



Student Number

St. Catherine's School, Waverley
2014

**TRIAL HIGHER SCHOOL CERTIFICATE
EXAMINATION**

Mathematics Extension 1

General Instructions

- Reading Time – 5 minutes
- Working Time – 2 hours
- Write using black or blue pen
- Black pen is preferred
- Board-approved calculators may be used
- A table of standard integrals is provided at the back of this paper
- In Questions 11-14, show relevant mathematical reasoning and/or calculations
- Task weighting – 40%

Section I Pages 3-6

10 marks

- Attempt Questions 1 – 10
- Allow about 15 minutes for this section
- Answer on the multiple choice answer sheet provided.

Section II Pages 7-13

60 marks

- Attempt Questions 11-14
- Allow about 1 hour and 45 minutes for this section
- Answer each question in the booklet provided.

Total Marks – 70

STANDARD INTEGRALS

$$\int x^n dx = \frac{1}{n+1} x^{n+1}, \quad n \neq -1; \quad x \neq 0, \text{ if } n < 0$$

$$\int \frac{1}{x} dx = \ln x, \quad x > 0$$

$$\int e^{ax} dx = \frac{1}{a} e^{ax}, \quad a \neq 0$$

$$\int \cos ax dx = \frac{1}{a} \sin ax, \quad a \neq 0$$

$$\int \sin ax dx = -\frac{1}{a} \cos ax, \quad a \neq 0$$

$$\int \sec^2 ax dx = \frac{1}{a} \tan ax, \quad a \neq 0$$

$$\int \sec ax \tan ax dx = \frac{1}{a} \sec ax, \quad a \neq 0$$

$$\int \frac{1}{a^2 + x^2} dx = \frac{1}{a} \tan^{-1} \frac{x}{a}, \quad a \neq 0$$

$$\int \frac{1}{\sqrt{a^2 - x^2}} dx = \sin^{-1} \frac{x}{a}, \quad a > 0, \quad -a < x < a$$

$$\int \frac{1}{\sqrt{x^2 - a^2}} dx = \ln \left(x + \sqrt{x^2 - a^2} \right), \quad x > a > 0$$

$$\int \frac{1}{\sqrt{x^2 + a^2}} dx = \ln \left(x + \sqrt{x^2 + a^2} \right)$$

NOTE: $\ln x = \log_e x, \quad x > 0$

Extension 1 trials.

1 If $y = \sin^{-1}(x^2)$, then $\frac{dy}{dx} =$

A $\frac{2x}{\sqrt{1-x^4}}$

B $\frac{2x}{\sqrt{1-x^2}}$

C $2x \cos^{-1}(x^2)$

D $\frac{2x}{1-x^4}$

2 $\int \sin^2 3x \, dx$ is

A $\cos^2 3x + C$

B $2 \sin 3x \cos 3x + C$

C $\frac{1}{2} \left(x - \frac{\cos 6x}{6} \right) + C$

D $\frac{1}{2} \left(x - \frac{\sin 6x}{6} \right) + C$

3 The primitive of $\sqrt{e^{3x}}$ is given by

A $\frac{3}{2\sqrt{e^{3x}}} + C$

B $\frac{3}{2} e^{\frac{3x}{2}} + C$

C $\frac{2}{3} e^{\frac{3x}{2}} + C$

D $\frac{1}{2} e^{3x} + C$

4 $\int \frac{dx}{\sqrt{1-4x^2}}$ is given by

A $\sin^{-1} 2x + C$

B $\sin^{-1} \frac{x}{2} + C$

C $4 \sin^{-1} 2x + C$

D $\frac{1}{2} \sin^{-1} 2x + C$

5 The equation of motion of a particle moving in Simple harmonic Motion is given by

$x = 1 - 3x$, which of the following statements is true?

A The period of motion is $\frac{2\pi}{3}$ and the centre is $x = \frac{1}{3}$

B The period of motion is $\frac{-2\pi}{3}$ and the centre is $x = 3$

C The period of motion is $\frac{2\pi}{3}$ and the centre is $x = 3$

D The period of motion is $\frac{2\pi}{\sqrt{3}}$ and the centre is $x = \frac{1}{3}$

- 6 Given that $\frac{f'(x)}{f(x)} = 1$, which of the following statements is true?

