Student Name:______________________________

St Catherine's School

Waverley

2013

PRELIMINARY ASSESSMENT TASK I

Weighting 25%

Extension I Mathematics

Reading Time:	5 minutes
Time allowed:	75 minutes
Total marks:	55 marks

INSTRUCTIONS

SECTION 1 – Multiple Choice (Questions 1-5) (5 marks)

- Answer either A, B, C or D to each question on the answer sheet provided.

SECTION 2 – Written Response (Questions 6 and 7) (50 marks)

- Marks for each part of a question are indicated.
- The two questions are of equal value.
- Both questions should be attempted in the booklets provided.
- All necessary working should be shown in each question.
- Start each question in a new booklet and clearly label all parts of a question.
- Approved scientific calculators may be used.
- Diagrams should be drawn using pencil and ruler.

SECTION 1 Multiple Choice (5 marks)

Please answer on the multiple choice answer sheet provided.

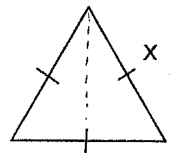
1. What are the coordinates of the point P that divides internally the interval joining the points $A(1,2)$ and $B(7,5)$ in the ratio $2:1$?

a) (3,3) b) (3,4) c) (5,3) d) (5,4)

2. What is the acute angle between the lines $3x+4y=8$ and $2x+3y=5$?

a) $3°11'$ b) $9°28'$ c) $70°36'$ d) $86°49'$

3. If the area of the triangle below is $64\sqrt{3}\ cm^2$, the value of x is:



a) 16 cm b) 8 cm

c) 14 cm c) $8\sqrt{3}$ cm

4) $\frac{1-sin^2\theta cos^2\theta}{cos^2\theta} =$

a) $cos^2\theta$ b) $sin^2\theta - tan^2\theta$

c) $tan^2\theta + cos^2\theta$ d) $1 + sin\theta$

5) Which of the following points is collinear with the points (4,6) and (−2,15)?

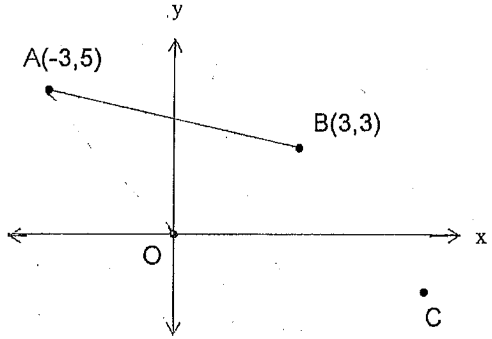
a) (6,4) b) $\left(7,\frac{3}{2}\right)$ c) (5,3) d) $\left(1,\frac{17}{2}\right)$

SECTION 2

Question 6 **(25 marks)** **(Start a new booklet)** **Marks**

a) Solve $\frac{2x-1}{x-2} \geq 1$ **3**

b)



Not to scale

The diagram shows the points A(-3,5), B(3,3), and O(0,0). The point C is the fourth vertex of the parallelogram OABC.

i) Show that the equation of AB is $x + 3y - 12 = 0$. **2**

ii) Show that the length of AB is $2\sqrt{10}$. **1**

iii) Calculate the perpendicular distance from O to the line AB. **2**

iv) Calculate the area of parallelogram OABC. **2**

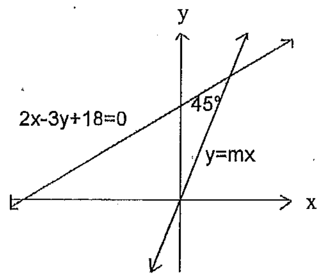
v) Find the perpendicular distance from O to the line BC. **2**

c) Find the ratio in which the point $P(-4,14)$ divides the interval BA, given $A(2,5)$ and $B(10,-7)$. **3**

d) If the point $(x, 7)$ is 5 units from the line $3x - 4y + 12 = 0$, find all possible values of x. **3**

e) Solve $2cos^2x - sinx = 1$ for $0° \leq x \leq 360°$. **4**

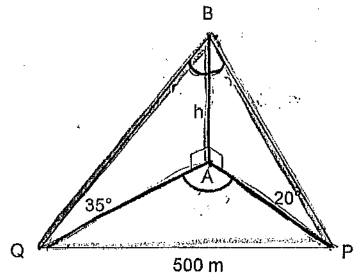
f)



