Name: ____________

Teacher: ____________

Class: ____________

FORT STREET HIGH SCHOOL

2014

HIGHER SCHOOL CERTIFICATE COURSE

ASSESSMENT TASK 3: TRIAL HSC

Mathematics

Time allowed: 3 hours
(plus 5 minutes reading time)

Outcomes Assessed	Questions
Chooses and applies appropriate mathematical techniques in order to solve problems effectively	1-10
Manipulates algebraic expressions to solve problems from topic areas such as inverse functions, trigonometry and polynomials	11,12
Uses a variety of methods from calculus to investigate mathematical models of real life situations, such as projectiles, kinematics and growth and decay	14,16
Synthesises mathematical solutions to harder problems and communicates them in appropriate form	13,15

Total Marks 100

Section I 10 marks
Multiple Choice, attempt all questions,
Allow about 15 minutes for this section

Section II 90 Marks
Attempt Questions 11-16,
Allow about 2 hours 45 minutes for this section

Section I	Total 10	Marks
Q1-Q10		
Section II	Total 90	Marks
Q11	/15	
Q12	/15	
Q13	/15	
Q14	/15	
Q15	/15	
Q16	/15	
	Percent	

General Instructions:

- Questions 11-16 are to be started in a new booklet
- The marks allocated for each question are indicated
- All necessary working should be shown in every question. Marks may be deducted for careless or badly arranged work.
- Board – approved calculators may be used

STUDENT NUMBER/NAME: ....

STANDARD INTEGRALS

$$\int x^n\,dx = \frac{1}{n+1}x^{n+1},\quad n\neq -1;\quad x\neq 0,\ \text{if } n<0$$

$$\int \frac{1}{x}\,dx = \ln x,\quad x>0$$

$$\int e^{ax}\,dx = \frac{1}{a}e^{ax},\quad a\neq 0$$

$$\int \cos ax\,dx = \frac{1}{a}\sin ax,\quad a\neq 0$$

$$\int \sin ax\,dx = -\frac{1}{a}\cos ax,\quad a\neq 0$$

$$\int \sec^2 ax\,dx = \frac{1}{a}\tan ax,\quad a\neq 0$$

$$\int \sec ax\ \tan ax\,dx = \frac{1}{a}\sec ax,\quad a\neq 0$$

$$\int \frac{1}{a^2+x^2}\,dx = \frac{1}{a}\tan^{-1}\frac{x}{a},\quad a\neq 0$$

$$\int \frac{1}{\sqrt{a^2-x^2}}\,dx = \sin^{-1}\frac{x}{a},\quad a>0,\ -a<x<a$$

$$\int \frac{1}{\sqrt{x^2-a^2}}\,dx = \ln\left(x+\sqrt{x^2-a^2}\right),\quad x>a>0$$

$$\int \frac{1}{\sqrt{x^2+a^2}}\,dx = \ln\left(x+\sqrt{x^2+a^2}\right)$$

NOTE: $\ln x = \log_e x,\quad x>0$

Section 1 Multiple Choice (10 Marks) Circle Correct Answer

Question 1

Find $log_4 32$

a) 1.5

b) 2

c) 2.5

d) 3

Question 2

Solve $x^2 + 4x - 1 = 0$

a) $x = -2 \pm \sqrt{5}$

b) $x = 2 \pm \sqrt{5}$

c) $x = -2 \pm 2\sqrt{5}$

d) $x = -4 \pm \sqrt{5}$

Question 3

Find the range of $y = 3 + 2\cos(2x - 3)$

a) $-2 \le y \le 2$

b) $-\frac{3}{2} \le y \le \frac{2}{3}$

c) $3 \le y \le 5$

d) $1 \le y \le 5$

Question 4

What is the derivative of $\frac{2x}{1+x^2}$

a) $\frac{2-x^2}{(1+x^2)^2}$

b) $\frac{2+2x^2}{(1+x^2)^2}$

c) $\frac{2-2x^2}{(1+x^2)^2}$

d) $\frac{-2-2x^2}{(1+x^2)^2}$

Question 5

What are the solutions of

$2cos\theta = -\sqrt{3}\ for\ 0 \le \theta \le 2\pi$?

