SOUTH SYDNEY HIGH SCHOOL

Number/Name: ..

2012

Year 12

HSC Half Yearly Exam

Friday 5th April 2013

HSC

Mathematics

Question	Mark
Multiple Choice	/10
11	/15
12	/15
13	/15
14	/15
TOTAL	

Weighting 25%

Working time: 2 hours

Total marks: 70

Outcomes:

P3 – P8
H3 – H9

Topics examined:

All topics

General Instructions:

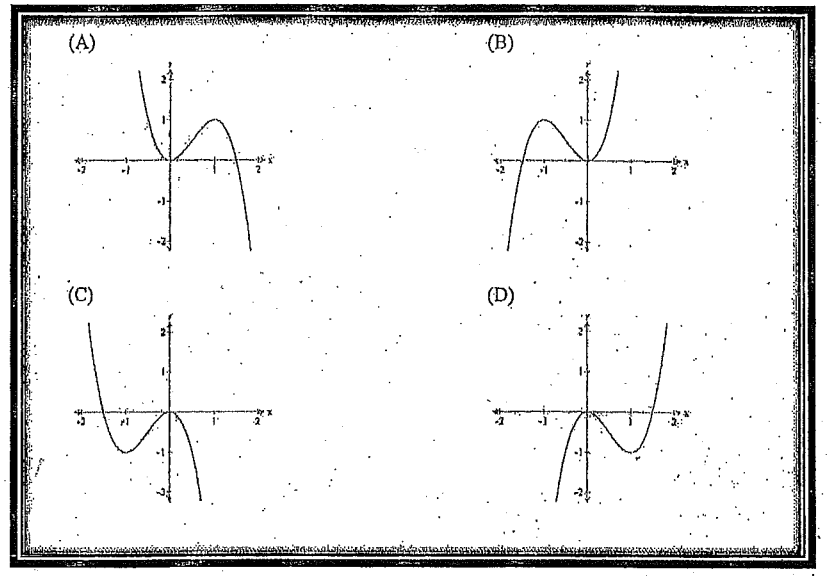
- Write using blue or black pen
- Board-approved calculators and templates may be used
- All necessary working should be shown in every question
- Questions are of equal value
- Full marks may not be awarded for careless or badly arranged work
- Questions are not necessarily arranged in order of difficulty

Total marks - 70

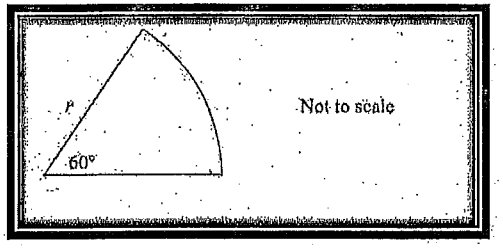
Section 1

Ten questions worth 10 marks
Answer on the answer sheet provided

1. Which of the following is the graph of $f(x)=2x^3-3x^2$?



2. The sector below has an area of 10π square units.



What is the value of r?

(A) $\sqrt{60}$ (B) $\sqrt{60\pi}$ (C) $\sqrt{\frac{\pi}{3}}$ (D) $\sqrt{\frac{1}{3}}$

3. For what values of x is the curve $f(x)=2x^3+x^2$ concave down?

(A) $x<-\frac{1}{6}$ (B) $x>-\frac{1}{6}$ (C) $x<-6$ (D)

4. What is the greatest value of the function $y = 4 - 2\cos 2x$?

(A) 2 (B) 4 (C) 6 (D) 8

5. If $4^{x-1} = 32$, then the value of x is ?

(A) 10 (B) 3.5 (C) 3 (D) 6

6. The values of x for which the curve $y = 2x^3 - 12x^2 + 18x + 7$ is decreasing are

(A) $x < 2$ (B) $x > 2$ (C) $1 < x < 3$ (D) $x < 1$ or $x > 3$

7. The angle which the straight line $3x + 5y + 2 = 0$ makes with the positive direction of the x-axis is closest to:

(A) 31° (B) 59° (C) 121° (D) 149°

8. If $\tan 2x = \sqrt{3}$ in the domain $-\pi \le x \le \pi$, the value of x is:

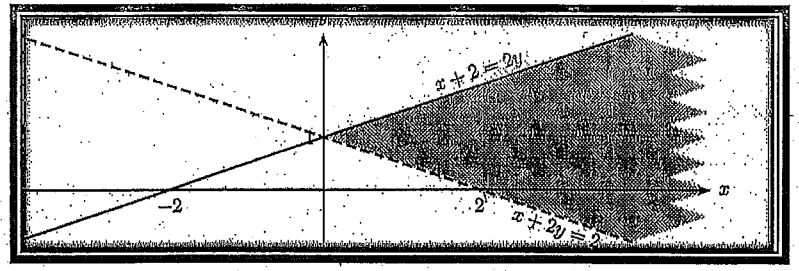
(A) $\frac{\pi}{6}, \frac{7\pi}{6}$ (B) $-\frac{5\pi}{6}, -\frac{11\pi}{6}$