(note: C is a constant in each case)

A $f(x) = \ln x + C$

B $f(x) = e^x + C$

C $f(x) = C e^x$

D $f(x) = C \ln x$

- 7 Consider the graph of the function $y = \frac{x^2+1}{x}$. The equations of the asymptotes are

A $x = 0$ and $x = 1$

B $x = 0$ and $y = x$

C $x = -1$ and $x = 1$

D No asymptotes

- 8 The solution to the inequality $x(x^2 - 4) > 0$ is

A $-2 \leq x \leq 0$ or $x > 2$

B $-2 < x < 2$

C $x < -2$ or $x > 2$

D $x < -2$ or $0 < x < 2$

- 9 Surface area S of a spherical balloon is given by the formula $S = 4\pi r^2$, where r is the radius.

A spherical balloon is being inflated so that, $\frac{dr}{dt} = 2$ cm/sec.

The value of $\frac{ds}{dt}$, when the surface area is 16π cm² is given by

A 32π

B $\frac{16}{\pi}$

C $256\pi^2$

D No sufficient information

- 10 $\cos^{-1}\left(\cos\frac{11\pi}{6}\right)$ is

A $\frac{11\pi}{6}$

B $\frac{\sqrt{3}}{2}$

C $\frac{7\pi}{6}$

D $\frac{\pi}{6}$

Question 11 Start a new page

- a) Solve for x : $\frac{1}{x-1} \geq 5$

3

- b) Find the value of k : $x^{3k+4} = e^{8\ln x}$

2

- c) Find the ratio in which P divides the interval AB, internally, where
P is $(2, \frac{20}{3})$ A is $(1, 5)$ and B is $(4, 10)$.

3

- d) The lines $y = 3mx + 1$ and $y = mx$ are inclined at an angle of α , where
 $\tan \alpha = \frac{1}{2}$. $m > 0$

- (i) Show that $3m^2 - 4m + 1 = 0$

2

- (ii) Hence find the possible values of m .

1

- e) If $x^2 - x - 2$ is a factor of the polynomial $(Px) = x^4 + 3x^3 + ax^2 - 2x - b$, find
the values of a and b .

4

Question 12 Start a new page

- a) Show that $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{5} = \tan^{-1} \frac{4}{7}$

3

- b) Use the substitution $x = 2 \sin \theta$, to evaluate $\int_0^2 \sqrt{4 - x^2} dx$

4

- c) Find the general solution to the equation $\sin 2\theta = \cos \theta$

3

- d) (i) Show that $\frac{1-x^2}{1+x^2} = -1 + \frac{2}{x^2+1}$

1

- (ii) Hence or otherwise clearly sketch the graph of the function

4

$$y = \frac{1-x^2}{1+x^2}, \text{ locating any stationary points and equations of asymptotes.}$$

(Use at least one third of a page)

Question 13 Start a new page

a) Find $\int \frac{5}{1+16x^2} dx$

2

b) Find the constant term in the expansion of $(2x - \frac{1}{x^2})^9$

3

c) Consider the expansion of $(3 + 4x)^{12}$ in ascending powers of x.

(i) Show that $\frac{\text{coefficient of } t_{r+1}}{\text{coefficient of } t_r} = \frac{4(13-r)}{3r}$, where t_r is the r^{th} term.

2

(ii) Hence or otherwise find the greatest coefficient in the expansion of $(3 + 4x)^{12}$

2

d) A particle is projected from a point O on level ground with velocity 20 metres per second.

Take the acceleration due to gravity as 10 m per sec^2

(i) Show that the equations of motion are given by

2

$x = 20t \cos \alpha$ and $y = -5t^2 + 20t \sin \alpha$, where α is the angle of projection. The axes are placed at the point of projection.

(ii) Show that the Cartesian equation of the motion is

2

$$y = x \tan \alpha - \frac{x^2}{80} \sec^2 \alpha$$

If this particle hits a target at a horizontal distance of 20 metres and a vertical height of 10 metres.

(iii) Find the values of α .