a) $\frac{\pi}{6}$ and $\frac{5\pi}{6}$

b) $\frac{5\pi}{6}$ and $\frac{7\pi}{6}$

c) $\frac{7\pi}{6}$ and $\frac{11\pi}{6}$

d) $\frac{\pi}{6}$ and $\frac{7\pi}{6}$

Question 6

What is the value of $\int_5^{15} \frac{1}{5x}\, dx$

a) $\frac{1}{5} ln5$

b) $\frac{1}{5} ln3$

c) $\frac{1}{5} ln10$

d) $\frac{3}{5} ln5$

Question 7

What is the perpendicular distance of the point (3, -2) from the line $y = 4 - 3x$

a) $\frac{4}{\sqrt{10}}$

b) $\frac{15}{\sqrt{10}}$

c) $\frac{3}{\sqrt{10}}$

d) $\frac{7}{\sqrt{10}}$

Question 8

The solution to $(2x - 5)(6 - x) \ge 0$ is

a) $\{x : -2.5 \le x \le 6\}$

b) $\{x : 2.5 \le x \le 6\}$

c) $\{x : x \le 2.5 ,\ x \ge -6\}$

d) $\{x : x \le -2.5 ,\ x \ge 6\}$

Question 9

For what values of x is the curve $y = 4x^3 - 3x^2$ concave down?

a) $x > \frac{1}{4}$

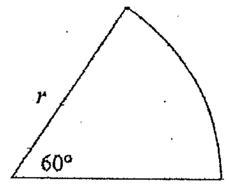
b) $x < \frac{1}{4}$

c) $x > \frac{3}{4}$

d) $x < 0$

Question 10

The sector below has an area of 30π square units



Not to scale

The value of r is

a) $5\sqrt{6}$

b) $\frac{6}{\sqrt{5}}$

c) $6\sqrt{5}$

d) $3\sqrt{2}$

Section II

90 Marks

Attempt Questions 11-16
Allow about 2 hours and 45 minutes for this section

Answer each question in the appropriate writing booklet. Extra writing booklets are available.
In Questions 11-16, your responses should include relevant mathematical reasoning and/or calculations

Question 11 (15 Marks) Use a SEPARATE writing booklet

a) Factorise $3x^2 - 16x + 5$. **2**

b) Solve $|5x + 2| < 3$ **2**

c) Differentiate $(3 - cos2x)^5$ **2**

d) Find the coordinates of the focus of the parabola $x^2 = 20(y + 3)$ **2**

e) Find the equation of the normal to the curve $y = \frac{2}{x}$ At the point where x=3. **3**

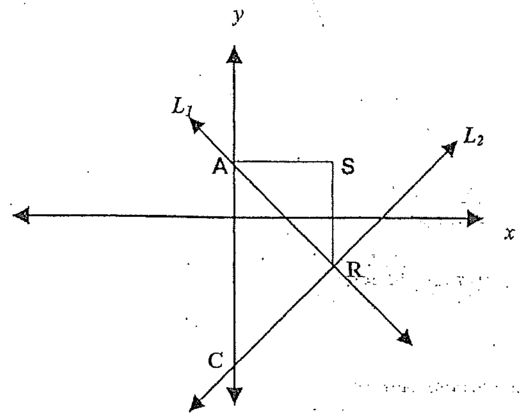
f) Evaluate $\int_1^3 \frac{4}{x^3}\, dx$ **2**

g) Sketch the region $(x + 3)^2 + (y - 2)^2 \geq 16$ 2

Question 12 (15 Marks) Use a SEPARATE writing booklet

a) Find the equation of the tangent to $y = xcosx \text{ where } x = \frac{\pi}{2}$ 3

b)



NOT TO SCALE

Line L_1 *has equation* $x + y = 4$ and intersects the y axis at point A.
Line L_2 *has equation* $x - y = 8$ and intersects the y axis at point C.
L_1 *and* L_2 Intersect at point R.

The horizontal line through A intersects the vertical line through R, at S.

