SOUTH SYDNEY H.S

2014

YEAR 12

ASSESSMENT 4

Student Name: ____________

Section 1	
Section II	
Question 6	
Question 7	

Mathematics

General Instructions

- Working time – 1 LESSON
- Write using black or blue pen
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- Show all necessary working in Questions 6 & 7

Total marks – 25
Weighting 15%

Section I
5 marks

- Attempt Questions 1-5
- Allow about 8 minutes for this section

Section II
20 marks

- Attempt Questions 6 & 7
- Allow about 30 minutes for this section

Section I

5 marks
Attempt Questions 1 - 5
Allow about 8 minutes for this section

Use the multiple-choice answer sheet for Questions 1-5

1 What is the solution to the equation $4^x = 32$?

(A) 0.4
(B) 2.5
(C) 3
(D) 8

2 What is the solution to the equation $\log_e(x+2) - \log_e x = \log_e 4$?

(A) $\frac{2}{5}$
(B) $\frac{2}{3}$
(C) $\frac{3}{2}$
(D) $\frac{5}{2}$

3 Ten kilograms of chlorine is placed in water and begins to dissolve. After t hours the amount A kg of undissolved chlorine is given by $A = 10e^{-kt}$. What is the value of k given that $A = 3.6$ and $t = 5$?

(A) −0.717
(B) −0.204
(C) 0.204
(D) 0.717

4 A circular metal plate of area A cm^2 is being heated. It is given that $\frac{dA}{dt} = \frac{\pi t}{32}$ cm^2/h.

What is the exact area of the plate after 8 hours, if initially the plate had a radius of 6 cm?

(A) π (B) 0.25π
(C) 36π (D) 37π

5 A particle moves along the x-axis with acceleration $3t-2$. Initially it is 4 units to the right of the origin, with a velocity of 2 units per second. What is the position of the particle after 5 seconds?

(A) 37.5 units to the right

(B) 37.5 units to the left

(C) 51.5 units to the right

(D) 51.5 units to the left

Section II

20 marks
Attempt Questions 6 & 7
Allow about 35 minutes for this section

Answer each question in the appropriate writing booklet.

All necessary working should be shown in every question.

Question 6 (10 marks) **Marks**

(a) Differentiate **2**

$$\left(e^{2x}+2\right)^3$$

(b) Integrate **1**

$$\int e^{4x}dx$$

(c) It is assumed that the number $N(t)$ of ants in a certain nest at time $t \geq 0$ is given by

$$N(t)=\frac{A}{1+e^{-t}}$$

where A is a constant and t is measured in months.

(i) At time $t=0$, $N(t)$ is estimated at 2×10^5 ants. What is the value of A? **1**

(ii) What is the value of $N(t)$ after one month? **1**

(iii) How many ants would you expect to find in the nest when t is very large? **1**

(iv) Find an expression for the rate at which the number of ants increases an any time t. **1**

(d) A swimming pool is being topped up automatically.

The rate $\frac{dc}{dt}=\frac{300}{t+1}$ where c is the capacity in litres and t is the time in seconds

(i) How fast is the pool initially being filled? **1**

The pool initially contained 90 000 litres of water.

(ii) Find, to the nearest litre, the amount of water in the pool after 10 minutes. **2**

Question 7 (10 marks) **Marks**

(a) Find the coordinates of the point on the curve $y = 2e^{3x} + 1$, where the tangent to this curve is parallel to the line $12x - y + 1 = 0$. **3**

(b) A particle moves along a straight line so that its distance x, in metres from a fixed point O is given by $x = \cos t + t$, where t is the time measured in seconds.

