frac{-3}{2(3x-4)^{3/2}}$

2] a) $y = x^3 - 3x + 4$

Stationary Pts $\frac{dy}{dx} = 0$

$\frac{dy}{dx} = 3x^2 - 3$

$3x^2 - 3 = 0$

$x^2 - 1 = 0$

$x^2 = 1$

$x = \pm 1$

Stationary pts $(1, 2)$, $(-1, 6)$

Type of Stationary pt
Check $\frac{d^2y}{dx^2}$

$\frac{d^2y}{dx^2} = 6x$

at $x = 1$ $\frac{d^2y}{dx^2} > 0$

Min at $(1, 2)$

at $x = -1$ $\frac{d^2y}{dx^2} < 0$

Max at $(-1, 6)$

b) Possible Pt of inflexion.

when $\frac{d^2y}{dx^2} = 0$

$6x = 0$

$x = 0$

Test points on each side

at $x = -\frac{1}{2}$, $\frac{d^2y}{dx^2} < 0$

at $x = \frac{1}{2}$ $\frac{d^2y}{dx^2} > 0$

$\Big\}$ Change of sign.

$\therefore$ Pt of inflexion at $(0,4)$

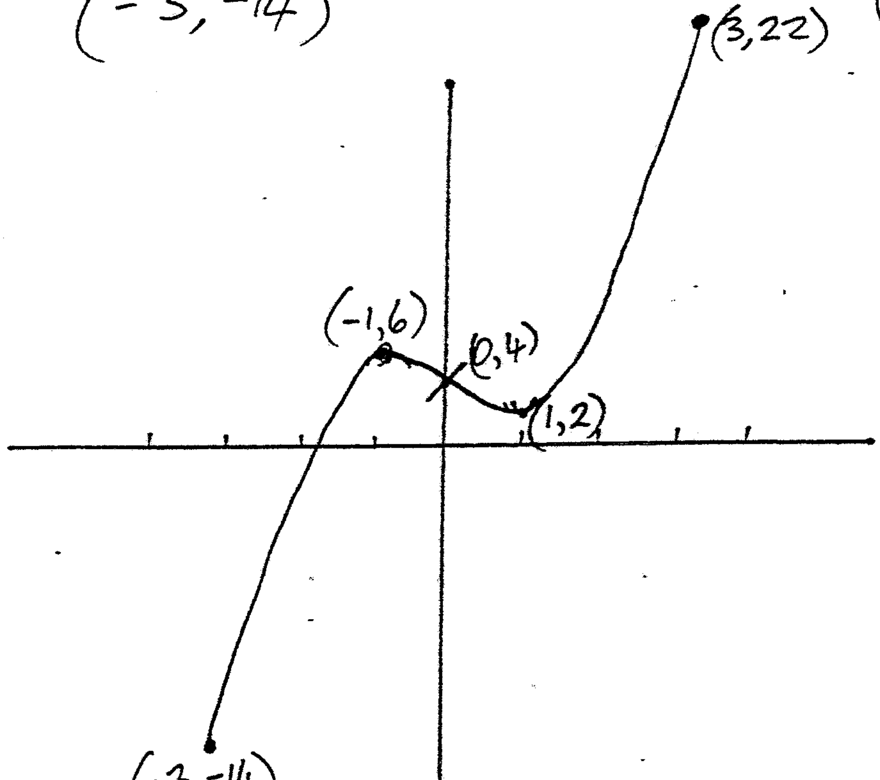
c) END Points $-3 \le x \le 3$

at $x = -3$ $y = -14$

$(-3, -14)$

at $x = 3$, $y = 22$

$(3, 22)$




d) Maximum Value is $\underline{22}$.

3] a) $P = \$2000$, $I = 12\%$ p.a or $3\%$ per quarter
$T = 3$ yrs so $n = 12$ periods (since quarters need

$$A = P\left(1+\frac{r}{100}\right)^n$$
$$= 2000 \times 1.03^{12}$$
$$= \underline{\$2851.52}$$

b) G.P. $T_2 = 52$, $T_4 = 13$
$T_2 = ar$ $\quad$ $T_4 = ar^3$
$52 = ar$ — ① $\quad$ $13 = ar^3$ — ②
sub ① into ②

$$13 = ar.r^2$$
$$13 = 52.r^2$$
$$\frac{1}{4} = r^2$$
$$\pm\frac{1}{2} = r$$

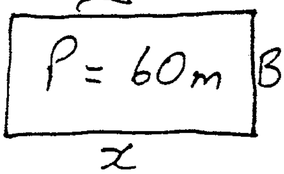
if $r = \frac{1}{2}$ $\quad a = 104$ $\quad$ Series $104 + 52 + 26 + 13 \ldots$

if $r = -\frac{1}{2}$ $\quad a = -104$ $\quad$ Series $-104 + 52 - 26 + 13 \ldots$

c) $y = 3x^2 - x^3$
$$\frac{dy}{dx} = 6x - 3x^2$$

at $x = -1$ $\quad m = -9$
$m = -9$ $\quad$ pt is $(-1, 4)$
Eq$^n$ of tangent
$$y - y_1 = m(x - x_1)$$
$$y - 4 = -9(x - {}^-1)$$
$$y - 4 = -9x - 9$$
$$\underline{9x + y + 5 = 0}$$

