



# Mathematics

## Instructions

1. Working time - 65 minutes
2. Reading time - 5 minutes
2. All questions should be attempted.
3. Show all working.
4. Start each question in a new booklet.
5. Marks will be deducted for careless work or poorly presented solutions.
6. On the cover sheet of the answer booklet clearly show:
  - a) your name
  - b) your mathematics class and teacher
7. Question 1 - parts a) to e) are multiple choice. Use the multiple choice answer sheet provided.

## Question 1 (5 Marks) -

Marks

Multiple Choice - For Parts a) to e) are multiple choice. Use the multiple choice answer sheet provided.

- a) Which of the given numbers  $\sqrt{\frac{6}{12}}$ ,  $\sqrt{0.81}$ , 5, -3.26,  $0.\dot{5}$ ,  $\sqrt{\frac{3}{12}}$ ,  $\frac{\pi}{5}$  are rational? 1
- (A)  $\sqrt{0.81}$ , 5, -3.26,  $0.\dot{5}$ , and  $\sqrt{\frac{3}{12}}$
- (B)  $\sqrt{\frac{6}{12}}$  and  $\frac{\pi}{5}$
- (C)  $\sqrt{\frac{6}{12}}$ ,  $\sqrt{0.81}$  and  $\sqrt{\frac{3}{12}}$
- (D) 5, -3.26 and  $\sqrt{\frac{6}{12}}$
- b) Which of the following is not a quadratic equation? 1
- (A)  $x^2 - 1 = 0$
- (B)  $x^2 - 1 + 2x = 0$
- (C)  $x^2 - \frac{1}{2} = 0$
- (D)  $x^2 - \frac{1}{x} = 0$
- c) The factorised form of  $a^3 + b^3$  is 1
- (A)  $(a + b)^3$
- (B)  $(a + b)(a^2 - 2ab + b^2)$
- (C)  $(a + b)(a^2 + b^2)$
- (D)  $(a + b)(a^2 - ab + b^2)$

Question 1 (cont'd)

Marks

d) Complete  $m^2 + 50m + \square = (m + \triangle)^2$ . The value of  $\square$  and  $\triangle$  are

1

(A)  $\square = 25, \triangle = 5$

(B)  $\square = 625, \triangle = 25$

(C)  $\square = 25, \triangle = -5$

(D)  $\square = 625, \triangle = 5$

e) The equation  $x^2 - ax - 15 = 0$  has one solution  $x = -5$ . The value of  $a$  and the other solution is:

1

(A)  $a = 2, x = 3$

(B)  $a = -2, x = -3$

(C)  $a = 2, x = -3$

(D)  $a = -2, x = 3$

Question 2 (10 Marks) – Start A New Booklet

Marks

a) Find the value of  $\pi^2$  correct to two decimal places.

1

b) A garden consists of a 40 metre by 25 metre rectangular garden bed surrounded by a concrete path 2 metre wide. What percentage of the yard is the garden?

2

c) Solve for  $a$   $7(a - 4) = 2(a + 11)$

2

d) Solve for  $x$  in  $3x(x - 2) = 0$

1

e) Find the value of

$$\frac{23.1}{56.3 \times \sqrt{25.04}}$$

correct to 3 significant figures

2

f) Factorise  $ab - a - bx + x$

2

Question 3 (10 Marks) – Start A New Booklet

Marks

- a) Express  $3\sqrt{18} - 4\sqrt{8}$  in its simplest surd form 2
- b) Express  $0.2\dot{3}$  as a fraction in simplest form 2
- c) Factorise  $x^3 - 27$  2
- d) Solve  $x^2 + 4x - 1 = 0$  leaving your answer in simplest surd form 2
- e) Simplify  $\frac{v^2 - u^2}{u - v}$  2

Question 4 (10 Marks) – Start A New Booklet

Marks

- a) Simplify  $\frac{2}{x} + \frac{3}{x^2 - 2x}$  2
- b) Solve the quadratic equation  $5x^2 - 18x + 16 = 0$  2
- c) Write the expression as a product of three factors  $a^4 + a^3 + a^2 + a$  2
- d) Expand and simplify  $(\sqrt{6} + \sqrt{11})(2\sqrt{6} - \sqrt{11})$  2
- e) Solve for  $x$   $\frac{x-2}{2} + \frac{x+1}{5} = 2$  2