2

Question 14 Start a new page

a) A particle moves with an acceleration given by the expression $a = 7x$. Initially the particle starts from the origin with a velocity of $-3 \text{ metres per second}$. Find an expression for the velocity v , in terms of the displacement, x

3

b) The displacement of a particle (in cm) from a point O on a line after t seconds is given by $x = 3 \sin(2t + \alpha)$. Initially the particle is at $x = 1.5$

3

(i) Find the value of α

1

(ii) Find the acceleration \ddot{x} in terms of the displacement x

1

(iii) Find the time the particle takes to reach the point $x = 0$, for the first time

2

(iii) Find the time it takes to reach an acceleration of -12 cm per sec^2 , for the first time.

2

c) A function is defined as $f(x) = 1 - \cos \frac{x}{2}$

1

(i) State the period of this function

1

(ii) Find the largest domain of the function for which the inverse function $f^{-1}(x)$ exists. Include $x = 0$ in the domain.

1

(iii) Find the equation of $y = f^{-1}(x)$

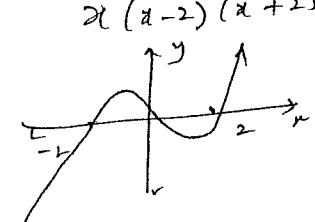
2

(i) Sketch $y = f^{-1}(x)$

2

End of Task

| Qn | Solutions | Marks | Comments: Criteria |
|----|-------------------------------------------------------------------------------------------------------------------------------------------|-------|--------------------|
| 1. | multiple choice. $y = \sin^{-1}(x^2)$ $y' = \frac{(2x)}{\sqrt{1-x^4}}$ A | | |
| 2. | $\int \sin^2 3x \, dx = \frac{1}{2} \int (1 - \cos 6x) \, dx$ $= \frac{1}{2} \left(x - \frac{\sin 6x}{6} \right) + C$ D. | | |
| 3. | $\int e^{\frac{3x}{2}} \, dx = \frac{2}{3} e^{\frac{3x}{2}} + C$. C. | | |
| 4. | $\int \frac{dx}{\sqrt{1-4x^2}} = \int \frac{dx}{\sqrt{4(\frac{1}{4}-x^2)}}$ $= \frac{1}{2} \cdot \sin^{-1} 2x + C$. D. | | |
| 5. | $x = 1 - 3x$ $= -3(x - \frac{1}{3})$ $\therefore \text{center } x = \frac{1}{3}$. D. | | |
| 6. | $\int \frac{f'(x)}{f(x)} \, dx = \int \frac{1}{x} \, dx$ $\ln f(x) = x + C$. $f(x) = e^{x+C}$ $= Ae^x$ $\text{or } Ce^x$. C | | |

| Qn | Solutions | Marks | Comments: Criteria |
|-----|------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|-------------------------------------|
| 7. | $y = \frac{x^2+1}{x}$ $= x + \frac{1}{x}$. $x \neq 0$, $y \neq x$. | | |
| 8. | (B). $x(x-2)(x+2) > 0$ $-2 < x < 0 \text{ or } x > 2$  (A). | | |
| 9. | $S = 4\pi r^2$ $\frac{dS}{dt} = 8\pi r \frac{dr}{dt}$. $= 8\pi r \times 2 \times 2$ $= 32\pi$. | | $6\pi = 4\pi$ $4 = r$ $2 = r$ |
| 10. | (A). $\cos^{-1}(\cos 11\pi/6)$ $= \cos^{-1}(\cos(2\pi - \pi/6))$ $= \cos^{-1} \cos(11\pi/6)$ $= \frac{\pi\pi}{6}$ D | | |

