South Sydney
High School

Student Name: ______________________

Section 1	
Section 2	
Question 1	
Question 2	
Question 3	
Question 4	
Question 5	
TOTAL	

2014

YEAR 12

ASSESSMENT TASK 2

Mathematics

General Instructions

- Working time – 1 lesson
- Write using black or blue pen
- Board-approved calculators may be used
- Show all necessary working in Section 2 Questions 1 - 5

Total marks - 30

Section 1

4 marks

- Attempt Questions 1-4
- Answer on the multiple choice response sheet provided at the back of this paper .

Section 2

26 marks

- Attempt Questions 1 - 5
- Allow about 45 minutes for this section

Section 1

4 marks
Attempt Questions 1 - 4
Use the multiple-choice answer sheet for Questions 1-4

1 *What are the coordinates of the vertex of the parabola* $x^2-4x-12=8y$?

(A) $(-2,2)$

(B) $(0,2)$

(C) $(2,0)$

(D) $(2,-2)$

2 *What are the coordinates of the focus of the parabola* $x^2=2(y-1)$?

(A) $(0,\frac{1}{2})$

(B) $(0,\frac{3}{2})$

(C) $(\frac{1}{2},0)$

(D) $(\frac{3}{2},0)$

3 *What is the equation of the tangent to the curve* $y=x^2-5x$ *at the point* $(1,-4)$?

(A) $y=-3x-1$

(B) $y=-3x-7$

(C) $y=3x+7$

(D) $y=3x-7$

4 *Consider the curve given by* $y=\frac{1}{2}x^4-x^3$. *What are the points of inflexion?*

(A) *only (0, 0)*

(B) *(0, 0) and* $(-1,\frac{5}{8})$

(C) *(0, 0) and* $(1,-\frac{1}{2})$

(D) *(0, 0) and* $(\frac{3}{2},-\frac{27}{32})$

Section 2

26 marks 1 – 5
Attempt Questions ~~11—16~~
Allow about ~~2 hours and~~ 45 minutes for this section

Begin each question on a separate sheet or sheets.

All necessary working should be shown in every question.

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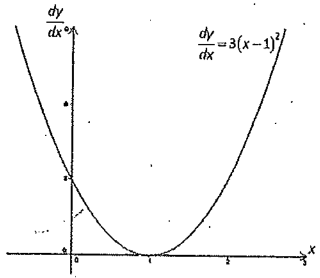
1. *For the parabola $x^2 = -16y$ find the*
   a. *focal length* 1
   b. *co-ordinates of the focus* 1
   c. *equation of the directrix* 1

2. *For the parabola $(x+2)^2 = 8(y-4)$*
   a. *Draw a neat sketch of the curve indicating the coordinates of the vertex, the coordinates of the focus and the equation of the directrix.* 5
   b. *Write down the equation of the line through the focus and parallel to the x-axis.* 1
   c. *Hence find the coordinates of the points where this line cuts the parabola.* 2

3. *The curve $f(x) = x^3 + 3x^2 - 9x - 1$ is defined in the domain $-4 \le x \le 2$*
   a. *Find the coordinates of the two stationary points and determine their nature.* 4
   b. *Show a point of inflexion occurs at $x = -1$.* 2
   c. *Sketch this curve in the domain.* 3

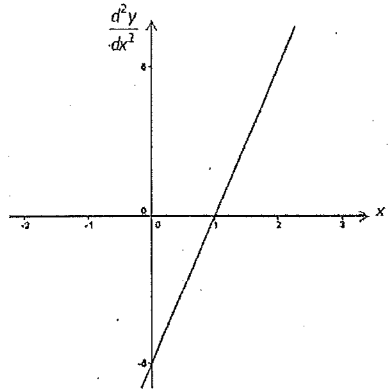
4. *The gradient function $\frac{dy}{dx}$ of a curve is illustrated in the graph below:*

   A stationary point is located at $x = 1$.