(i) Find the coordinates of point A and C. 2
(ii) Show that R has coordinates (6, -2). 1
(iii) State the equation of the line SR. 1
(iv) Find the gradient of the line L_1. 1
(v) Find the distance AR. 1
(vi) Show that triangle ARC is a right-angled isosceles triangle. 2
(vii) Find the equation of the circle with centre R, passing through the points A and C. 2

c) Sketch the graph of $y = 4cosx \text{ for } 0 \leq \theta \leq 2\pi$ 2

Question 13 (15 Marks) Use a SEPARATE writing booklet

a) Differentiate with respect to x:

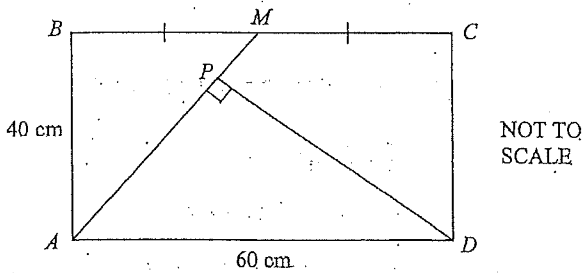
(i) $x\sqrt{x}$ 1

(ii) $x^2 lnx^2$ 2

(iii) $\frac{e^{-2x}}{sin3x}$ 2

b) Find $\int \frac{3sec^2 2x}{1+tan2x}\,dx$ 2

c)



ABCD is a rectangle in which $AB = 40$ cm and $AD = 60$ cm. *M* is the midpoint of *BC* and *DP* is perpendicular to *AM*.

Draw a neat sketch on your answer sheet. Hence:

(i) Prove that triangles *ABM* and *APD* are similar. 2

(ii) Calculate the length of *PD*. 2

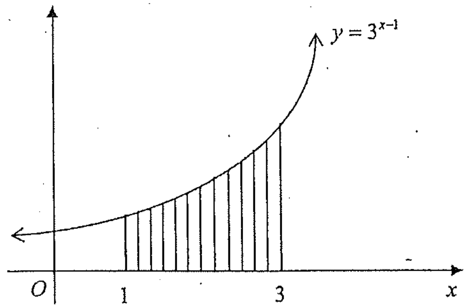
(iii) Using Pythagoras' Theorem in triangle *APD* show that $AP = 36$ cm. 1

(iv) By finding the two areas of the triangles *ABM* and *APD*, prove that the area of the quadrilateral *PMCD* is 936 cm^2. 3

Question 14 (15 Marks) Use a SEPARATE writing booklet

a)

The diagram below shows the shading of a region bounded by the graph $y = 3^{x-1}$ and the lines $x = 1$ and $x = 3$.



(i) Copy and complete the following table giving your answer correct to three decimal places: 1

x	1	1.5	2	2.5	3
$y = 3^{x-1}$	1	1.732			

(ii) Use Simpson's Rule with five function values to approximate the shaded area to three decimal places. 2

b) Consider the curve given by the equation $y = x^3 - 6x^2 + 9x + 4$

(i) Find the coordinates of the stationary points and determine their nature 4

(ii) Find the coordinates of any point of inflexion. 2

(iii) Sketch the curve, showing only the above information. 2

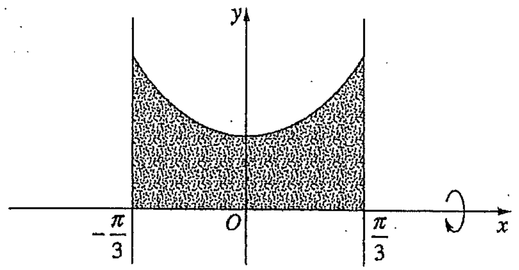
(iv) Determine the values of x for which $\frac{dy}{dx} > 0$ 1

Question 14 continues on the next page

Question 14 continued

c)

The diagram shows the region bounded by the curve $y = \sec x$, the lines $x = \frac{\pi}{3}$ and $x = -\frac{\pi}{3}$, and the x-axis. **3**



The region is rotated about the x-axis. Find the volume of the solid of revolution formed.

