SOUTH SYDNEY HIGH SCHOOL

Year 12

TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION

1996

MATHEMATICS

2/3 UNIT (COMMON)

Time Allowed - Three hours
(Plus 5 minutes reading time)

DIRECTIONS TO CANDIDATES

- Attempt ALL questions.
- ALL questions are of equal value.
- Write your student Name / Number on every page of the question paper and your answer sheets.
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Standard integrals are supplied.
- Board approved calculators may be used.

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x}\,dx = \ln x, \quad x > 0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax}, \quad a \neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax, \quad a \neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax, \quad a \neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax\,dx = \frac{1}{a}\sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

QUESTION 1 | Marks

(a) Calculate $\dfrac{2.3}{\sqrt[3]{2.45-1.09}}$ rounding off your answer to one decimal place. 2

(b) Find those values of x which satisfy the inequality : $|2x-1| \le 5$ 2

(c) Solve the equation : $\dfrac{2-x}{3} - \dfrac{3-x}{2} = \dfrac{1}{5}$ 2

(d) Factorise $x^3 - 64x$ 2

(e) If $\dfrac{3}{2\sqrt{3}-1} = A + B\sqrt{3}$ find the values of A and B. 2

(f) The fine for exceeding the speed limit by more than 50 kilometres per hour has increased by 12.5% to \$558. What was the fine before the increase ? 2

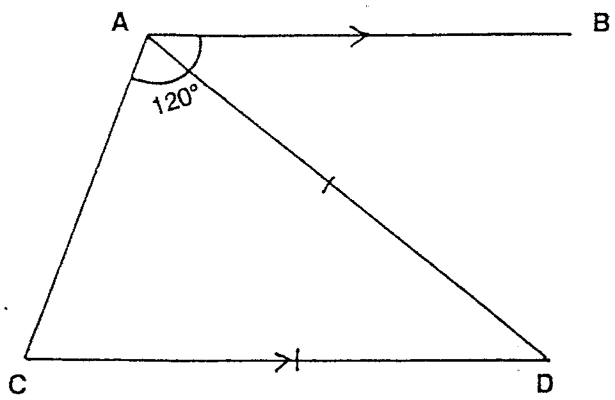
QUESTION 2 *(Begin a new sheet)* **Marks**

(a) Differentiate: 3

(i) $\sin(3x - 2)$

(ii) $\dfrac{e^x}{x}$

(b) 3



NOT TO SCALE

In the diagram, $AB \parallel CD$, $AD = CD$ and $\angle BAC = 120°$.

Copy the diagram onto your answer sheet.

(i) Explain why $\angle ACD = 60°$.

(ii) Show that $\Delta\, ADC$ is equilateral, giving reasons.

(c) Find 6

(i) $\int \sec^2 3x \, dx$

(ii) $\int (5x - 3)^5 \, dx$

(iii) $\displaystyle\int_{-1}^{0} \frac{dx}{2x + 3}$

Question 3 (Start a new page) Marks

(a) Given that $\sin X = \frac{2}{5}$ and X is an obtuse angle, find the EXACT values of $\tan X$ and $\cos X$. 3

(b) A woman commenced paying into a superannuation fund on January 1st, 1996. On this day she paid $2 000 into the fund and arranged to pay an additional $2 000 into the fund on January 1st, every subsequent year until the year 2015 which would be her final payment of $2 000. The superannuation fund has guaranteed that her investment can be locked in at a fixed interest rate of 9% per annum, compounded annually and that it will mature (be available for collection) on January 1st 2016. 5

(i) What is the value of her initial $2 000 investment after 1 year?
(ii) What is the value of her initial $2 000 investment upon maturity?
(iii) Calculate the total value of her investments upon maturity.

(c.) Simplify $\sin^3 A + \sin A \cos^2 A$ 1

(d.) Express $0{\cdot}31\dot{5}$ as a fraction. 2

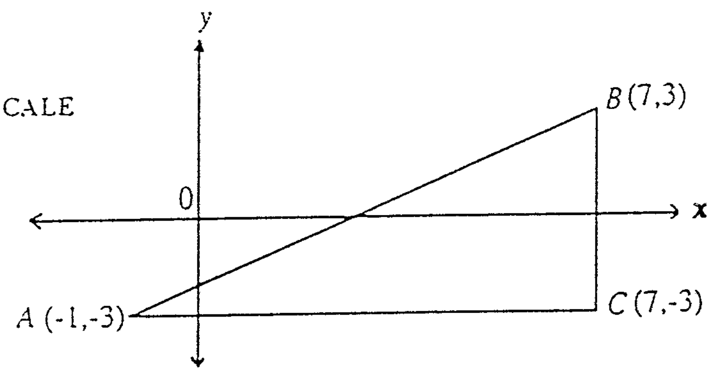
(e) Factorise $5x^2 - 2x - 3$ 1

Question 4 (Start a new page)

Marks

The points $A(-1,-3)$, $B(7,3)$ and $C(7,-3)$ determine the vertices of triangle ABC as shown in the diagram below

NOT TO SCALE



Copy this diagram onto your writing paper.