**Question 5 (10 Marks) – Start A New Booklet**

Marks

- a) Solve simultaneously for  $x$  and  $y$

2

$$\begin{aligned}x - 2y &= 8 \\ 5x + 3y &= 1\end{aligned}$$

- b) Find  $p$  and  $q$  if

2

$$\frac{6}{2 - \sqrt{3}} = p + q\sqrt{3}$$

- c) Rearrange the formula so that  $b$  is the subject of the formula

2

$$a = \frac{b + 5}{b - 4}$$

- d) Simplify

4

$$\frac{1}{x^2 - 4x + 3} + \frac{1}{x^2 - 5x + 6} + \frac{1}{x^2 - 3x + 2}$$

**Question 6 (10 Marks) – Start A New Booklet**

Marks

- a) The perimeter of a rectangle is 40 cm and its area is 84 cm<sup>2</sup>

- (i) If the width of the rectangle is  $x$  cm, express the length in terms of  $x$

1

- (ii) Write down the area of the rectangle in terms of  $x$

1

- (iii) Form a quadratic equation in  $x$  and solve it to find the length and width

2

- b) Simplify

3

$$\frac{a^2}{a^3 + b^3} + \frac{a - b}{a^2 - ab + b^2} + \frac{1}{a + b}$$

- c) Solve simultaneously for  $x$  and  $y$

3

$$2x + y = 5$$

$$4x^2 + y^2 = 17$$

SOLUTIONS  
Mathematics

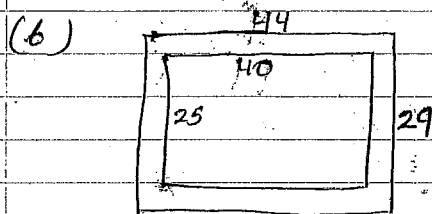
Common Test 1 2013

Question 1      Multiple Choice

- (a) A  
(b) D  
(c) D  
(d) B  
(e) D

Question 2

(a) 9.87 (2 d.p)



$$\frac{(40 \times 25) \times 100}{(44 \times 29)}$$

$$= 78.4\% \text{ (1 d.p.)}$$

(c)  $7(a-4) = 2(a+11)$   
 $7a - 28 = 2a + 22$   
 $7a - 2a = 22 + 28$   
 $5a = 50$   
 $a = 10$

(d)  $3x(x-2) = 0$   
 either  $3x = 0$       or  $x-2 = 0$   
 $x = 0$                        $x = 2$   
 there are 2 answers  $x=0$  &  $x=2$

Question 2 (Cont'd)

(e)  $\frac{23.1}{56.3 \times \sqrt{25.04}}$   
 $\approx 0.0819948$   
 $\approx 0.0820$  (3 sig. fig)

(f)  $ab - a - bx + x$   
 $= a(b-1) - x(b-1)$   
 $= (a-x)(b-1)$

Question 3

(a)  $3\sqrt{18} - 4\sqrt{8}$   
 $= 3 \cdot \sqrt{9} \cdot \sqrt{2} - 4 \cdot \sqrt{4} \cdot \sqrt{2}$   
 $= 9\sqrt{2} - 8\sqrt{2}$   
 $= \sqrt{2}$

(b) let  $x = 0.2\dot{3}$       (i)  
 $\therefore 10x = 2.\dot{3}$       (ii)  
 $100x = 23.\dot{3}$       (iii)  
 taking away (ii) from (iii)  
 $100x = 23.\dot{3}$   
 $- 10x = 2.\dot{3}$   
 $90x = 21$

$$x = \frac{21}{90}$$

$$x = \frac{7}{30}$$

### Question 3 cont'd

$$\begin{aligned} (c) \quad x^3 - 27 \\ = x^3 - 3^3 \\ = (x-3)(x^2+3x+9) \end{aligned}$$

