SOUTH SYDNEY HIGH SCHOOL

Year 12 Assessment Task
March 1999

MATHEMATICS

4 Unit

Instructions : **Time Allowed:** *2 periods*

1. All questions may be attempted.
2. Start each question on a new sheet of paper.
3. All necessary working should be shown.
4. Marks may be deducted for poorly arranged or missing working.
5. Approved calculators may be used.

QUESTION 1 **MARKS**

(a) Given that $z_1 = \frac{1+i}{1-i}$ and $z_2 = \frac{\sqrt{2}}{1-i}$.

(i) Express z_1 and z_2 in the form $a+ib$, and find their modulus and arguments. **3**

(ii) By plotting z_1 and z_2 on the Argand diagram, and without the calculator, show that the arguments of $z_1 + z_2$ is $\frac{3\pi}{8}$. **3**

(b) Describe geometrically the locus of z given that $\arg\left(\frac{z-2}{z+2}\right) = \frac{\pi}{3}$. **4**

(c) By using De Moivre's theorem, solve for z in $z^5 = \left(\sqrt{3}+i\right)^4$. **5**

QUESTION 2

(a) If $z = \frac{8}{1+\sqrt{3}\,i}$, find $\overline{z}$ **2**

(b) For what values of n is $\left(3+\sqrt{3}\,i\right)^n$ purely imaginary ? **3**

(c) Shade the region of the Argand diagram defined by $|z-(2+i)| \le 3$ and $-3 \le Re[(1+2i)z] \le 2$ **3**

(d) Solve the equation $z^2 + (1+2i)z + 1 - 5i = 0$ **4**

(e) Find $|z_1 z_2|$ and $\arg(z_1 z_2)$ if z_1 and z_2 are the roots of **3**

$(5+3i)z^2 - (1+4i)z + (8-2i) = 0$

QUESTION 3 (18 marks) **MARKS**

(a) (i) On a single Argand diagram, sketch the loci of $|z| = 1$ and of $|z-1| = 1$. **2**

(ii) Hence or otherwise find, in the form of $a + ib$, all complex numbers satisfying the simultaneous equations : **2**

$$|z| = 1 \text{ and } |z-1| = 1.$$

(b) $\alpha, \beta, \gamma, \delta$ are the roots of the equation $x^4 - 2x^3 + 7x^2 + 4x - 18 = 0$. **3**

Given that $\alpha + \beta = 0$, find all the roots of the equation.

(c) The equation $8x^4 + 12x^3 - 30x^2 + 17x - 3 = 0$ has a three-fold root. **2**

Find all the roots.

(d) Given that for $P(z) = z^4 - 4z^3 + 12z^2 + 4z - 13 = 0$, factorise completely given that $(2-3i)$ is a root of $P(z)$. **3**

(e) Consider the equation $x^4 + x^2 + 1 = 0$ which is known to have four complex roots.

(i) If w is a root of this equation, show that $w^6 = 1$. **2**

(ii) Deduce that w^2 is also a root of $x^4 + x^2 + 1 = 0$. **2**

(iii) Write down the other two complex roots of **2**

$$x^4 + x^2 + 1 = 0 \text{ in terms of } w.$$

(f) If $z_1 = r_1 cis\theta_1$ and $r_2 = r_2 cis\theta_2$ where $cis\theta = \cos\theta + i\sin\theta$, **2**

prove that : (i) $\left|\frac{z_1}{z_2}\right| = \frac{|z_1|}{|z_2|}$ and (ii) $\arg\left(\frac{z_1}{z_2}\right) = \arg z_1 - \arg z_2$.

End of Assessment Task !!!

Answers to 4U Assessment Task - March 1999

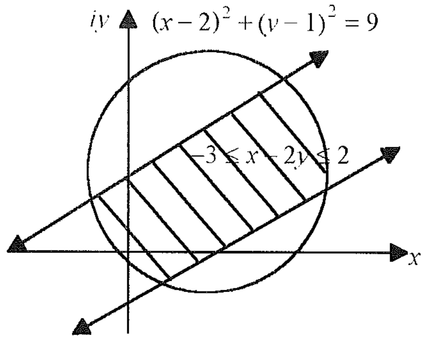
(1) (a) (i) $z_1 = i = cis\left(\frac{\pi}{2}\right)$, $z_2 = \frac{\sqrt{2}}{2}(1+i) = cis\left(\frac{\pi}{4}\right)$ (ii) Geometrical proof is best.

(b) Major arc of a circle with centre $\left(0, \frac{2\sqrt{3}}{3}\right)$ and radius $\frac{4\sqrt{3}}{3}$

(c) $z_k = 16^{\frac{1}{5}} cis\left(\frac{2\pi}{3} + 2k\pi\right)^{\frac{1}{5}}$ for $k = 0, 1, 2, 3, 4$.

(2) (a) $2\left(1+\sqrt{3}\,i\right)$ (b) $n = 6k - 3$ where $k = 1, 2, \ldots$

(c)

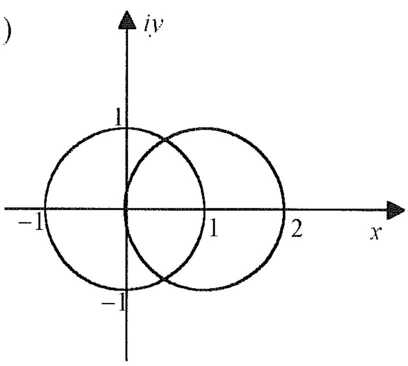


(d) $z = 1 + i$ or $-2 - 3i$

(e) $|z_1 z_2| = \sqrt{2}$,

$\arg(z_1 z_2) = -\frac{\pi}{4}$

(3) (a) (i)



(ii) $\left(\frac{1}{2}, -\frac{\sqrt{3}}{2}\right);\left(\frac{1}{2}, \frac{\sqrt{3}}{2}\right)$

(b) $\pm\sqrt{2}, \pm\sqrt{3}\,i$ (c) $\frac{1}{2}, \frac{1}{2}, \frac{1}{2}, -3$ (d) $2 \pm 3i$, ± 1

(e) (i) Proof (ii) Deduce (iii) w, w^2

(f) (i), (ii) Proofs