| Qn | Solutions | Marks | Comments: Criteria |
|-----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|-----------------------------------------------------------------------------------------------------------------------------------------|
| 11. | $\frac{1}{x-1} \geq 5 ; x \neq 1$ $(x-1)^2 \cdot \frac{1}{x-1} \geq 5(x-1)^2$ $x-1 \geq 5(x-1)^2$ $(x-1) - 5(x-1)^2 \geq 0$ $(x-1)(1-5(x-1)) \geq 0$ $(x-1)(6-5x) \geq 0$ $1 < x \leq \frac{6}{5}$ | | $x \neq 1$ $(-\frac{1}{5})$ |
| 12. | $x = e^{\frac{3k+4}{\ln x}}$ $x = e^{\ln x^{\frac{3k+4}{8}}}$ $x = e^{\frac{3k+4}{8}}$ $x = 8^{\frac{3k+4}{8}}$ $3k+4 = 8$ $3k = 4$ $k = \frac{4}{3}$ | 1 | Taking logs. on both sides. ① sol ① |
| 13. | A(1, 5) B(4, 10) k $(2, \frac{20}{3})$ $\frac{4k+1}{k+1} = 2$ $4k+1 = 2k+2$ | 1 | Ensure you take k : 1 rather than $k: k$ for ease of calculation. A number of students had the wrong formula. |
| | $2k = 1$ $k = \frac{1}{2}$ | | |

| Qn | Solutions | Marks | Comments: Criteria |
|-----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|--------------------|
| | <u>Note</u> : works for y. coordinate no need to do this as well. | | |
| 14. | $\frac{10k+5}{k+1} = \frac{20}{3}$ $30k+15 = 20k+20$ $5 = 10k$ $k = \frac{1}{2}$ | | |
| 15. | $y = 3m^2 + 1$ $y = m^2$ grad $m_1 = 3m$ grad $m_2 = m$ $\text{tan } \alpha = \frac{ 3m-m }{1+3m^2}$ $\frac{1}{2} = \frac{3m}{1+3m^2}$ $3m^2 + 1 = 4m$ $3m^2 - 4m + 1 = 0$ $(3m-1)(m-1) = 0$ $m = 1$. $m = \frac{1}{3}$ | | |
| 16. | $(x-2)(x+1)$ is a factor $\therefore (x-2)$ is a factor of $P(x)$ $\therefore P(2) = 0$ $16 + 24 + 4a - 4 - b = 0$ $4a - b = -36$ — (1) $(x+1)$ is a factor: $1 - 3 + a + 2 - b = 0$ $a - b = 0$ — (2) | | |

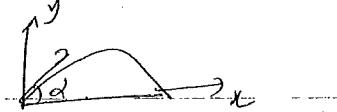
| Qn | Solutions | Marks | Comments: Criteria |
|----|--------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|--------------------|
| | $3Q = -36$ $Q = -12$ $b = -12$. $\tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{5} = \tan^{-1} \frac{4}{7}$ Ques. $\alpha = \tan^{-1} \frac{1}{3}$ $\tan \alpha = \frac{1}{3}$ $\beta = \tan^{-1} \frac{1}{5}$ $\tan \beta = \frac{1}{5}$ Consider $\tan(\alpha + \beta)$ $= \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$ $= \frac{\frac{1}{3} + \frac{1}{5}}{1 - \frac{1}{15}} = \frac{8}{14} = \frac{4}{7}$ $\therefore \tan^{-1} \frac{4}{7} = \alpha + \beta$ $= \tan^{-1} \frac{1}{3} + \tan^{-1} \frac{1}{5}$ | 1 | |
| b) | $x = 2 \sin \theta$ $dx = 2 \cos \theta d\theta$ $\alpha = 0 : \theta = 0$ $\sin \theta = 1$ $\theta = \frac{\pi}{2}$ $\therefore \int_0^2 \sqrt{4-x^2} dx = \int_0^{\pi/2} \sqrt{4-4\sin^2 \theta} 2 \cos \theta d\theta$ $= 4 \int_0^{\pi/2} \cos^2 \theta d\theta$ $= 2 \int_0^{\pi/2} (1 + \cos 2\theta) d\theta$ | 2 | |