   a. *Justify this statement by reference to the graph of $\frac{dy}{dx}$* 1
   b. *What is the sign of $\frac{dy}{dx}$ for all x, $x \ne 1$?* 1
   c. *What information does this give about the curve $y = f(x)$?* 1

5. *The graph of $\frac{d^2y}{dx^2}$ is shown below:*



   a. *Copy and complete the table* 1

x	0	1	2
sign of $\frac{d^2y}{dx^2}$			

   b. *What is the nature of the stationary point at $x = 1$? Give a reason for your answer.* 2

END OF PAPER

1	$x^2-4x-12=8y$ Vertex is (2, -2) $(x-2)^2-4-12=8y$ $(x-2)^2=8y+16$ $(x-2)^2=4\times2\times(y+2)$	1 Mark: D
2	$x^2=2(y-1)$ $(x-0)^2=4\times\frac{1}{2}\times(y-1)$ Vertex is $(0,1)$ and focal length is $\frac{1}{2}$ Focus is $(0,\frac{3}{2})$	1 Mark: B
3	Concave down when $f''(x)<0$ $f'(x)=6x^2+2x$ $f''(x)=12x+2$ $12x+2<0$ $x<-\frac{1}{6}$	1 Mark: A
4	Possible points of inflexion $\frac{d^2y}{dx^2}=0$ $6x^2-6x=0$ $6x(x-1)=0$ $x=0,\ x=1$ Check for change in concavity When $x=-0.1$ then $\frac{d^2y}{dx^2}=6\times-0.1\times(-0.1-1)>0$ When $x=0.1$ then $\frac{d^2y}{dx^2}=6\times0.1\times(0.1-1)<0$ When $x=1.1$ then $\frac{d^2y}{dx^2}=6\times1.1\times(1.1-1)>0$ Hence (0, 0) and $(1,-\frac{1}{2})$ are points of inflexion.	1 Mark: C

1 For the parabola $x^2=-16y$ find the

a focal length — 1

For the parabola $x^2=-4ay$ focal length is a units

$x^2=-16y$

$4a=16$

$a=4$

The focal length is 4 units

b co-ordinates of the focus — 1

For the parabola $x^2=-4ay$

The parabola is concave down

The focus is at $(0,-a)$

$x^2=-16y$

The focus is at $(0,-4)$

c equation of the directrix — 1

For the parabola $x^2=-4ay$ the equation of the directrix is $y=a$

$x^2=-16y$

The directrix is $y=4$

2 For the parabola $(x+2)^2=8(y-4)$

Draw a neat sketch of the curve indicating the

a. coordinates of the vertex
b. coordinates of the focus
c. equation of the directrix

$(x+2)^2=8(y-4)$

$4a=8$

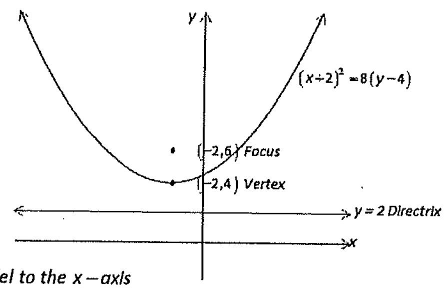
$a=2$

Focal length is 2 units

The vertex is at $(-2,4)$

The focus is at $(-2,6)$

The directrix is $y=2$



The line through the focus at $(-2,6)$ parallel to the x-axis is $y=6$

To find the points of intersection

Sub $y=6$ in $(x+2)^2=8(y-4)$

$(x+2)^2=8(6-4)$

$(x+2)^2=16$

$x+2=\pm4$

$x=2,\ -6$

Points of intersection $(2,6)$ and $(-6,6)$

a MAX 5 marks		
Sketch	neat	1
	labelled	1
Focal length		1
Vertex		1
Focus		1
Directrix		1
b Equation y=6		1
c Intersection	both correct	2
	sub correctly	1

3 The curve $f(x)=x^3+3x^2-9x-1$ is defined in the domain $-4\le x\le 2$

a Find the coordinates of the two stationary points and determine their nature.