(a.) Find the distance between the points A and B 1

(b.) Calculate the gradient of the line passing through the points A and B. 1

(c.) Determine, to the nearest whole degree, the size of the acute angle formed between the line AB and the x axis. 1

(d.) M is the midpoint of the interval AB. Find the co-ordinates of M. 1

(e.) Find the equation of the line AB. 2

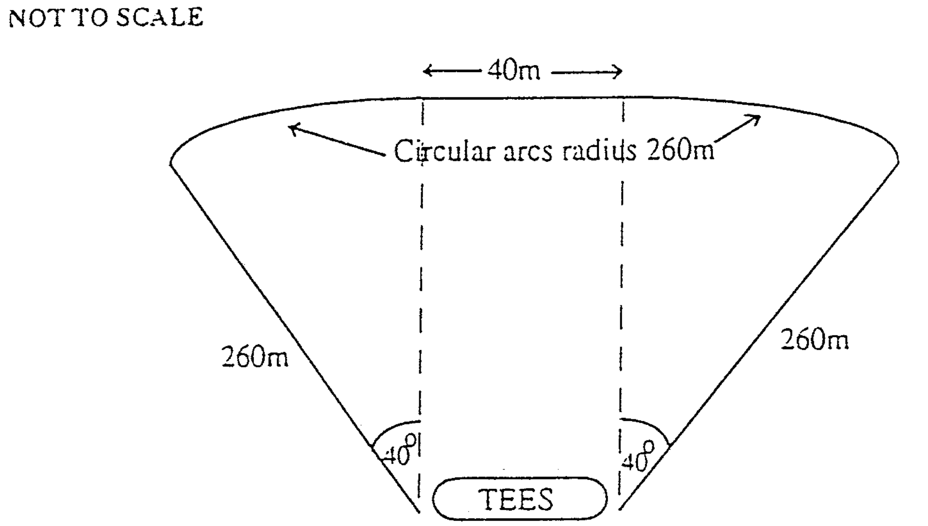
(f.) Find the area of triangle ACM. 2

(g.) Calculate the perpendicular distance of the point C from the line AB. 2

(h.) Show that a circle, with centre at point M, can be drawn to pass through all the points A, B, and C. 2

Question 5 (Start a new page) Marks

(a.) A developer is building a practise golf driving range according to the plan shown below. A boundary fence three metres high is to be constructed along the sides and end of the driving area. This is indicated by the dark lines in the plan. 5



Using the information supplied in the plan, find:

(i.) the total length of boundary fence required. Give your answer to the nearest whole metre.

(ii.) the total cost of constructing the fence if materials cost $70 per metre.

(iii.) the total area of the driving range to the nearest square metre.

(b.) Consider the curve given by $y = x^3 - 3x^2 - 9x + 2$ 7

(i.) Find the co-ordinates of the stationary points for this curve.

(ii.) Determine and describe the nature of these stationary points.

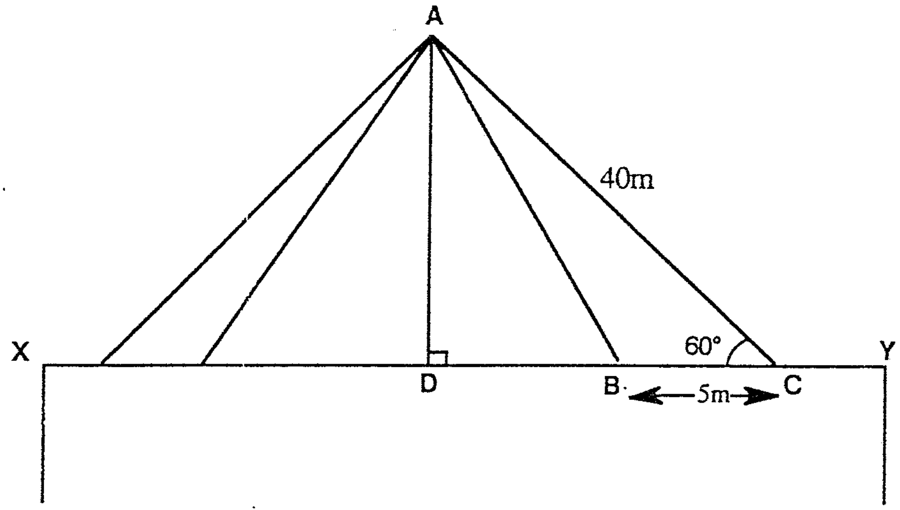
(iii.) Find the co-ordinates of any point of inflection.