| Qn | Solutions | Marks | Comments: Criteria |
|----|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|-----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|
| | $= 2 \left[\theta + \frac{\sin 2\theta}{2} \right]_0^{\pi/2}$ $= 2 \left[\frac{\pi}{2} \right] = \pi$. | 1 | $\textcircled{-1}$ if 15 ^o didn't use $\cos^2 \theta = \frac{1+\cos 2\theta}{2}$. |
| c) | $\sin 2\theta = \cos \theta$ $2 \sin \theta \cos \theta = \cos \theta$ $\cos \theta (2 \sin \theta - 1) = 0$ $\cos \theta = 0 \quad \left[\begin{array}{l} \sin \theta = \frac{1}{2} \\ \theta = n\pi + (-1)^n \cdot \frac{\pi}{6} \end{array} \right]$ $\theta = 2n\pi \pm \frac{\pi}{2}$ | 1 | $\sin \theta$ - losing $\cos \theta$ - $\alpha + \beta$ elimination $2 \sin \theta \cos \theta = \cos \theta$ $2 \sin \theta = 1$ $\sin \theta = \frac{1}{2}$ $\theta = \frac{\pi}{6}$ $n = \frac{1}{3}$ |
| d) | $\frac{1-x^2}{1+x^2} = -1 + \frac{2}{x^2+1}$ $-1 + \frac{2}{x^2+1} = \frac{-x^2-1+2}{x^2+1}$ $= \frac{1-x^2}{1+x^2}$ $y = -1 + \frac{2}{x^2+1}$ | 1 | $\frac{1-x^2}{1+x^2}$ - wrong approach but correct graph. Marks were awarded for the question asked for the graph. If the question was calculated, for the stat. pt. + number of terms would have got full part marking. |
| | $y^1 = -\frac{4x}{(x^2+1)^2}$ At stat. pt.: $y^1 = 0$ $x = 0$ $y = 1$. $(0, 1) \rightarrow$ a stat. pt. | 1 | |

| Qn | Solutions | Marks | Comments: Criteria |
|----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|--------------------|
| | $\begin{array}{ccccc} x & 0^- & 0 & 0^+ \\ y' & > 0 & 0 & < 0 \end{array}$ <p>$\therefore (0, 1)$ is a stat. max. p.r.</p> <p>$y = 0 : x = \pm 1$</p> <p>$x = 0 : y = 1$</p> <p>$y = -1 + \frac{2}{x^2+1}$</p> <p>$\frac{2}{x^2+1} \neq 0 \quad \therefore y = -1$ is an asymptote</p> | | |

| Qn | Solutions | Marks | Comments: Criteria |
|----|----------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|------------------------------------------------------------------------|
| | $\begin{aligned} 8.13 \\ a) \int \frac{5}{1+16x^2} dx \\ &= \frac{5}{16} \int \frac{1}{\frac{1}{16}+x^2} dx \\ &= \frac{5}{16} \left[4 \tan^{-1} 4x + C \right] \\ &= \frac{5}{16} \tan^{-1}(4x) + C \\ &\quad (2x - \frac{1}{x^2})^9. \end{aligned}$ | 2 | 1 FOR $\tan^{-1}(4x) + C$ |
| b) | $\begin{aligned} &= q_{c_0} (2x)^9 + q_{c_1} (2x)^8 (-\frac{1}{x^2}) + \dots \\ &= q_{c_0} (2x)^9 + q_{c_1} (-\frac{1}{x^2})^{9-r} (-\frac{1}{x^2})^r \\ T_{r+1} &= q_{c_r} (2x)^{9-r} (-\frac{1}{x^2})^r \end{aligned}$ | 1M | |
| | $\text{when } T_{r+1} \text{ is a const:}$ $T_{r+1} = q_{c_r} \cdot 2^{9-r} (-1)^r \cdot x^{9-r-2r}$ $T_{r+1} \text{ is a const. for } 9-3r=0 \quad r=3$ | 1M | (3) |
| c) | $\begin{aligned} \text{Constant term is} \\ (-1)^3 q_{c_3}^2 = -q_{c_3}^2 = -5376 \\ (3+4x)^{12} = {}_{C_0}^{12} 3^{12} (4x) + {}_{C_1}^{12} 3^{12-r} (4x)^r \\ T_{r+1} = {}_{C_r}^{12} 3^{12-r} (4x)^r \\ t_r = {}_{C_{r-1}}^{12} 3^{12-(r-1)} (4x)^{r-1}. \end{aligned}$ | 1M | $-\frac{1}{2}$ for incorrect $\frac{1}{2}$ sign. |
| | | | $\frac{1}{2}$ FOR CORRECT EXPRESSIONS FOR t_{r+1} AND t_r |