$f(x)=x^3+3x^2-9x-1$

$f'(x)=3x^2+6x-9$

$f''(x)=6x+6$

For stationary points $f'(x)=0$

$3x^2+6x-9=0$

$x^2+2x-3=0$

$(x+3)(x-1)=0$

$x=-3, 1$

Derivatives	first	1
	second	1
Stationary pts	solved correctly	2
	correct attempt	1
TP tested	one correct	1

At $x=-3$

$f(x)=(-3)^3+3(-3)^2-9(-3)-1$

$=26$

$f''(x)=6x+6$

$=6(-3)+6$

$=-12$

<0

$\therefore$ $(-3, 26)$ is a maximum TP

At $x=1$

$f(x)=(1)^3+3(1)^2-9(1)-1$

$=-6$

$f''(x)=6x+6$

$=6(1)+6$

$=12$

>0

$\therefore$ $(1, -6)$ is a minimum TP

b Show a point of inflexion occurs at $x = -1$.

For points of inflexion $f''(x)=0$

$6x+6=0$

$x=-1$

Test

$x=-2$ $\quad f''(x)=-6$

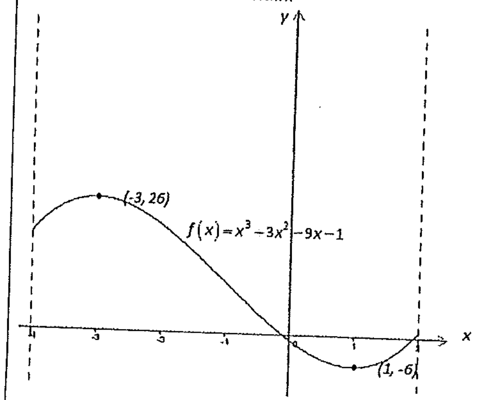
$x=0$ $\quad f''(x)=6$

$\therefore$ concavity changes

There is a point of inflexion at $x=-1$

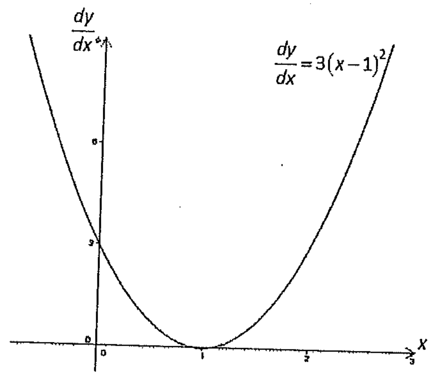
Second derivative 0	1
Test concavity	1

c Sketch this curve in the domain.



Sketch	neat, accurate	1
	correct domain	1
	labelled	1

4 The gradient function $\frac{dy}{dx}$ of a curve is illustrated in the graph below:



a A stationary point is located at $x = 1$. (1)

Justify this statement by reference to the graph of $\frac{dy}{dx}$

The graph of $\frac{dy}{dx}$ shows that $\frac{dy}{dx}=0$ at $x=0$ which is the condition that all stationary points must meet.

b What is the sign of $\frac{dy}{dx}$ for all x, $x\neq 1$? (1)

For all x values except $x = 1$ $\frac{dy}{dx}$ is positive because the graph of $\frac{dy}{dx}$ lies entirely above the x-axis.

c What information does this give about the curve $y = f(x)$? (1)

The curve $y = f(x)$ is increasing for all $x < 1$, stationary at $x = 1$ and then increasing again for all $x > 1$

5 The graph of $\frac{d^2y}{dx^2}$ is shown:

a Copy and complete the table (1)

x	0	1	2
sign of $\frac{d^2y}{dx^2}$	-	0	+

b What is the nature of the stationary point at $x = 1$? (2)

The stationary point at $x = 1$ is a point of inflexion because the sign of $\frac{d^2y}{dx^2}$ changes when moving from the LHS to the RHS of the stationary point.