(C) A and B (D) None of the above

9. What is the value of k, if the expression $4x^2 - 6x + k$ is a perfect square?

(A) $\frac{4}{9}$ (B) $\frac{9}{4}$ (C) 4 (D) 9

10.



The shaded region in the diagram satisfies:

(A) $x + 2 \ge 2y$ and $x + 2y > 2$

(B) $x + 2 \ge 2y$ and $x + 2y < 2$

(C) $x + 2 \le 2y$ and $x + 2y > 2$

(D) $x + 2 \le 2y$ and $x + 2y < 2$

Section 2

Attempt questions 11 – 14

All questions are of equal value (15 marks each)

Question 11 (15 marks) **Marks**

(a) Factorise $64 - x^3$ **2**

(b) Find the values of x for which $|3 - 2x| \le 5$ **3**

(c) Find the coordinates of the vertex of the parabola $y = x^2 + 4x - 3$. **2**

(d) Rationalise the denominator of $\frac{4}{\sqrt{5} - \sqrt{3}}$ **2**
Give your answer in simplest form

(e) Express $\frac{x}{x-2} - \frac{8}{x^2-4}$ as a single fraction in its simplest form **2**

(f) Solve $\frac{x-5}{3} - \frac{x+1}{4} = 5$ **2**

(g) Solve $x^2 = 5x$ **2**

Question 12 (15 marks)

(a) Find the equation of the parabola with vertex $(-1, 3)$ and the directrix $y = -1$. **2**

(b) Differentiate

i) $3x^2 - 4x^{-1}$ **2**

ii) $x^2 \sin(2x + 1)$ **3**

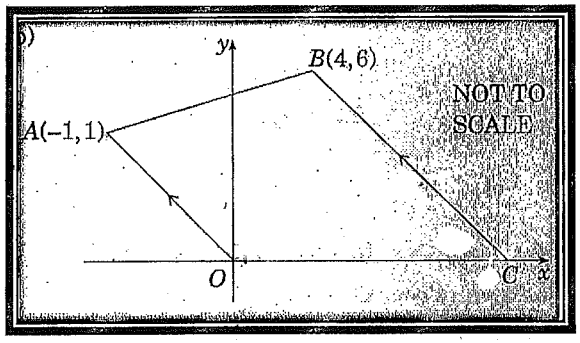
iii) $\frac{3x}{\tan 2x}$ **3**

(c) Find i) $\int (x^2 - \frac{x}{4} + \frac{3}{x^2})dx$ **3**

ii) $\int \sec^2(3x + 1)dx$ **2**

Question 13 (15 marks)

(a)



In the diagram $OABC$ is a trapezium with $OA \parallel CB$. The coordinates of O, A and B are (0,0), (-1,1) and (4,6) respectively.

i) Calculate the length of OA. **1**

ii) Write down the gradient of the line OA. **1**

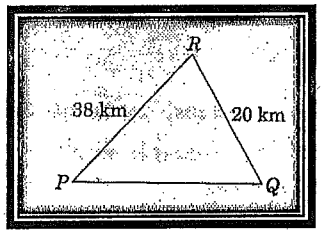
iii) What is the size of $\angle AOC$? **1**

iv) Find the equation of the line BC, and hence find the coordinates of C. **2**

v) Show that the perpendicular distance from O to the line BC is $5\sqrt{2}$ **2**

vi) Hence, or otherwise calculate the area of the trapezium OABC. **2**

(b)



NOT TO SCALE

In the diagram, the point Q is due east of P.

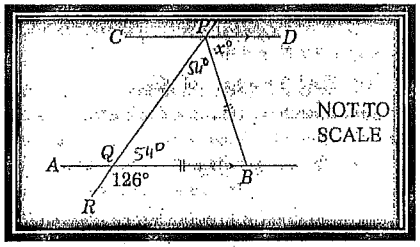
The point R is 38 km from P and 20 km from Q.