| Qn | Solutions | Marks | Comments: Criteria |
|----|-------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|---------------------------------------------------------------------|
| | <p><u>Coeff. of tr^{r+1}</u> = $\frac{12c_r}{12c_{r-1}} \cdot \frac{3}{13-r} \cdot \frac{4^r}{4^{r+1}}$</p> <p><u>Coeff. of tr^r</u> = $\frac{12c_r}{12c_{r-1}} \cdot \frac{3}{3} \cdot \frac{4^r}{4^{r+1}}$</p> <p>$= \frac{12!}{r!(12-r)!} \cdot \frac{(r-1)!(13-r)!}{12!} \cdot \frac{4}{3}$</p> <p>$= \frac{13-r}{r} \cdot \frac{4}{3}$</p> <p><u>Coeff. of tr^{r+1}</u> \geq <u>Coeff. of tr^r</u></p> <p>$4(13-r) \geq 3^r$</p> <p>$52 - 4r \geq 3^r$</p> <p>$7r \leq 52$ $(\frac{52}{7} = 7\frac{3}{7})$</p> <p>$r \leq 7$</p> <p><u>Coeff. of tr^{r+1}</u> \geq <u>Coeff. of tr^r</u></p> <p>for $r \geq 8$.</p> <p><u>Coeff. of t_8</u> \geq <u>Coeff. of t_7</u> $\geq \dots$ <u>Coeff. of t_6</u> \dots <u>Coeff. of t_8</u>.</p> <p><u>Coeff. of t_8</u> \geq <u>Coeff. of t_7</u> $<$ <u>Coeff. of t_{12}</u> \dots <u>Coeff. of t_8</u>.</p> <p>Also. <u>Coeff. of t_{13}</u> $<$ <u>Coeff. of t_{12}</u> \dots <u>Coeff. of t_8</u>.</p> <p>$\therefore T_8 = \frac{12c_7}{12c_6} \cdot 3^5 \cdot (4x)^7 \cdot \frac{1}{2}$</p> <p><u>Coeff. of T_8</u> = $\frac{12}{12} \cdot 3^5 \cdot 4^7$</p> <p>or greater coeff. ≈ 3153199104</p> | 1 | 1 for $\frac{12c_r}{12c_{r-1}} \cdot \frac{4^r}{4^{r+1}}$ (2) |

| Qn | Solutions | Marks | Comments: Criteria |
|------|---------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------------|-------|----------------------------------------------------------------------------------------------------------------------|
| 13a. |  <p>The only acceleration on the particle is gravity.</p> <p>Initially $x = 20 \cos \alpha$, $y = 20 \sin \alpha$. $\dot{x} = 0$, $\dot{y} = 0$.</p> <p>$\ddot{y} = -10$</p> <p>$\ddot{x} = -10t + C$</p> <p>$20 \sin \alpha = -10t + C$</p> <p>$\therefore y = -10t + 20 \sin \alpha$</p> <p>$y = -5t^2 + (20 \sin \alpha)t + C$</p> <p>$\therefore C = 0$</p> <p>$y = -5t^2 + 20t \sin \alpha$ - (2)</p> <p>eliminate t in (1) & (2)</p> <p>Corrasian equation.</p> <p>(1) $t = \frac{x}{20 \cos \alpha}$</p> <p>Sub in (2)</p> <p>$y = -5 \frac{x^2}{400 \cos^2 \alpha} + 20 \frac{x}{20 \cos \alpha}$</p> <p>$= -\frac{1}{80} \sec^2 \alpha x^2 + x \tan \alpha$</p> <p>$= -\frac{1}{80} (1 + \tan^2 \alpha) x^2 + x \tan \alpha$</p> <p>$= x \tan \alpha - \frac{x^2}{80} \sec^2 \alpha$</p> | 2 | $\frac{-1}{3}$ PERCENTAGE $-\frac{1}{2}$ if didn't start from $\dot{x}=0, \dot{y}=0$. $-\frac{1}{2}$ IF NO C. |