End of Question 14

Question 15 (15 Marks) Use a SEPARATE writing booklet

a)

On being retrenched from his job, Kevin receives a cash payment of \$20 000.

One year later, he receives his first annual payout of \$10 000. He continues to receive annual payouts of \$10 000 every year thereafter.

He places all of this money in his suitcase as he receives it, and spends none.

At the end of every year, just before the next payout, Kevin spends 20% of the money in his suitcase on a holiday.

Let A_n be the amount Kevin has in his suitcase immediately after his n^{th} annual payout.

(i) Show that Kevin has \$26 000 in his suitcase immediately after his first annual payout. **1**

(ii) Show that the money in Kevin's suitcase immediately after his 3rd annual payout is given by **2**

$$A_3 = 20000(0.8)^3 + 10000(1 + 0.8 + 0.8^2).$$

(iii) Show that $A_n = 50000 - 30000(0.8^n)$. **3**

(iv) After how many years will the amount in Kevin's suitcase first exceed \$48 000? **2**

(v) What is the most money Kevin will ever have in his suitcase? **1**

b)

Two particles, A and B, move along a straight line so that their displacements, x_A and x_B, in metres, from the origin at time t seconds are given by the following equations respectively:

$$x_A = 12t + 5 \qquad x_B = 6t^2 - t^3$$

(i) Find two expressions for the velocities of particles A and B. **2**

(ii) Which of the two particles is travelling faster at $t = 1$ second? **1**

(iii) At what time does particle B come to rest? **1**

(iv) Find the maximum positive displacement of particle B. **2**

Question 16 (15 Marks) Use a SEPARATE writing booklet

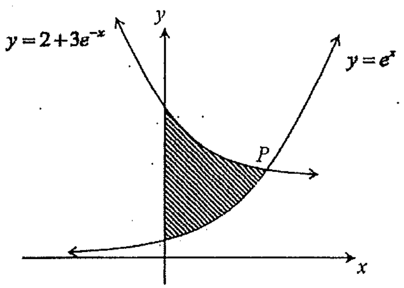
a) A 250mg tablet is dissolved in a glass of water. After t minutes the amount of undissolved tablet, U in mg, is given by the formula:

$U = 250\ e^{-kt}$, where k is a constant.

(i) Calculate the value of k, correct to 4 decimal places, given that 10mg of the tablet remain after 15 minutes. **2**

(ii) Find the rate at which the tablet is dissolving in the glass of water after 10 minutes.
Give your answer correct to two decimal places. **2**

b)



The diagram shows the graphs of $y = e^x$ and $y = 2 + 3e^{-x}$ intersecting at the point P.

(i) Show that the curves intersect when **1**

$$e^{2x} - 2e^x - 3 = 0.$$

(ii) Hence show that the x-coordinate of the point P is $\ln 3$. **2**

(iii) Hence find the exact area of the shaded region. **3**

Question 16 continues on the next page

Question 16 continued

c) There are 5 red marbles and 4 blue marbles in a bag. Bill and Ben are playing a game in which they take turns drawing a marble from the bag and then replacing it.

To win the game, Ben must draw a red marble and for Bill to win he must draw a blue marble. They continue taking turns until there is a winner. Ben goes first.

(i) Find the probability that Ben wins on his first draw. **1**

(ii) Find the probability that Ben wins in three or less of his turns. **2**

(iii) Find the probability that Ben wins the game. **2**

End of Question 16

END OF EXAMINATION

Section I M/C

SOLUTIONS
2 UNIT TRIAL
2014

Q1	C	Q6	B
Q2	A	Q7	C
Q3	D	Q8	B
Q4	C	Q9	B
Q5	B	Q10	C

Section II

Q11

a) $3x^2 - 16x + 5$ $3x \quad -1$ / $x \quad -5$ $\therefore (3x-1)(x-5)$ ✓ ✓ [2]

Well done

b) $|5x+2| < 3 \quad \therefore \pm(5x+2) < 3$

(+) $5x + 2 < 3$, $5x < 1$, $x < \frac{1}{5}$ ✓   (−) $-5x - 2 < 3$, $-5x < 5$, ✓ $x > -1$

