SOUTH SYDNEY HIGH SCHOOL

Year 12 Assessment Task 1
March 2001

MATHEMATICS

Extension 2

Instructions :

Time Allowed: *2 periods*

1. All questions may be attempted.
2. Questions are **not** of equal value.
3. All necessary working should be shown.
4. Marks may be deducted for poorly arranged or missing working.
5. Approved calculators may be used.

STANDARD INTEGRALS

$$\int x^n \, dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1;\ x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} \, dx = \ln x, \ x > 0$$

$$\int e^{ax} \, dx = \frac{1}{a}e^{ax}, \ a \neq 0$$

$$\int \cos ax \, dx = \frac{1}{a}\sin ax, \ a \neq 0$$

$$\int \sin ax \, dx = -\frac{1}{a}\cos ax, \ a \neq 0$$

$$\int \sec^2 ax \, dx = \frac{1}{a}\tan ax, \ a \neq 0$$

$$\int \sec ax \tan ax \, dx = \frac{1}{a}\sec ax, \ a \neq 0$$

$$\int \frac{1}{a^2 + x^2} \, dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \ a \neq 0$$

$$\int \frac{1}{\sqrt{(a^2 - x^2)}} \, dx = \sin^{-1}\frac{x}{a}, \ a > 0, \ -a < x < a$$

$$\int \frac{1}{\sqrt{(x^2 - a^2)}} \, dx = \ln\left\{x + \sqrt{(x^2 - a^2)}\right\}, \ |x| > |a|$$

$$\int \frac{1}{\sqrt{(x^2 + a^2)}} \, dx = \ln\left\{x + \sqrt{(x^2 + a^2)}\right\}$$

NOTE : $\ln x = \log_e x; \quad x > 0$

Question 1 (15 marks) **Marks**

(a) Solve the following equations over the complex field. **4**

(i) $x^2+5x+10=0$

(ii) $x^3+x^2-2=0$

(b) Simplify, expressing each answer in the form $a+ib$ **4**

(i) $(i-2)^2+(i+3)^2$

(ii) $3-2i+\dfrac{1}{2+i}$

(c) Find the modulus and argument of each complex number **4**

(i) $1-3i$

(ii) $1+i\tan\alpha$

(d) If $z=2-3i$ evaluate $\overline{z}$, $z+4$ and $\overline{z}-4$. **3**
Plot points, to represent these four complex numbers, in the Argand diagram.
Interpret these results geometrically.

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Question 2 (15 marks) **Marks**

(a) Find the square roots of $7-24i$. **2**

(b) $ABCD$ is a square described in an anticlockwise sense. If A and B respectively Represent $4-2i$ and $3+2i$, find the complex numbers represented by C and D. **3**

(c) Shade the region in the Argand diagram defined by the inequalities: **3**

$$-\frac{\pi}{4} < \arg z < \frac{\pi}{4} \text{ and } |z| \leq 2.$$

(d) If w is a non-real cube root of unity, evaluate $(1+w)^3(1+2w+2w^2)$. **2**
(You may assume that $1+w+w^2=0$.)

(e) By expanding $(\cos\theta+i\sin\theta)^5$, show that $\sin 5\theta$ may be expressed in the form $a\sin^5\theta+b\sin^3\theta+c\sin\theta$, where a, b and c are constants and find a, b and c. **5**

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Question 3 (20 marks)

Marks

(a) Use De Moivre's theorem to solve $z^6 = 64$. Show that the points representing the six roots of this equation on an Argand diagram form the vertices of a regular hexagon. Find the area of this regular hexagon. **5**

(b) Solve the equation $x^4 - 3x^3 - 6x^2 + 28x - 24 = 0$ given that it has a ***triple*** root. **3**

(c) Use the factor theorem to show that $1+i$ is a zero of the polynomial **3**

$$P(z) = 2z^3 - 5z^2 + 6z - 2.$$

Hence factorise the polynomial function over the complex field.

(d) If $z_1 = r_1(\cos\theta_1 + i\sin\theta_1)$ and $z_2 = r_2(\cos\theta_2 + i\sin\theta_2)$,

(i) Show that $|z_1 z_2| = |z_1|.|z_2|$ and $\arg(z_1 z_2) = \arg z_1 + \arg z_2$. **2**

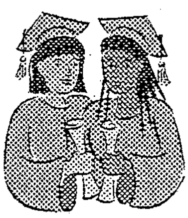
(ii) Hence deduce the result for $\left|\dfrac{z_1}{z_2}\right|$ and $\arg\left(\dfrac{z_1}{z_2}\right)$. **1**

(iii) Using the above properties, find $\left|\dfrac{1-i\sqrt{3}}{z}\right|$ and $\arg\left(\dfrac{1-i\sqrt{3}}{z}\right)$. **2**

(e) If $z = \cos\theta + i\sin\theta$,

(i) Show that $z^n + \dfrac{1}{z^n} = 2\cos n\theta$. **1**

(ii) Hence show that $\cos^5\theta = \dfrac{1}{16}(\cos 5\theta + 5\cos 3\theta + 10\cos\theta)$. **3**



End of Assessment task 1

(1) (a) (i) $x = \dfrac{-5 \pm \sqrt{15}i}{2}$

(ii) $x = 1, -1 \pm i$

(b) (i) $11 + 2i$

(ii) $\dfrac{17 - 11i}{5}$

(c) (i) $|z| = \sqrt{10}$, $\arg z = -71^0 33'$.

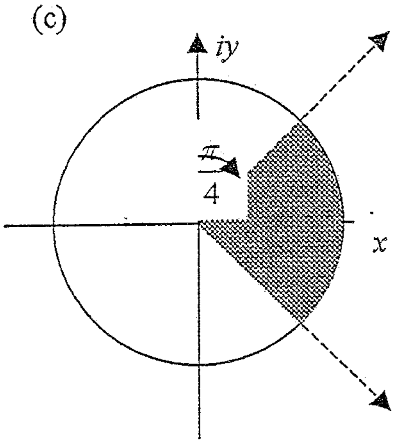
(ii) $|z| = \sec\alpha$, $\arg z = \alpha$

(d) Parallelogram.

(2) (a) $4 - 3i, -4 + 3i$

(b) $C(-1,1)$, $D(0,-3)$

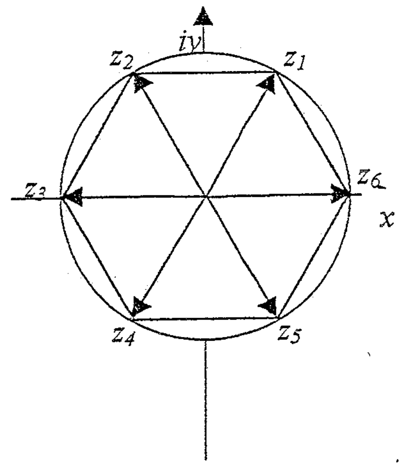
(c)



(d) 1

(e) $a = 16, b = -20, c = 5$

(3) (a)



Area $= 6\sqrt{3}\ \text{u}^2$

(b) $x = -3, 2, 2, 2$

(c) $(2z-1)(z-1-i)(z-1+i)$

(d) (i) Proof

(ii) $\left|\dfrac{z_1}{z_2}\right| = \dfrac{|z_1|}{|z_2|}$ and

$\arg\left\{\dfrac{z_1}{z_2}\right\} = \arg z_1 - \arg z_2$

(iii) $\dfrac{2}{|z|} = \dfrac{2}{r}$; $-\dfrac{\pi}{6} - \theta$

(e) (i) Proof

(ii) Proof