The bearing of R from Q is 325°

i) What is the size of $\angle PQR$? **1**

ii) What is the bearing of R from P? **3**

(c)



In the diagram, CD is parallel to AB, PB=QB, and $\angle BPD = x^\circ$.

Copy or trace this diagram.

Find the value of x, giving complete reasons. **2**

Question 14 (15 marks)

(a) Helen borrows \$25 000 from the bank. The loan plus interest and charges are to be repaid at the end of each month in equal monthly instalments of \$$F$ over 5 years. Interest is charged at 6% p.a. and is calculated on the balance owing at the beginning of each month. Furthermore, at the end of each month a bank charge of \$15 is added to the account valance.

Let A_n be the amount owing after n months

i) Write down an expression for A_1 **1**

ii) Deduce that the amount owing after three months is given by **2**

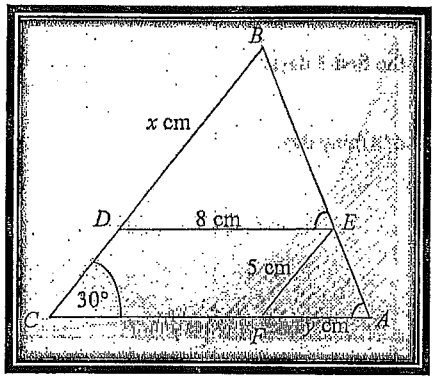
$$A_3 = 25000 \times 1.005^3 - (F-15)(1+1-005+1.005^2).$$

iii) Hence write an expression for A_n. **1**

iv) Find Helen's monthly instalment \$$F$ correct to the nearest cent. **3**

(b) In the diagram ABC is a triangle in which $\angle ACB = 30^\circ$ and D, E and F lie on the line BC, BA and AC respectively.

$CDEF$ is a parallelogram with $DE = 8$ cm and $EF = 5$ cm.



Let $BD = x$ cm and $AF = y$ cm,

i) Show triangles BDE and EFA are similar. **2**

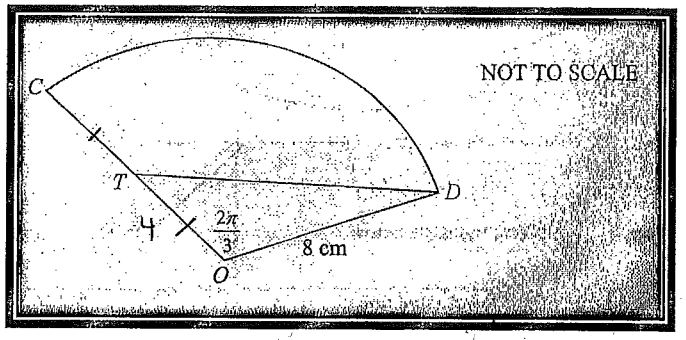
ii) Show that $xy = 40$. **1**

iii) Show that the area, A, of triangle ABC is given by **2**

$$A = 20 + 2x + \frac{50}{x}$$

iv) Find the values of and y which will minimise the area of triangle ABC. **3**

Justify your answer.



Find the perimeter of $CTDC$ in exact form.

BONUS QUESTION

(a) Consider the curve $y = 3\cos 2x$ in the domain $-\pi \le x \le \pi$.

i) State the amplitude of the curve. **1**

ii) Sketch the curve in the given domain **2**

(b) In the diagram CD is an arc of a circle with radius 8 cm and centre O. **3**

T is the midpoint of OC. Angle COD is $\frac{2\pi}{3}$.

Question 11:

2 (a) $64 - x^3 = (4-x)(16+4x+x^2)$

(b) $|3-2x| \leq 5$

$-5 \leq 3-2x \leq 5$

3 $-8 \leq -2x \leq 2$

$4 \geq x \geq -1$

$-1 \leq x \leq 4$

(c) Axis of symetry: $x = -\frac{b}{2a}$

$= -2$

when $x=-2$ $y = 4-8-3$

2 $= -7$

$\therefore$ Vertex $= (-2,-7)$

(d) $\frac{4}{\sqrt{5}-\sqrt{3}} = \frac{4}{\sqrt{5}-\sqrt{3}} \times \frac{\sqrt{5}+\sqrt{3}}{\sqrt{5}+\sqrt{3}}$