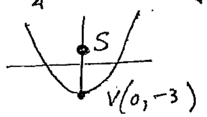
Soln $\{x : -1 < x < \frac{1}{5}\}$ [2]

Some students failed to swap inequality, due to negative sign, in 2nd inequality

c) $\frac{d}{dx}(3 - \cos 2x)^5 = 5(3 - \cos 2x)^4 \times -(-2\sin 2x)$ ✓ ✓
$= 10\sin 2x(3 - \cos 2x)^4$ [2]

some students forgot to multiply by −2

d) Focus of $x^2 = 20(y+3)$ $\therefore 4a = 20$ ie $a = 5$ ✓



$\therefore S(0, 2)$ (focus) ✓ [2]

Well done

e) Eq^n Normal $y = \frac{2}{x}$ at $x = 3$ $\therefore y = \frac{2}{3}$
$y' = -2x^{-2} = -\frac{2}{x^2}$ at $x = 3$ $\therefore y'(3) = -\frac{2}{9}$ ✓
$\therefore$ Normals gradient $\frac{9}{2}$ ✓ $\therefore$ eq^n Normal $y - \frac{2}{3} = \frac{9}{2}(x - 3)$ [3]
$\therefore y = \frac{9}{2}x - \frac{77}{6}$ or $27x - 6y - 77 = 0$ ✓

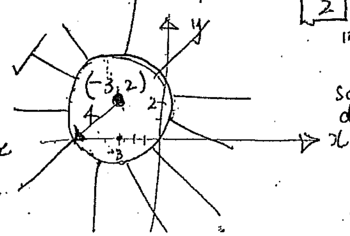
Many students thought $y' = 2\ln x$.

f) $\int_1^3 \frac{4}{x^3}\,dx = \int_1^3 4x^{-3}\,dx = \left[\frac{4x^{-2}}{-2}\right]_1^3 = \left[\frac{-2}{x^2}\right]_1^3$
$= -\frac{2}{9} + 2 = 1\frac{7}{9}$ ✓ [2]

Some students differentiated $\frac{4}{x^3}$ instead of integrating.

g) Centre $(-3, 2)$
radius $r = 4$
TP$(-3, 0)$ False

1 for circle correct centre + radius
1 (solid circle region



[2]

Some students did not shade correctly.

Q12

a) $y = x\cos x$ at $x = \frac{\pi}{2}$, $y = 0$
$y' = x \cdot -\sin x + \cos x \cdot 1 = -x\sin x + \cos x$ ✓
$y'(\frac{\pi}{2}) = -\frac{\pi}{2} \cdot 1 + 0 = -\frac{\pi}{2}$ ✓ [3]
$\therefore$ Eq^n of Tangent $y - 0 = -\frac{\pi}{2}(x - \frac{\pi}{2})$
$\therefore y = -\frac{\pi}{2}x + \frac{\pi^2}{4}$ or $2\pi x + 4y - \pi^2 = 0$ ✓

b) $L_1 : x + y = 4$ $\therefore L_1 + L_2 \to 2x = 12 \therefore x = 6$
$L_2 : x - y = 8$ sub $x = 6$ into L_1 $\therefore y = -2$

(i) $L_1 : x = 0 \; y = 4$
$L_2 : x = 0 \; y = -8$
$\therefore A(0, 4)$ ✓
$C(0, -8)$ ✓

(ii) See above (Solve simultaneously)
$R(6, -2)$ ✓

(iii) SR $x = 6$ ✓

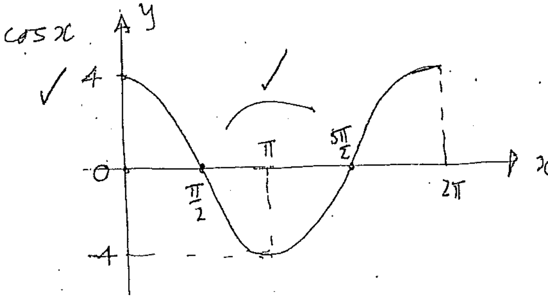
(iv) $L_1 \to y = -x + 4$
ie $m = -1$ ✓

(v) AR $(0, 4)$ to $(6, -2)$
$d = \sqrt{(6-0)^2 + (-2-4)^2} = \sqrt{36 + 36}$
$d = \sqrt{72}$ or $6\sqrt{2}$ ✓ [10]