(iv.) Sketch the graph of $y = x^3 - 3x^2 - 9x + 2$ over the domain $-2 \leq x \leq 4$.

Question 6 (Start a new page) **Marks**

(a) Find the equation of the tangent to the curve $y = x\ln x$ at the point (1,0). 4

(b) 4



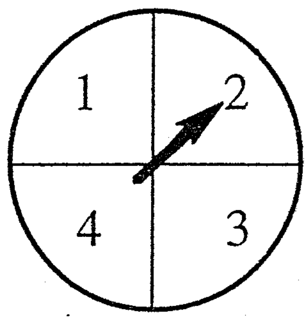
NOT TO SCALE

A horizontal bridge was built between points X and Y. Cables were used to support the bridge as shown in the diagram above. The distance between the cables AB and AC was 5 metres. Cable AC was 40 metres long and $\angle ACB = 60°$.

(i) Show that the height of A above the horizontal bridge is $20\sqrt{3}$ metres.

(ii) Use the cosine rule to show the exact length of the cable AB is $5\sqrt{57}$ metres.

(c) 4



Dino and Chris used the spinner shown above to play a game. Dino spun the spinner twice and added the results of the two spins to get his score. Chris then took his turn. The player with the highest score won the game.

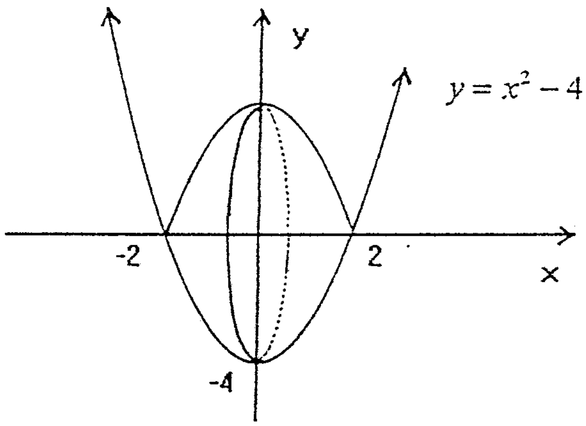
(i) Use a tree diagram or a sample space to show all the possible scores Dino could have achieved when he played the game.

(ii) What is the probability that Dino scored 6 in the game?

(iii) Dino's score was 6. What is the probability that Chris won the game?

Question 7 (Start a new page) **Marks**

(a) 3



A number of gold beads are made to form a necklace. They are formed by rotating the section enclosed by the parabola $y = x^2 - 4$ and the x-axis about the x-axis, as shown. Each bead is 4 millimetres wide.

Find the volume of gold used to make each bead.

(b) Consider the function $y = \log_e x$ 4

x	1	2	3	4	5
$\log_e x$					

(i) Copy and complete above the table giving values correct to 3 decimal places.

(ii) Using Simpson's Rule with these five function values, find an estimate for :

$$\int_1^5 \log_e x \, dx.$$

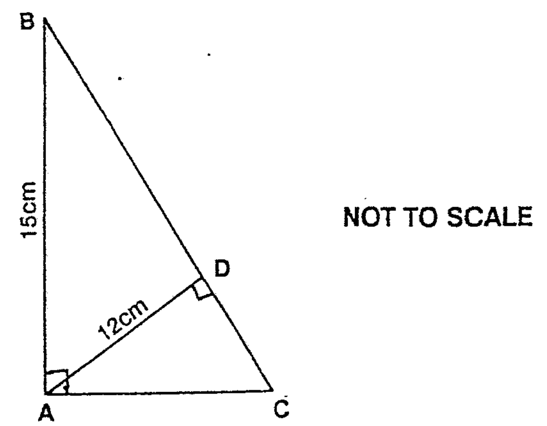
(c) (i) Evaluate $\int_0^2 \left(x^2 - 1\right) dx$ 5

(ii) Find the area enclosed by the curve $y = x^2 - 1$, the x-axis and the ordinates $x = 0$ and $x = 2$

(iii) By considering your answers to parts (i) and (ii) give reasons why $\int_a^b f(x)\, dx$ may not always represent the area between the curve $y = f(x)$, and the ordinates $x = a$ and $x = b$.