Find the values of m, if the line $y = mx$ makes an angle of 45° with the line $2x - 3y + 18 = 0$ **3**

Question 7 **(25 marks)** **(Start a new booklet)**

a)



From a point P, on a bearing of 110° from a tower AB, the angle of elevation to the top of the tower is 20°. From another point Q, on a bearing of 240° from the tower, the angle of elevation to the top of the tower is 35°. If the distance PQ is 500 m:

i) Show that $\angle PAQ = 130°$. **1**

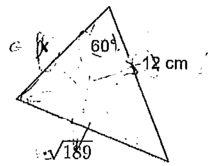
ii) Show that $AP = h \tan 70°$. **1**

iii) Find a similar expression for the length of AQ. **1**

iv) Show that $h = \frac{500}{\sqrt{tan^2 70° + tan^2 55° - 2tan 70° tan 55° cos 130°}}$ **2**

v) Hence find the height of the tower correct to the nearest metre. **1**

b) For the triangle shown:



i) Use the cosine rule to prove that the side length x cm satisfies the equation: **2**

$$x^2 - 12x - 45 = 0$$

ii) Hence find the side length marked x. **1**

c) Prove that $\frac{1+cot\theta}{cosec\theta} - \frac{sec\theta}{tan\theta+cot\theta} = cos\theta$ **3**

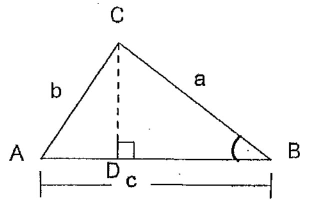
d) If $\tan x = \frac{2t}{1-t^2}$ where x is acute, **3**

find a simplified expression for $\sin x$ in terms of the variable t.

e) If $cot\theta = \frac{3}{x}$ and $cos\theta = \frac{2}{y}$, find an equation in x and y which is independent of θ. **2**

f) By making the substitution $x = 2tan\theta$, show that $\frac{x^2}{\sqrt{x^2+4}} = 2tan\theta sin\theta$ **3**

g)



The triangle ABC has side lengths a, b and c as shown in the diagram. The point D lies on AB, and CD is perpendicular to AB.

i) Show that $asinB = bsinA$ **1**

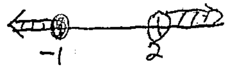
ii) Show that $c = a\,cosB + b\,cosA$ **1**

iii) Given that $c^2 = 4ab\,cosA\,cosB$, show that $a = b$. **3**

END OF TEST

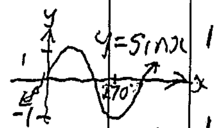
Qn	Solutions	Marks	Comment: Criteria
	SECTION 1 MULTIPLE CHOICE		
1	D		
2	A		
3	A		
4	C		
5	B		
	WORKING		
1	$(x,y)=\left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$ $=\left(\frac{2(7)+1(1)}{2+1}, \frac{2(5)+1(2)}{2+1}\right)$ $=(5,4)$		
2	$m_1=-\frac{3}{4}\quad m_2=-\frac{2}{3}$ $\tan\theta=\left\|\frac{m_1-m_2}{1+m_1m_2}\right\|$ $=\left\|\frac{-\frac{3}{4}+\frac{2}{3}}{1+\left(-\frac{3}{4}\right)\left(-\frac{2}{3}\right)}\right\|$ $=\frac{1}{18}$ $\theta=3^\circ 11'$		