$$\begin{aligned} (d) \quad x^2 + 4x - 1 &= 0 \\ x^2 + 4x &= 1 \\ x^2 + 4x + 4 &= 1 + 4 \quad (\text{completing square}) \\ (x+2)^2 &= 5 \\ x+2 &= \pm\sqrt{5} \\ x &= \pm\sqrt{5} - 2 \\ \boxed{x = \sqrt{5} - 2} \quad \text{and} \quad \boxed{x = -\sqrt{5} - 2} \end{aligned}$$

$$\begin{aligned} (e) \quad \frac{v^2 - u^2}{u - v} \\ = \frac{(v-u)(v+u)}{u-v} \\ = \frac{(v-u)(v+u)}{-1(v-u)} \\ = -(v+u) \end{aligned}$$

### Question 4

$$\begin{aligned} (a) \quad \frac{2}{x} + \frac{3}{x^2 - 2x} \\ = \frac{2(x-2) + 3}{(x)(x-2)} \\ = \frac{2x - 4 + 3}{x(x-2)} \\ = \frac{2x - 1}{x(x-2)} \quad \text{or} \quad \frac{2x-1}{x^2-2x} \end{aligned}$$

$$\begin{aligned} (b) \quad 5x^2 - 18x + 16 &= 0 \\ 5x^2 - 10x - 8x + 16 &= 0 \\ 5x(x-2) - 8(x-2) &= 0 \\ (5x-8)(x-2) &= 0 \\ 5x-8=0 \quad \text{or} \quad x-2=0 \\ \boxed{x = \frac{8}{5}} \quad \text{and} \quad \boxed{x = 2} \end{aligned}$$

$$\begin{aligned} (c) \quad a^4 + a^3 + a^2 + a \\ = a[a^3 + a^2 + a + 1] \\ = a[a^2(a+1) + 1(a+1)] \\ = a(a^2+1)(a+1) \end{aligned}$$

$$\begin{aligned} (d) \quad (\sqrt{6} + \sqrt{11})(2\sqrt{6} - \sqrt{11}) \\ = 2 \times 6 - \sqrt{66} + 2\sqrt{66} - 11 \\ = 12 + \sqrt{66} - 11 \\ = 1 + \sqrt{66} \end{aligned}$$

### Question 4 cont'd

$$(c) \frac{x-2}{2} + \frac{x+1}{5} = 2$$

$$\frac{5(x-2)}{5 \times 2} + \frac{2(x+1)}{2 \times 5} = 2$$

$$\frac{5x-10}{10} + \frac{2x+2}{10} = 2$$

$$\frac{5x-10+2x+2}{10} = 2 \quad (\times 10)$$

$$7x-8 = 20$$

$$7x = 28$$

$$\boxed{x = 4}$$

### Question 5

$$(a) \begin{array}{r} x-2y = 8 \quad \text{--- (1)} \\ 5x+3y = 1 \quad \text{--- (2)} \end{array}$$

$\begin{array}{l} \nearrow \times 3 \\ \nwarrow \times 2 \end{array}$

$$\begin{array}{r} 3x-6y = 24 \quad \text{--- (1)} \\ + 10x+6y = 2 \quad \text{--- (2)} \\ \hline 13x = 26 \end{array}$$

$$\boxed{x = 2}$$

$$x-2y = 8$$

$$2-2y = 8$$

$$-2y = 6$$

$$\boxed{y = -3}$$

$$(b) \frac{6}{2-\sqrt{3}} \times \frac{(2+\sqrt{3})}{(2+\sqrt{3})} \quad \left( \begin{array}{l} \text{rationalizing denominator} \\ \text{of L.H.S.} \end{array} \right)$$

$$\frac{12+6\sqrt{3}}{2^2-3}$$

$$12+6\sqrt{3} = p+q\sqrt{3} \quad \left( \begin{array}{l} \text{equating with} \\ \text{R.H.S.} \end{array} \right)$$