| Qn | Solutions | Marks | Comments: Criteria |
|-------|-------------------------------------------------------|---------------|----------------------------------------------------------|
| | if $t = 10\pi$ | 2 | |
| | $y = xt - \frac{x^2}{80}(1+t^2)$ | | |
| | $10 = 20t - \frac{400(1+t^2)}{80}$ | | $\frac{1}{2}$ FOR CORRECT SUBSTITUTION $x=20 y=10$ |
| | $10 = 20t - 5(1+t^2)$ | | $\frac{1}{2}$ FOR SIGNIFICANT SIMPLIFICATION |
| | $2 = 4t - 1 - t^2$ | | |
| | $t^2 - 4t + 3 = 0$ | | |
| | $(t-3)(t-1) = 0$ | | |
| | $t = 1$ | | |
| | $t = 3$ | | $\tan \alpha = 1$ |
| | $\tan \alpha = 3$ | | |
| | $\alpha = \tan^{-1} 3; 45^\circ$ $71^\circ 34'$ | | |
| | $x = 3 \sin(2t+\alpha)$ | | |
| 14(b) | $t=0 \quad x=1.5$ | | |
| | $1.5 = 3 \sin \alpha \quad \sin \alpha = \frac{1}{2}$ | | |
| | $\alpha = 30^\circ$ | 1 | |
| | $\ddot{x} = 6 \cos(2t+\alpha)$ | | |
| 11. | $\ddot{x} = -12 \sin(2t+\alpha)$ | | |
| | $= -4(3 \sin(2t+\alpha))$ | | |
| | $= -4x$ | $\frac{1}{2}$ | |

| Qn | Solutions | Marks | Comments: Criteria |
|------|----------------------------------------------------------------|---------------|-------------------------|
| (ii) | $0 = 3 \sin(2t+\alpha)$ | $\frac{1}{2}$ | (2) |
| | $2t+\alpha = 0, \pi, 2\pi,$ | | |
| | $2t + \frac{\pi}{6} = \pi$ | 1 | |
| | $2t = \frac{5\pi}{6}$ | | |
| | $t = \frac{5\pi}{12}$ | $\frac{1}{2}$ | |
| (iv) | $-12 = -12 \sin(2t + \frac{\pi}{6})$ | $\frac{1}{2}$ | (2) |
| | $\sin(2t + \frac{\pi}{6}) = 1$ | | |
| | $2t + \frac{\pi}{6} = \frac{\pi}{2}$ | 1 | |
| | $t = \frac{\pi}{6}$ | $\frac{1}{2}$ | |
| (a) | $a = 7x$ | | $t=0$ $x=0$ $v=3$ |
| | $\frac{d}{dx} \left(\frac{1}{2}v^2 \right) = 7x$ | 1 | |
| | $\frac{1}{2}v^2 = \frac{7x^2}{2} + C$ | | $x=0$ $v=3$ |
| | $\frac{9}{2} = C$ | | |
| | $\sqrt{2} = \sqrt{7x^2 + 9}$ | | |
| | $\therefore v = \pm \sqrt{7x^2 + 9}$, reconcile $x=0, v=3$ | | |
| | $\therefore v = \sqrt{7x^2 + 9}$ | $\frac{1}{2}$ | (3) |

| On | Solutions | Marks | Comments: Criteria |
|------|-------------------------------------------------------------------------------------------------------------------------------------------------|---------------|--------------------|
| 14c. | $f(x) = 1 - \cos \frac{x}{2}$ $T = \frac{2\pi}{\frac{1}{2}} = 4\pi$ $f^{-1}(x)$ exists $0 \leq x \leq 2\pi$ $(0 \leq x \leq 2\pi)$ | 1 | |
| | $y = 1 - \cos \frac{x}{2}$ | | |
| | $x = 1 - \cos \frac{y}{2}$ | 1 | |
| | $\cos \frac{y}{2} = 1 - x$ | | |
| | $\frac{y}{2} = \cos^{-1}(1-x)$ | $\frac{1}{2}$ | |
| | $y = 2 \cos^{-1}(1-x)$ | $\frac{1}{2}$ | |
| | $\text{Shape: } \text{C}\text{orrect }$ $\text{Domain/Range: } \text{C}\text{orrect }$ $\text{Endpoints: } \text{C}\text{orrect }$ | 2 | |