$= \frac{4(\sqrt{5}+\sqrt{3})}{2}$

2 $= 2(\sqrt{5}+\sqrt{3})$

(e) $\frac{x}{x-2} - \frac{8}{x^2-4} = \frac{x}{x-2} - \frac{8}{(x-2)(x+2)}$

$= \frac{x(x+2)-8}{(x-2)(x+2)}$

$= \frac{x^2+2x-8}{(x-2)(x+2)}$

2 $= \frac{(x+4)(x-2)}{(x-2)(x+2)}$

$= \frac{x+4}{x+2}$

(f) $\frac{x-5}{3} - \frac{x+1}{4} = 5$

$4(x-5) - 3(x+1) = 60$

2 $4x - 20 - 3x - 3 = 60$

$x - 23 = 60$

$x = 83$

(g) $x^2 = 5x$

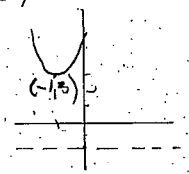
$x^2 - 5x = 0$

2 $x(x-5) = 0$

$x = 0$ or $x = 5$

Question 12:

a)



$(x-p)^2 = 4a(y-q)$

$a = 4$

2 $\therefore (x+1)^2 = 16(y-3)$

b) i) $y = 3x^2 - 4x^{-1}$

$y' = 6x + 4x^{-2}$

$= 6x + \frac{4}{x^2}$

2

ii) $y = x^2 \sin(2x+1)$ (u, v)

$y' = uv' + vu'$

$= x^2 \cdot 2\cos(2x+1) + \sin(2x+1) \cdot 2x$

$= 2x^2\cos(2x+1) + 2x\sin(2x+1)$

3 $= 2x[x\cos(2x+1) + \sin(2x+1)]$

(iii) $y = \frac{3x}{\tan 2x}$ (u, v)

$y' = \frac{vu' - uv'}{v^2}$

$= \frac{\tan 2x(3) - 3x(2\sec^2 2x)}{\tan^2 2x}$

3 $= \frac{3\tan 2x - 6x\sec^2 2x}{\tan^2 2x}$

c) i) $\int (x^2 - \frac{x}{4} + \frac{3}{x^2})\,dx$

$= \int (x^2 - \frac{1}{4}x + 3x^{-2})\,dx$

$= \frac{x^3}{3} - \frac{1}{8}x^2 + \frac{3x^{-1}}{-1} + C$

3 $= \frac{x^3}{3} - \frac{x^2}{8} - \frac{3}{x} + C$

ii) $\int \sec^2(3x+1)\,dx = \frac{1}{3}\tan(3x+1) + C$

2

Question 13:

a) i) $OA = \sqrt{3^2+4^2} = \sqrt{5}$

1 ii) $m_{OA} = -1$

1 iii) $\angle AOC = \tan^{-1}(-1)$

$= 135°$

iv) $y - y_1 = m(x - x_1)$

$y - 6 = -1(x-4)$

$y - 6 = -x + 4$

$x + y = +10$

$x + y - 10 = 10$

When $y=0$ $x = 10$

$\therefore$ Coord of C are $(10,0)$

2

v) $p = \frac{|ax_1 + by_1 + c|}{\sqrt{a^2+b^2}}$

$= \frac{|1.0 + 1.0 - 10|}{\sqrt{2}}$

$= \frac{10}{\sqrt{2}} \times \frac{\sqrt{2}}{\sqrt{2}}$

2 $= \frac{10\sqrt{2}}{2} = 5\sqrt{2}$

vi) $BC = \sqrt{6^2 + (-6)^2}$

$= \sqrt{72} = 6\sqrt{2}$

$\therefore$ Area $= \frac{1}{2}(a+b)h$

$= \frac{1}{2}(\sqrt{72} + \sqrt{2}) \cdot 5\sqrt{2}$

$= \frac{1}{2}(6\sqrt{2} + \sqrt{2}) \cdot 5\sqrt{2}$

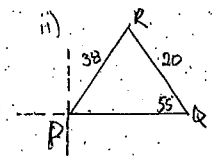
2 $= \frac{1}{2}(60 + 10)$

$= 35$

b) i) $\angle PQR = 55°$

1

ii)