(vi) $AR = 6\sqrt{2}$ RC, $(6, -2)$ to $(0, -8)$
$d = \sqrt{(6-0)^2 + (-2+8)^2} = \sqrt{36 + 36}$
$d = \sqrt{72}$ or $6\sqrt{2}$ ✓
$\therefore \triangle ARC$ is isosceles as $RC = AR = 6\sqrt{2}$
$m_{L_1} = -1$, $m_{L_2} = 1$ as $m_{L_1} \times m_{L_2} = -1$ $\angle R$ is $90°$ ✓
$\therefore \triangle ARC$ is right-angled isosceles triangle. ✓

(vii) $r = 6\sqrt{2} = \sqrt{72}$ $\therefore$ eq^n of circle Centre $(6, -2)$ $r = \sqrt{72}$
is $(x-6)^2 + (y+2)^2 = 72$ ✓

c) $y = 4\cos x$



[2]

Q13

a) (i) $y = x^{\frac{3}{2}}$ ∴ $y' = \frac{3}{2}x^{\frac{1}{2}}$ ✓ [1]

(ii) $y = x^2 \ln x^2$ (product rule) ∴ $y' = x^2 \cdot \frac{2}{x} + 2\ln x \cdot 2x$ ✓

∴ $y' = 2x + 4x\ln x$ or $2x(1 + 2\ln x)$ ✓ [2]

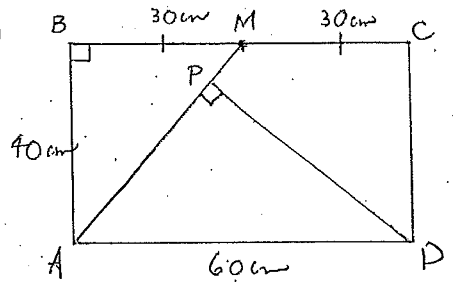
(iii) $y = \dfrac{e^{-2x}}{\sin 3x}$ (Quotient rule) $y' = \dfrac{\sin 3x \cdot -2e^{-2x} - e^{-2x} \cdot 3\cos 3x}{\sin^2 3x}$ ✓

∴ $y' = \dfrac{-e^{-2x}(2\sin 3x + 3\cos 3x)}{\sin^2 3x}$ ✓ [2]

b) $\displaystyle\int \frac{3\sec^2 2x}{1+\tan 2x}\,dx = \frac{3}{2}\int \frac{2\sec^2 2x}{1+\tan 2x}\,dx$ ✓

$= \frac{3}{2}\ln(1+\tan 2x) + C$ ✓ [2]

c)



(i) Prove △ABM ||| △APD

① ∠B = ∠P (90° given) ✓

② ∠BAM + ∠AMB = 90° (anglesum of △) ✓

also ∠BAM + ∠PAD = 90° (rectangle)

∴ ∠AMB = ∠PAD

(or alternate) → 3rd angle equal

∴ △ABM ||| △APD (equiangular) [2]

(ii) PD ? → AM = 50 cm (Pythagoras)

∴ $\frac{50}{60} = \frac{40}{PD}$ (corresponding sides similar △s) [2]

∴ $PD = \frac{40 \times 60}{50} = 48\text{ cm}$

(iii) $AP^2 = AD^2 - PD^2 = 60^2 - 48^2$

∴ $AP^2 = 1296$ ∴ $AP = \sqrt{1296} = 36\text{ cm}$ ✓ [1]

(iv) Area of PMCD = Total − [sum of 2 △s] ✓

$= 40 \times 60 - [\frac{1}{2} \times 40 \times 30 + \frac{1}{2} \times 36 \times 48]$

$= 2400 - [600 + 864]$ ✓

$= 936\text{ cm}^2$ ✓ [3]