(i) Where is the particle initially? **1**

(ii) When does the particle first come to rest? **2**

(iii) Where does the particle first come to rest? **1**

(iv) When does the particle next come to rest? **1**

(v) What is the acceleration of the particle after $\frac{\pi}{3}$ seconds? **2**

End of paper

1 What is the solution to the equation $4^x = 32$?

$$4^x = 32$$
$$(2^2)^x = 2^5$$
$$2x = 5x$$
$$x = 2.5$$

1 Mark: B

2 What is the solution to the equation $\log_e(x+2) - \log_e x = \log_e 4$?

$$\log_e\left(\frac{x+2}{x}\right) = \log_e 4$$
$$\left(\frac{x+2}{x}\right) = 4$$
$$x+2 = 4x$$
$$3x = 2$$
$$x = \frac{2}{3}$$

1 Mark: B

3 Ten kilograms of chlorine is placed in water and begins to dissolve. After t hours the amount A kg of undissolved chlorine is given by $A = 10e^{-kt}$. What is the value of k given that $A = 3.6$ and $t = 5$?

$$A = 10e^{-kt}$$
$$3.6 = 10e^{-k\times 5}$$
$$e^{-5k} = 0.36$$
$$-5k \log_e e = \log_e 0.36$$
$$k = \frac{\log_e 0.36}{-5}$$
$$= 0.2043302495 \approx 0.204$$

1 Mark: C

4 A circular metal plate of area A cm2 is being heated. It is given that $\frac{dA}{dt} = \frac{\pi t}{32}$ cm^2/h. What is the exact area of the plate after 8 hours, if initially the plate had a radius of 6 cm?

$$A = \pi r^2 = \pi \times 6^2 = 36\pi \text{ cm}^2$$
$$A = \int \frac{\pi t}{32} dt$$
$$= \frac{1}{64}\pi t^2 + c$$

When $t = 0, A = 36\pi$ $\quad 36\pi = \frac{1}{64}\pi 0^2 + c$

$$c = 36\pi$$

Hence $A = \frac{1}{64}\pi t^2 + 36\pi$

$$= \frac{1}{64}\pi \times 8^2 + 36\pi = 37\pi$$

1 Mark: D

5 A particle moves along the x-axis with acceleration $3t-2$. Initially it is 4 units to the right of the origin, with a velocity of 2 units per second. What is the position of the particle after 5 seconds?

$$a = 3t - 2$$
$$v = \frac{3t^2}{2} - 2t + c$$

When $t = 0$ then $v = 2$

$$2 = \frac{3\times 0^2}{2} - 2\times 0 + c \text{ or } c = 2$$
$$v = \frac{3t^2}{2} - 2t + 2$$
$$x = \frac{t^3}{2} - t^2 + 2t + k$$

When $t = 0$ then $x = 4$

$$4 = \frac{0^3}{2} - 0^2 + 2\times 0 + k \text{ or } k = 4$$
$$x = \frac{t^3}{2} - t^2 + 2t + 4$$

When $t = 5$

$$x = \frac{5^3}{2} - 5^2 + 2\times 5 + 4$$
$$= 51.5 \text{ units}$$

1 Mark: C

Question 6			
(a)	(i)	Differentiate $\left(e^{2x}+2\right)^3$ $\frac{d}{dx}\left(e^{2x}+2\right)^3=3\left(e^{2x}+2\right)^2\times 2e^{2x}$ $=6e^{2x}\left(e^{2x}+2\right)^2$	2 Marks: Correct answer. 1 Mark: chain rule used correctly
(b)	(ii)	$\int e^{4x}dx$ $\int e^{4x}dx=\frac{1}{4}e^{4x}+c$	1 Mark: Correct answer
(c)	It is assumed that the number $N(t)$ of ants in a certain nest at time $t\geq 0$ is given by $N(t)=\frac{A}{1+e^{-t}}$ where A is a constant, t is measured in months.		
	(i)	At time $t=0$, $N(t)$ is estimated at 2×10^5 ants. What is the value of A? $N(t)=\frac{A}{1+e^{-t}}$ $2\times10^5=\frac{A}{1+e^0}$ $A=4\times10^5$	1 Mark: Correct answer.
	(ii)	What is the value of $N(t)$ after one month? When $t=1$ $N(t)=\frac{A}{1+e^{-t}}$ $=\frac{4\times10^5}{1+e^{-1}}$ $=292423$ ants	1 Mark: Correct answer.
	(iii)	How many ants would you expect to find in the nest when t is very large? When $t\to\infty$ $N(t)=\frac{A}{1+e^{-t}}$ $=\frac{4\times10^5}{1+e^{-\infty}}$ $=400000$ ants	1 Mark: Correct answer.