Question 8 (Start a new page) **Marks**

(a) 5



Δ ABC is right-angled at A and AD is drawn perpendicular to BC.
AB = 15 cm and AD = 12 cm.

Copy the given diagram onto your answer sheet.

(i) Show that BD = 9 cm.

(ii) Prove that ΔABC is similar to ΔDBA.

(iii) Hence find the length of AC.

(b) Alex accepted a job that pays an initial salary of \$28 000 per annum. After each year of service she will receive an increment of \$950 until she reaches the maximum salary of \$40 350. 4

(i) What will her salary be after 5 years of service?

(ii) How long will she have to work before she reaches her maximum salary?

(iii) Calculate her total earnings for the first 10 years of service.

(c) For the parabola $x^2 - 2x - 15 = 4y$ 3

(i) Find (α) the vertex

(β) the focus

(ii) Hence sketch the curve, clearly labelling the vertex and focus.

Question 9 (Start a new page) Marks

(a) Whilst driving through the town of Bathurst a motorist encounters four sets of traffic lights. Assuming that the lights are randomly operated and alternate green for 2 minutes and red for 1 minute, 4

find the probability that:

(i.) the first light the motorist encounters is red.

(ii.) all four lights he encounters are red.

(iii.) he encounters three red lights and only one green light.

(iv.) at least one of the lights he encounters is green.

(b) If $\cos A = p$, express $\dfrac{1-\sin^2 A}{\operatorname{cosec}^2 A}$ in terms of p. 3

(c) Solve the equation $\sin 2\theta = \frac{1}{2}$ for $0 \le \theta \le 2\pi$. 3

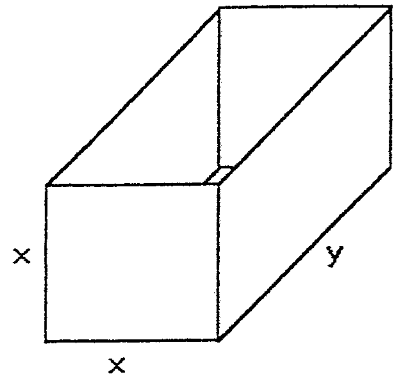
(d) For what values of k does the equation $x^2 - 2x + 3 = k$ have real roots? 2

Question 10 (Start a new page) **Marks**

(a) The diagram below shows an open rectangular box with square ends, to hold $8m^3$. 6

It is to be made at a cost of \$10 per m^2 for the base and \$5 per m^2 for the sides.

The dimensions shown on the box are in metres.



(i) Write down an expression for the volume of the box.

(ii) Show that the cost \$$C$ to build this box is given by : $C = 10x^2 + \dfrac{160}{x}$

(iii) Find the most economical dimensions.

(b) The position, x cm, of a particle moving along an x-axis is given by: 6

$$x = 3t + e^{-2t}$$

where t is the time in seconds.

(i) What is the position of the particle when $t = ½$ second?

(ii) What is the initial velocity of the particle?

(iii) Show the initial acceleration of the particle is 4 cm/s^2.

(iv) Explain why the particle will never come to rest.

(1)(a) 2.1 (to 1 d.p.)

(b) $-2 \leq x \leq 3$

(c) $x = 6\frac{1}{5}$

(d) $x(x-8)(x+8)$

(e) $A = \frac{3}{11}$, $B = \frac{6}{11}$

(f) \$496

(2) (a) (i) Graph

(ii) $\angle BCF = 60^0$

(b) (i) $\frac{3\sqrt{x}}{2}$

(ii) $e^{2x}(1+2x)$

(iii) $2\sec^2(2x+3)$

(c) (i) $\ln(2x-5)+c$

(ii) $\frac{\sqrt{3}}{4}$

(3)(a) $5\sqrt{5}$

(b) Proof

(c) $\frac{13\sqrt{5}}{5}$

(d) 32.5 cm^2

(e) $\left(-2, 3\frac{1}{2}\right)$

(f) $4x+2y+1=0$

(g) $\left(\frac{1}{2}, -\frac{3}{2}\right)$

(4) (a) $\frac{512\pi}{15}$ mm^3

(b) (i)

x	1	2	3	4	5
ln x	0	0.693	1.099	1.386	1.609

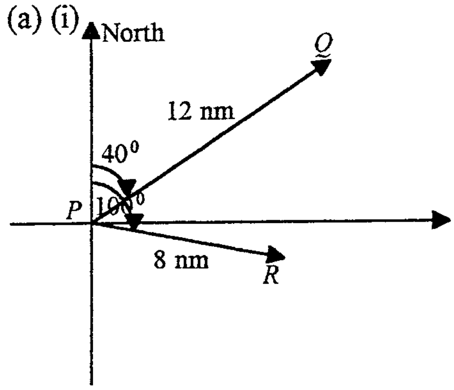
(ii) 4.041 cm^2

(c) (i) $\pm\sqrt{11}$

(ii) $x = 0$ or -3

(d) $2e$

(5) (a) (i)



(ii) 10.6 (to 1 d.p.)