Q14

a) (i) $y = 3^{x-1}$ (3DP) [1]

x	1	1.5	2	2.5	3
y	1	1.732	3	5.196	9

Well done

(ii) $h = 0.5$ (width)

$\displaystyle\int_1^3 3^{x-1}\,dx \doteq \frac{h}{3}[1^{st} + last + 4\cdot odds + 2\,evens]$

$\doteq \frac{0.5}{3}[1 + 9 + 4[1.732 + 5.196] + 2\times 3]$

$\doteq \frac{1}{6}[10 + 6 + 27.712] \doteq \frac{1}{6} \times 43.712$

$= 7.285$ (3DP) [2]

Some wrong formulas

b) $y = x^3 - 6x^2 + 9x + 4$

(i) $y' = 3(x^2 - 4x + 3)$

$= 3(x-3)(x-1)$

$f'(x) = 0$ when $x = 1, 3$

$y'' = 6x - 12$

When $x = 1$, $y = 8$, $f'(1) = 0$

and $f''(1) = -6 < 0$

∴ (1, 8) is a max turning point

when $x = 3$ $y = 4$ $f'(3) = 0$

and $f''(3) = 6 > 0$

∴ (3, 4) is a min turning pt

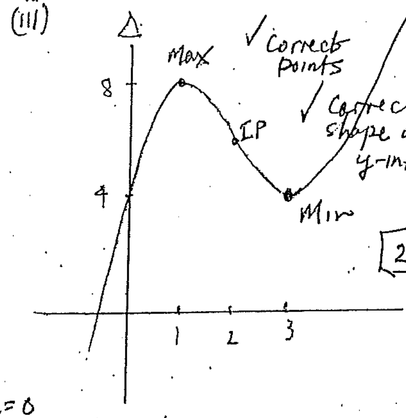
(ii) Inflexions $f''(x) = 0$ when $6x - 12 = 0$

∴ $f''(2) = 0$ check for concavity change

$f''(2-\epsilon) = f''(1) < 0$, $f''(2+\epsilon) = f''(3) > 0$ ∴ (2, 6) inflexion

Some students did not test for P.O.I

(iii)



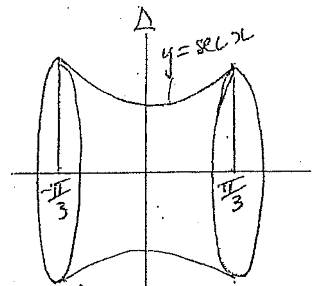
✓ Correct points

✓ Correct shape and y-intercept [2]

(iv) $\frac{dy}{dx} > 0$ when $x > 3$, $x < 1$ [1]

mostly well done

c)



As $y = \sec x$ is even ie $\sec(-\frac{\pi}{3}) = \sec(\frac{\pi}{3})$

$\displaystyle\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} \rightarrow 2\int_0^{\frac{\pi}{3}}$

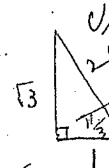
$V = \pi \displaystyle\int_{-\frac{\pi}{3}}^{\frac{\pi}{3}} [f(x)]^2\,dx$ ✓

∴ $V = 2\pi \displaystyle\int_0^{\frac{\pi}{3}} \sec^2 x\,dx$

$= 2\pi [\tan x]_0^{\frac{\pi}{3}}$ ✓

$= 2\pi [\tan\frac{\pi}{3} - \tan 0]$

$V = 2\sqrt{3}\pi\ u^3$ ✓ [3]



Use exact values where possible

Well done

Q15 a) (i) $A_1 = 20000 \times 0.8 + 10000 = \$26{,}000$ [1]

(ii) $A_2 = A_1 \times 0.8 + 10{,}000$

$= [20000 \times 0.8 + 10000] \times 0.8 + 10{,}000$

$= 20000(0.8)^2 + 10000[1 + 0.8]$ ✓ Well done

$A_3 = A_2 \times 0.8 + 10000$

$= [20000(0.8)^2 + 10000[1+0.8]] \times 0.8 + 10000$

$= 20000(0.8)^3 + 10000[0.8 + 0.8^2] + 10000$ [2]