	(iv)	Find an expression for the rate at which the number of ants increases an any time t. $N(t)=\frac{4\times10^5}{1+e^{-t}}=4\times10^5\times(1+e^{-t})^{-1}$ $\frac{dN(t)}{dt}=4\times10^5\times-1\times(1+e^{-t})^{-2}\times-1e^{-t}$ $=\frac{(4\times10^5)e^{-t}}{(1+e^{-t})^2}$ or $\frac{Ae^{-t}}{(1+e^{-t})^2}$	1 Mark: Correct answer.
(d)	A swimming pool is being topped up automatically. The rate $\frac{dc}{dt}=\frac{300}{t+1}$ where c is the capacity in litres and t is the time in seconds		
	(i)	How fast is the pool initially being filled? $\frac{dc}{dt}=\frac{300}{t+1}$ $t=0$ $\therefore \frac{dc}{dt}=300\text{L/s}$	1 Mark: Correct answer.
	(ii)	The pool initially contained 90 000 litres of water. Find, to the nearest litre, the amount of water in the pool after 10 minutes. $c=300\ln(t+1)+k$ $t=0, c=90000$ $90000=300\ln(1)+k$ $k=90000$ $\therefore c=300\ln(t+1)+90000$ $t=10\text{ mins}=600\text{ secs}$ $\therefore c=300\ln(601)+90000$ $c=91919.58\text{ L}$ $=91920\text{L}$	2 Marks: Correct answer. 1 Mark: correct integration

Question 7		
(a)	Find the coordinates of the point on the curve $y = 2e^{3x} + 1$, where the tangent to this curve is parallel to the line $12x - y + 1 = 0$. $y' = 6e^{3x} \quad m = 12$ $\therefore 6e^{3x} = 12$ $e^{3x} = 2$ $3x = \ln 2$ $x = \frac{\ln 2}{3}$ $f\left(\frac{\ln 2}{3}\right) = 2e^{3\left(\frac{\ln 2}{3}\right)} + 1$ $= 5$ $\text{Pt} = \left(\frac{\ln 2}{3}, 5\right)$	Differentiates correctly 1 Achieves 1 $x = \frac{\ln 2}{3}$ Correct answer 1
(b)	A particle moves along a straight line so that its distance x, in metres from a fixed point O is given by $x = \cos t + t$, where t is the time measured in seconds.	

	(i)	Where is the particle initially? Initially $t = 0$ $x = \cos t + t = \cos 0 + 0 = 1$	1 Mark: Correct answer.
	(ii)	When does the particle first come to rest? Particle comes to rest when $v = 0$ $v = \frac{dx}{dt}$ $0 = -\sin t + 1$ $\sin t = 1$ $t = \frac{\pi}{2}$ seconds	2 Marks: Correct answer. 1 Mark: Finds an expression for the velocity
	(iii)	Where does the particle first come to rest? When $t = \frac{\pi}{2}$ $x = \cos\frac{\pi}{2} + \frac{\pi}{2} = \frac{\pi}{2}$ metres	1 Mark: Correct answer.
	(iv)	When does the particle next come to rest? $\sin t = 1$ Next comes to rest at $t = \frac{\pi}{2}, \frac{5\pi}{2}, \ldots$seconds $\frac{5\pi}{2}$ seconds	1 Mark: Correct answer.
	(v)	What is the acceleration of the particle after $\frac{\pi}{3}$ seconds? $a = \frac{dv}{dt}$ $= -\cos t$ When $t = \frac{\pi}{3}$ $\quad a = -\cos\frac{\pi}{3} = -0.5$	2 Marks: Correct answer. 1 Mark: Finds exp for acceleration