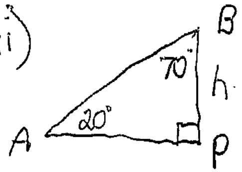
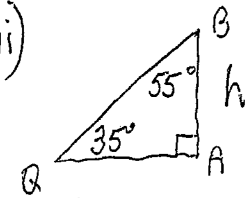
Qn	Solutions	Marks	Comment: Criteria
3	$A=\frac{1}{2}ab\sin C$ $64\sqrt{3}=\frac{1}{2}x^2\sin 60^\circ$ $64\sqrt{3}=\frac{\sqrt{3}}{4}x^2$ $x^2=256$ $x=16\text{cm}$		
4	$\frac{1-\sin^2\theta\cos^2\theta}{\cos^2\theta}=\frac{1}{\cos^2\theta}-\sin^2\theta$ $=\sec^2\theta-\sin^2\theta$ $=1+\tan^2\theta-(1-\cos^2\theta)$ $=\tan^2\theta+\cos^2\theta$		
5	$m=\frac{15-6}{-2-4}$ $=-\frac{3}{2}$ FOR $(7,\frac{3}{2})$ AND $(4,6)$ $m=\frac{\frac{9}{2}}{-3}$ $=-\frac{3}{2}$ SINCE THE GRADIENTS ARE THE SAME AND (4,6) IS COMMON THE 3 POINTS ARE COLLINEAR.		

Qn	Solutions	Marks	Comment: Criteria
6	SECTION 2 a) $(x-2)^2 \frac{2x-1}{x-2} \geq 1(x-2)^2$ $(2x-1)(x-2) \geq (x-2)^2$ $(x-2)[2x-1-(x-2)] \geq 0$ $(x-2)(x+1) \geq 0$  $\therefore x \leq -1, x > 2$	3	$+\frac{1}{2}$ for $x \neq 2$ 1 for single solution $-\frac{1}{2}$ for $x \geq 2$
	b) i) $m_{AB} = \frac{5-3}{-3-3}$ $= -\frac{1}{3}$ USING (3,3) $y - y_1 = m(x - x_1)$ $y - 3 = -\frac{1}{3}(x-3)$ $3y - 9 = -x + 3$ $x + 3y - 12 = 0$	2	
	ii) $d = \sqrt{(x_2-x_1)^2 + (y_2-y_1)^2}$ $= \sqrt{(-3-3)^2 + (5-3)^2}$ $= \sqrt{40}$ $= 2\sqrt{10}$ UNITS	1	

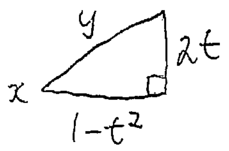
Qn	Solutions	Marks	Comment: Criteria
	iii) $d = \frac{\lvert ax_1 + by_1 + c \rvert}{\sqrt{a^2+b^2}}$ $= \left\lvert \frac{1(0) + 3(0) - 12}{\sqrt{1^2+3^2}} \right\rvert$ $= \frac{12}{\sqrt{10}}$ u $= \frac{6\sqrt{10}}{5}$ u	2	
	iv) $A = BH$ $= 2\sqrt{10} \times \frac{12}{\sqrt{10}}$ $= 24u^2$	2	
	v) $d_{OA} = \sqrt{(-3-0)^2 + (5-0)^2}$ $= \sqrt{34}$ $A = BH$ $24 = \sqrt{34}h$ $h = \frac{24}{\sqrt{34}}$ $= \frac{12\sqrt{34}}{17}$ u	$\frac{1}{2}$ $\frac{1}{2}$ 1 2	

Qn	Solutions	Marks	Comment: Criteria
	c) $(x,y) = \left(\frac{mx_2+nx_1}{m+n}, \frac{my_2+ny_1}{m+n}\right)$ $(-4,14) = \left(\frac{2m+10n}{m+n}, \frac{5m-7n}{m+n}\right)$ $\frac{2m+10n}{m+n} = -4$ $2m+10n = -4m-4n$ $6m = -14n$ $\therefore \frac{m}{n} = -\frac{7}{3}$ ie $m:n = -7:3$ or $7:-3$ So P cuts BA <u>EXTERNALLY</u> in the ratio $7:3$.	3	$-\frac{1}{2}$ for $m:n$ AB not $m:n$ BA
	d) $d = \frac{\lvert ax_1+by_1+c\rvert}{\sqrt{a^2+b^2}}$ $5 = \frac{\lvert 3x-28+12\rvert}{\sqrt{3^2+(-4)^2}}$ $\lvert 3x-16\rvert = 25$ $3x-16=25$ $3x-16=-25$ $3x=41$ $3x=-9$ $x=\frac{41}{3}$ $x=-3$ $\therefore x = \frac{41}{3}, -3$.	3	