$\frac{38}{\sin 55} = \frac{20}{\sin P}$

$38\sin P = 20\sin 55°$

$\sin P = \frac{20\sin 55}{38}$

$P = 25°32'$

3 $\therefore$ Bearing $= 90 - 25°32'$

$= 64°$

$\angle PQB = 180 - 126$ (Angles along st line)
$= 54°$

$\angle PBQ = 180 - (54 + 54)$
$= 72$ (Angle sum of isosceles $\triangle$)

$\therefore x = 72°$ (corresponding $CD \parallel AB$) 2

$\therefore F - 15 = 25000 \times 1.005^{60} \times \frac{0.005}{1.005^{60} - 1}$

$\therefore F = \$498.32$ 3

Question 14:

a)

i) $A_1 = 25000 \times 1.005 - F + 15$ 1

ii) $A_2 = A_1 \times 1.005 - F + 15$

$= (25000 \times 1.005 - F + 15) \times 1.005 - F + 15$

$= 25000 \times 1.005^2 - F \times 1.005 + 15 \times 1.005 - F + 15$

$A_3 = A_2 \times 1.005 - F + 15$

$= (25000 \times 1.005^2 - F \times 1.005 + 15 \times 1.005 - F + 15) \times 1.005 - F + 15$

$= 25000 \times 1.005^3 - F \times 1.005^2 + 15 \times 1.005^2 - F \times 1.005 + 15 \times 1.005 - F + 15$

$= 25000 \times 1.005^3 - F(1.005^2 + 1.005 + 1) + 15(1.005^2 + 1.005 + 1)$ 2

$= 25000 \times 1.005^3 - (F - 15)(1.005^2 + 1.005 + 1)$

$= 25000 \times 1.005^3 - (F - 15)(1 + 1.005 + 1.005^2)$

(iii) $A_n = 25000 \times 1.005^n - (F - 15)(1 + 1.005 + \dots + 1.005^{n-1})$ 1

(iv) $A_{60} = 0$

$\therefore 25000 \times 1.005^{60} - (F - 15)(1 + 1.005 + \dots + 1.005^{59}) = 0$

$S_n = \frac{a(r^n - 1)}{r - 1}$

$= \frac{1(1.005^{60} - 1)}{0.005}$

i) $\angle BED = \angle EAF$ (corresp $\angle$s, $DE \parallel CA$)

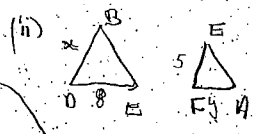
$\angle BDE = \angle DCF$ (" ")

$\angle DCF = \angle EFA$ (" $DC \parallel EF$) 2

$\therefore \angle BDE = \angle EFA = 30°$

$\therefore \triangle BDE \equiv \triangle EFA$ (AAA)

(ii)



$\therefore \frac{x}{5} = \frac{8}{y}$ 1

$xy = 40$

iii) $A = \frac{1}{2}ab\sin C$

$= \frac{1}{2}(y + 8)(x + 5)\sin 30°$

$= \frac{1}{2}\left[xy + 5y + 8x + 40\right] \times \frac{1}{2}$

$= \frac{1}{4}\left[40 + 5\left(\frac{40}{x}\right) + 8x + 40\right]$

$= \frac{1}{4}\left[80 + \frac{200}{x} + 8x\right]$ 2

$= 20 + \frac{50}{x} + 2x$

iv) $A = 2b + 2x + 50x^{-1}$

$A' = 2 - 50x^{-2}$

For st pts $A' = 0$

$2 - \frac{50}{x^2} = 0$

$2x^2 = 50$

$x^2 = 25$

$x = 5$

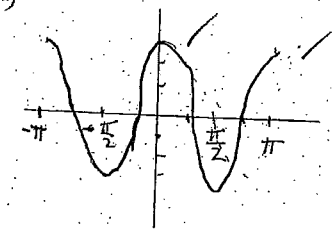
$A'' = 100x^{-3}$

$= \frac{100}{x^3} > 0$ 3

$\therefore$ min when $x = 5$, $y = 8$

a) i) Amplitude $= 3$ 1

ii)



2

b) $CD = FD$

$= 8 \times \frac{2\pi}{3}$

$= \frac{16\pi}{3}$

$TD^2 = 4^2 + 8^2 - 2(4)(8)\cos\frac{2\pi}{3}$

$= 16 + 64 - 64\cos\frac{2\pi}{3}$

$= 112$ $\frac{1}{2}$

$TD = \sqrt{112}$

$= 4\sqrt{7}$

$\therefore CTDC = 4\sqrt{7} + \frac{16\pi}{3}$ 3