(iii) 140 km^2

(b) (i) $\frac{2}{3}$

(ii) 2 $units^2$

(iii) Part (i) does not consider area below the x–axis as this area has to be considered separately.

(c) $k \geq 2$

(6)(a)(i) Diagram

(ii) to (iv) Proofs

(b) (i) $x = 300.5$ m (to 1 d.p.)

(ii) $v = 74.5\ ms^{-1}$; $a = 0.505\ ms^{-2}$

(iii) As $t \to \infty, e^{-t} \to 0, v \to 75\ ms^{-1}$

(7) (a) (i) 1.05 m

(ii) $1+1.05+1.1+1.15$

(iii) 1.85 m

(iv) $S_{21}=40.5$ m, long enough to reach the ground.

(b) $p^2(1-p^2)$

(c) $\frac{\pi}{12}, \frac{5\pi}{12}, \frac{13\pi}{12}, \frac{17\pi}{12}$

(8)(a)(i) $\dfrac{1}{1000}$ (ii) $\dfrac{1}{200}$

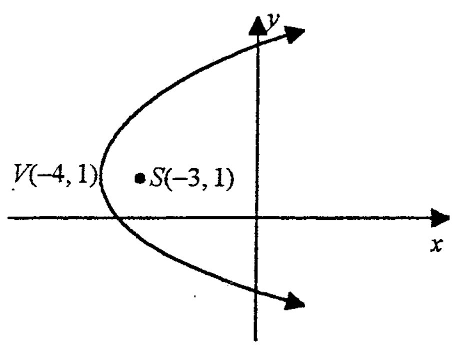
(b)(i) (α) $x<-2$ or $x>2$ (β) $x<0$

(ii) $x=-2$ max. t.p.

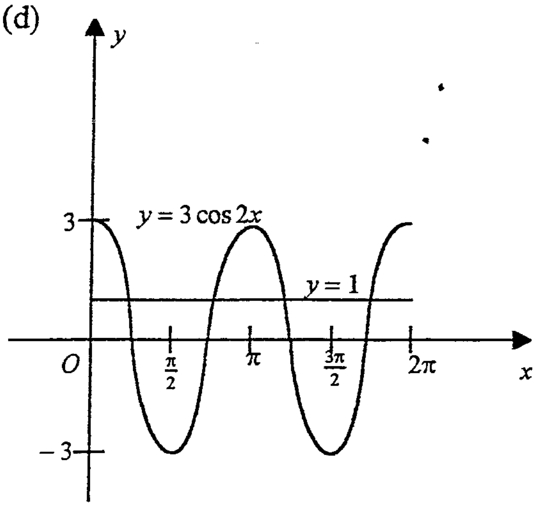
(c) (i) (α) Vertex $(-4,1)$

(β) Focus $(-3,1)$

(ii)



(d)



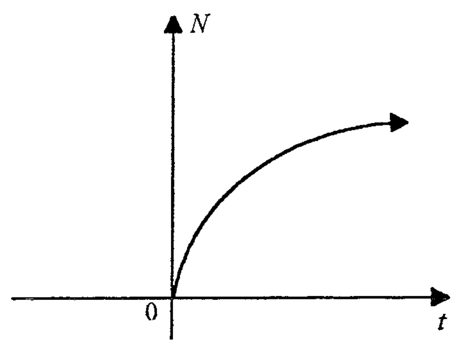
(d) In solving $\cos 2x=\frac{1}{3}$ use the graphs $y=3\cos 2x$ and $y=1$, there are **four** solutions in the given domain.

(9)(a) (i) and (ii) Proofs

(iii) $E=7071$

(b) (i) The number of vehicles has increased over the 10 years

(ii) The rate of change in the number is on the increase but heading towards a maximum as $\dfrac{d^2N}{dt^2}<0$ suggests.



(c) (i) ΔBOC is equilateral.

(ii) $2\sqrt{3}$ m (iii) $3\sqrt{3}+\frac{2\pi}{3}$ m^2

(10) (a) (i) $V=x^2y$ (ii) Proof

(iii) $x=2, y=2$ i.e a cube

(b) (i) Proof

(ii) $h=17.3$ hours

(iii) $B=211035$

(iv) after 76 hours