Qn	Solutions	Marks	Comment: Criteria
	e) $2\cos^2 x - \sin x = 1$ $2(1-\sin^2 x) - \sin x = 1$ $2\sin^2 x + \sin x - 1 = 0$ $(2\sin x - 1)(\sin x + 1) = 0$ $\sin x = \frac{1}{2}$ $\sin x = -1$ $x = 30°, 150°, 270°$  	4	1 1 1 1
	f) $\tan\theta = \left\lvert \frac{m_1-m_2}{1+m_1m_2} \right\rvert$ $\tan 45° = \left\lvert \frac{m-\frac{2}{3}}{1+\frac{2m}{3}} \right\rvert$ $1 = \left\lvert \frac{3m-2}{3+2m} \right\rvert$ $\frac{3m-2}{3+2m} = 1$ $\frac{3m-2}{3+2m} = -1$ $3m-2 = 3+2m$ $3m-2 = -3-2m$ $m = 5$ $5m = -1$ $m = -\frac{1}{5}$ $\therefore m = -\frac{1}{5}, 5$	3	

Qn	Solutions	Marks	Comment: Criteria
7	a) i) $\angle PAQ = 240° - 110°$ ($\angle$ BETWEEN 2 BEARINGS) $= 130°$	1	
	ii)  $\tan 70° = \frac{AP}{h}$ $AP = h\tan 70°$	1	
	iii)  $\tan 55° = \frac{AQ}{h}$ $AQ = h\tan 55°$	1	
	iv) $c^2 = a^2 + b^2 - 2ab\cos C$ $500^2 = h^2\tan^2 70° + h^2\tan^2 55° - 2(h\tan 70°)(h\tan 55°)\cos 130°$ 1 $500^2 = h^2(\tan^2 70° + \tan^2 55° - 2\tan 70°\tan 55°\cos 130°)$ $\frac{1}{2}$ $h^2 = \dfrac{500^2}{\tan^2 70 + \tan^2 55 - 2\tan 70\tan 55\cos 130°}$ $\therefore h = \dfrac{500}{\sqrt{\tan^2 70° + \tan^2 55 - 2\tan 70\tan 55\cos 130°}}$ $\frac{1}{2}$	2	

Qn	Solutions	Marks	Comment: Criteria
	v) $h \doteqdot 131$ m	1	
	b) i) $c^2 = a^2 + b^2 - 2ab\cos C$ $(\sqrt{189})^2 = x^2 + 12^2 - 2(x)(12)\cos 60°$ 1 $189 = x^2 + 144 - 12x$ $\frac{1}{2}$ $x^2 - 12x - 45 = 0$ $\frac{1}{2}$	2	
	ii) $(x-15)(x+3) = 0$ $x = 15, -3$ SINCE $x > 0$ (LENGTH OF SIDE) $\therefore x = 15$ cm	1	
	c) PROVE $\dfrac{1+\cot\theta}{\operatorname{cosec}\theta} - \dfrac{\sec\theta}{\tan\theta + \cot\theta} = \cos\theta$ LHS $= \dfrac{1 + \frac{\cos\theta}{\sin\theta}}{\frac{1}{\sin\theta}} \times \frac{\sin\theta}{\sin\theta}$ $\frac{1}{2}$ $- \dfrac{\frac{1}{\cos\theta}}{\frac{\sin\theta}{\cos\theta} + \frac{\cos\theta}{\sin\theta}} \times \frac{\cos\theta\sin\theta}{\cos\theta\sin\theta}$ $\frac{1}{2}$ $= \sin\theta + \cos\theta$ $\frac{1}{2}$ $- \dfrac{\sin\theta}{\sin^2\theta + \cos^2\theta}$ $\frac{1}{2}$ $= \sin\theta + \cos\theta - \sin\theta$ $\frac{1}{2}$ $= \cos\theta = RHS$ $\frac{1}{2}$	3	