$$\therefore p=12 \text{ \& } q=6$$

### Question 5

$$(c) a = \frac{b+5}{b-4}$$

$$a(b-4) = b+5$$

$$ab-4a = b+5$$

$$ab-b = 4a+5$$

$$b(a-1) = 4a+5$$

$$\boxed{b = \frac{4a+5}{a-1}}$$

$$(d) \frac{1}{x^2-4x+3} + \frac{1}{x^2-5x+6} + \frac{1}{x^2-3x+2}$$

$$\frac{1}{x^2-3x-x+3} + \frac{1}{x^2-3x-2x+6} + \frac{1}{x^2-2x-x+2}$$

$$\frac{1}{x(x-3)-1(x-3)} + \frac{1}{x(x-3)-2(x-3)} + \frac{1}{x(x-2)-1(x-2)}$$

$$\frac{1}{(x-1)(x-3)} + \frac{1}{(x-2)(x-3)} + \frac{1}{(x-1)(x-2)}$$

$$\frac{(x-2) + (x-1) + (x-3)}{(x-1)(x-2)(x-3)}$$

$$\frac{3x-6}{(x-1)(x-2)(x-3)}$$

$$\frac{3x-6}{(x-1)(x-2)(x-3)}$$

$$\frac{3(x-2)}{(x-1)(x-2)(x-3)}$$

$$\frac{3}{(x-1)(x-3)}$$

### Question 6

(a) (i)  $2x + 2y = 40$  (let length =  $y$  & width =  $x$ )

$$x + y = 20$$

$$y = \boxed{20 - x}$$

(ii)  $A = x(20 - x)$   
 $= 20x - x^2$

(iii)  $20x - x^2 = 84$

$$-x^2 + 20x - 84 = 0$$

$$x^2 - 20x + 84 = 0$$

$$x^2 - 14x - 6x + 84 = 0$$

$$x(x - 14) - 6(x - 14) = 0$$

$$(x - 6)(x - 14) = 0$$

$$\boxed{x = 6} \text{ or } \boxed{x = 14}$$

if length = 6 & if width = 16  
width = 14 & length = 14

If we accept the logic that length is longer than width then only answer will be length = 14 & width = 6

(b)  $\frac{a^2}{a^3 + b^3} + \frac{(a-b)}{a^2 - ab + b^2} + \frac{1}{a+b}$

$$\frac{a^2}{(a+b)(a^2 - ab + b^2)} + \frac{(a-b)}{(a^2 - ab + b^2)} + \frac{1}{(a+b)}$$

$$\frac{a^2 + (a+b)(a-b) + 1(a^2 - ab + b^2)}{(a+b)(a^2 - ab + b^2)}$$

$$\frac{a^2 + a^2 - b^2 + a^2 - ab + b^2}{(a+b)(a^2 - ab + b^2)}$$

$$\frac{3a^2 - ab}{(a+b)(a^2 - ab + b^2)}$$

$$\frac{3a^2 - ab}{(a+b)(a^2 - ab + b^2)} \text{ or } \frac{a(3a - b)}{a^3 + b^3}$$

### Question 6 Contd

(c)  $2x + y = 5$  — (i)

$$4x^2 + y^2 = 17$$
 — (ii)

$$y = 5 - 2x \text{ from (i)}$$

$$y^2 = 25 - 20x + 4x^2 \text{ (squaring both sides)}$$

$$y^2 = 17 - 4x^2 \text{ — from (ii)}$$

$$\therefore 25 - 20x + 4x^2 = 17 - 4x^2 \text{ (Putting } y^2 \text{ value from (i) \& (ii))}$$

$$8x^2 - 20x + 8 = 0$$

$$2x^2 - 5x + 2 = 0$$

$$2x^2 - 4x - x + 2 = 0$$

$$2x(x - 2) - 1(x - 2) = 0$$

$$(2x - 1)(x - 2) = 0$$

$$2x - 1 = 0 \text{ or } x - 2 = 0$$

$$x = \frac{1}{2} \text{ or } x = 2$$

When  $\boxed{x = \frac{1}{2}}$  & when  $\boxed{x = 2}$   
 $\boxed{y = 4}$   $\boxed{y = 1}$

There are 2 sets of solutions.