4] a) i) Length is $x$




$$P = 2L + 2B$$
$$60 = 2x + 2B$$
$$30 = x + B$$
$$30 - x = B$$

So $\text{Area} = L \times B$
$$= x(30 - x)$$
$$\underline{\underline{A = 30x - x^2}}$$

ii) Max $A$ when $\dfrac{dA}{dx} = 0$
$$30 - 2x = 0$$
$$\underline{x = 15}$$

When $x = 15$
$L = 15$ & $B = 15$
So Max Area
$$= 15 \times 15$$
$$= \underline{\underline{225\,m^2}}$$

b) $1 + (\sqrt{2} - 1) + (\sqrt{2} - 1)^2 + \ldots$

$a = 1 \quad r = \sqrt{2} - 1 \quad \text{Lim}\, S_n = ?$

$$\text{Lim}\, S_n = \frac{a}{1 - r}$$
$$= \frac{1}{1 - (\sqrt{2} - 1)}$$
$$= \frac{1}{1 - \sqrt{2} + 1}$$
$$= \frac{1}{2 - \sqrt{2}} = \underline{\underline{\frac{2 - \sqrt{2}}{2}}}$$

c) Let $x = 1.2\dot{5} = 1.25555$
Then $10x = 12.55555$
Subtracting
$$9x = 11.3$$
$$x = \frac{11.3}{9}$$
$$x = \frac{113}{90}$$
$$\underline{\underline{x = 1\tfrac{23}{90}}}$$

5] a) i) AP $T_{10} = 29$ $T_{15} = 44$

$T_{10} = a + 9d$ $T_{15} = a + 14d$

$29 = a + 9d$ — ① $44 = a + 14d$ — ②

② − ① $15 = 5d$

$\underline{3 = d}$ & $\underline{a = 2}$

ii) $S_n = \frac{n}{2}[2a + (n-1)d]$

$S_{75} = \frac{75}{2}[4 + 74 \times 3]$

$S_{75} = \underline{8475}$

b) $f(x) = \sqrt{9 - x^2}$

$f(x) = (9 - x^2)^{\frac{1}{2}}$

$f'(x) = \frac{1}{2}(9 - x^2)^{-\frac{1}{2}} \cdot -2x$

$= -x(9 - x^2)^{-\frac{1}{2}}$

$= -\frac{x}{\sqrt{9 - x^2}}$

$f(2) = \sqrt{9 - 2^2}$ $f'(2) = \frac{-2}{\sqrt{9 - 2^2}}$

$= \sqrt{5}$ $= \frac{-2}{\sqrt{5}}$

So $f(2) - f'(2) = \sqrt{5} - \frac{-2}{\sqrt{5}}$

$= \sqrt{5} + \frac{2}{\sqrt{5}}$

$= \sqrt{5} + \frac{2\sqrt{5}}{5}$

$= \underline{\frac{7\sqrt{5}}{5}}$

c) $y = \frac{x^2}{4-x}$

$$\frac{dy}{dx} = \frac{(4-x)\cdot 2x - x^2 \cdot (-1)}{(4-x)^2}$$

$$= \frac{8x - 2x^2 + x^2}{(4-x)^2}$$

$$= \frac{8x - x^2}{(4-x)^2}$$

at $x = 2$

$$m = \frac{16-4}{2^2}$$

$$= 3$$

Normal has $m = -\frac{1}{3}$. Pt is $(2,2)$

Eq$^n$ of normal

$$y - y_1 = m(x - x_1)$$

$$y - 2 = -\frac{1}{3}(x-2)$$

$$y - 2 = -\frac{x}{3} + \frac{2}{3}$$

$$3y - 6 = -x + 2$$

$$\underline{\underline{x + 3y - 8 = 0}}$$

6] a) $4, 6, 8, \ldots 60$

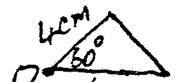
AP $a=4,\ d=2,\ T_n=60,\ n=?$

$$T_n = a+(n-1)d$$
$$60 = 4+(n-1)2$$
$$60 = 4+2n-2$$
$$60 = 2+2n$$
$$58 = 2n$$
$$\underline{29 = n}$$

b) Each side of hexagon is 4 cm (Smallest)

So Perimeter $= 6\times 4$

$= \underline{24\text{ cm}}$




Equilateral $\Delta$

c) Perimeter of hexagons

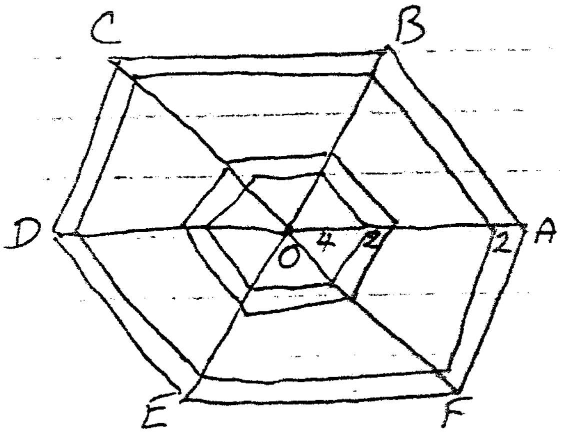
$24 + 36 + 48 + \ldots$

AP $a=24,\ d=12,\ n=29$

$$S_n = \frac{n}{2}\left[2a+(n-1)d\right]$$
$$= \frac{29}{2}\left[2\times 24+(29-1)12\right]$$
$$= 5568\text{ cm}$$

+

the Six rays each of 60 cm




Total Thread used by spider

$= 5568 + 6\times 60$

$= 5928\text{ cm}$

$\underline{(59.28\text{ m})}$