Qn	Solutions	Marks	Comment: Criteria
	d) $\tan x = \dfrac{2t}{1-t^2}$  $y^2 = (1-t^2)^2 + (2t)^2$ 1 $y = \sqrt{1-2t^2+t^4+4t^2}$ $= \sqrt{1+2t^2+t^4}$ $= \sqrt{(1+t^2)^2}$ $= 1+t^2$ 1 $\therefore \sin x = \dfrac{2t}{1+t^2}$ 1	3	
	e) $\cot\theta = \dfrac{3}{x}$ $\cos\theta = \dfrac{2}{y}$ ½ ½ $\therefore \tan\theta = \dfrac{x}{3}$ $\sec\theta = \dfrac{y}{2}$ $1+\tan^2\theta = \sec^2\theta$ $1+\left(\dfrac{x}{3}\right)^2 = \left(\dfrac{y}{2}\right)^2$ 1 $1+\dfrac{x^2}{9} = \dfrac{y^2}{4}$ $36+4x^2 = 9y^2$	2	

Qn	Solutions	Marks	Comment: Criteria
	f) $LHS = \dfrac{x^2}{\sqrt{x^2+4}}$ $x = 2\tan\theta$ $LHS = \dfrac{4\tan^2\theta}{\sqrt{4\tan^2\theta+4}}$ 1 $= \dfrac{4\tan^2\theta}{\sqrt{4(1+\tan^2\theta)}}$ $= \dfrac{4\tan^2\theta}{\sqrt{4\sec^2\theta}}$ 1 $= \dfrac{4\tan^2\theta}{2\sec\theta}$ ½ $= 2\tan\theta\,\dfrac{\sin\theta}{\cos\theta}\cdot\cos\theta$ ½ $= 2\tan\theta\sin\theta = RHS$	3	

Qn	Solutions	Marks	Comment: Criteria
	9) i) IN $\triangle ABC$		
	$\frac{a}{\sin A} = \frac{b}{\sin B}$		
	$a\sin B = b \cdot \sin A$	1	
	ii) $c = AD + BD$		
	$\cos A = \frac{AD}{b}$ $\quad \cos B = \frac{BD}{a}$		
	$\therefore AD = b\cos A \quad BD = a\cos B$		
	$\therefore c = b\cos A + a\cos B$	1	
	iii) $c^2 = (a\cos B + b\cos A)^2$		
	$= a^2\cos^2 B + 2ab\cos A\cos B + b^2\cos^2 A$ (1)		1
	ALSO $c^2 = 4ab\cos A\cos B$ (given) (2)		
	SOLVING (1) AND (2) SIMULTANEOUSLY		
	$4ab\cos A\cos B = a^2\cos^2 B + 2ab\cos A\cos B + b^2\cos^2 A$		
	$a^2\cos^2 B - 2ab\cos A\cos B + b^2\cos^2 A = 0$		$\frac{1}{2}$
	$(a\cos B - b\cos A)^2 = 0$		$\frac{1}{2}$
	$a\cos B - b\cos A = 0$		
	$a\cos B = b\cos A$		$\frac{1}{2}$
	SQUARING		
	$a^2\cos^2 B = b^2\cos^2 A$		

$(a\cos B + b\cos A)^2 = 4ab\cos A\cos B$ (1)

Qn	Solutions	Marks	Comment: Criteria
	$a^2(1-\sin^2 B) = b^2(1-\sin^2 A)$		
	$a^2 - [a\sin B]^2 = b^2 - [b\sin A]^2$		
	Since $a\sin B = b\sin A$ $\quad \frac{1}{2}$		
	$\therefore a^2 = b^2$		
	$a = b.$	3	