$= 20{,}000(0.8)^3 + 10000[1 + 0.8 + 0.8^2]$ as required

(iii) $A_n = 20{,}000(0.8)^n + 10000[1 + 0.8 + 0.8^2 + \dots + 0.8^{n-1}]$ ✓

$= 20000(0.8)^n + 10000\left[S_n = \frac{1(1-(0.8)^n)}{1-0.8}\right] \rightarrow S_n = \frac{a(1-r^n)}{1-r}$

$= 20000(0.8)^n + 50000(1-(0.8)^n)$ ✓ Mostly well done

$= 50000 - 30000(0.8)^n$ ✓ [3]

(iv) $50000 - 30000(0.8)^n > 48000$

$-30000(0.8)^n > -2000$ ✓

$\therefore (0.8)^n < \frac{2}{30}$ take logs $n \log 0.8 < \log \frac{1}{15}$

and as $\log(0.8) < 0$ $\therefore n > \frac{\log(\frac{1}{15})}{\log(0.8)} > 12.1359\dots$

Some inequality signs wrong. $\therefore$ **$n = 13$ yrs** (required to exceed \$48,000) ✓ [2]

(v) As $n \to \infty$ $A_n = 50000 - 30000(0.8)^n$

$0.8^n \to 0$ $\therefore A_n \to 50{,}000$ ✓ (most kevin will ever have in suitcase) [1]

b) $x_A = 12t + 5$ ✓ $x_B = 6t^2 - t^3$ ✓

(i) $\therefore v_A = \frac{dx_A}{dt} = 12$ m/s $\therefore v_B = \frac{dx_B}{dt} = 12t - 3t^2$ m/s [2]

(ii) $v_A(1) = 12$ m/s $v_B(1) = 12 - 3 = 9$ m/s

$\therefore$ Particle A travelling faster at $t = 1$ sec ✓ [1]

(iii) $v_B = 0$ $\therefore 12t - 3t^2 = 0$ ie $3t(4-t) = 0$

$\therefore$ $t = 0, 4$ secs Particle B at rest $t = 0, 4$ secs [1]

(iv) Maximum displacement (pos ≥ 0) of particle B

$x_B(4) = 6 \cdot 4^2 - 4^3 = 32$ m ✓ [2]

($v_B = 0$) ✓

Well done overall

Q16 a) $U = 250e^{-kt}$ (i) $10 = 250e^{-15k}$ $t = 15$ mins, $U = 10$ mg

$\frac{dU}{dt} = -250ke^{-kt}$ $\therefore \frac{1}{25} = e^{-15k}$ ✓ (take logs)

$\ln e^{-15k} = \ln\frac{1}{25}$, $-15k = \ln 25$, $k = -\frac{1}{15}$

errors
- wrong substitution: $k = -\frac{1}{15}\ln\frac{1}{25} = -\frac{1}{15}\ln 5^{-2} = \frac{2}{15}\ln 5$
- incorrect calc. work $\therefore k = 0.2146$ ✓ (4DP) ✓ [2]

(ii) $\frac{dU}{dt}$ $(t = 10)$ $\frac{du}{dt} = -ku$ decay

$\therefore \frac{dU}{dt} = -250 \cdot \frac{2}{15}\ln 5 \, e^{-10 \times \frac{2}{15}\ln 5}$ ✓ $\doteq$ ~~3.90 mg~~/min -6.27 ✓ [2]

b) When $y = e^x$ and $y = 2 + 3e^{-x}$ intersects

(i) $e^x = 2 + 3e^{-x}$ must be explicit $2 + 3e^{-x} = e^x$ $\therefore e^{2x} = 2e^x + 3$ or $e^{2x} - 2e^x - 3 = 0$ [1]

(ii) let $u = e^x$ $\therefore u^2 - 2u - 3 = 0$ ie $(u-3)(u+1) = 0$ ✓

$\therefore e^x = 3$ or $e^x = -1$ no soln

Take $\log_e$: $\ln e^x = \ln 3$ ie $x \ln e = \ln 3$ (as $\ln e = 1$)

$\therefore x = \ln 3$ generally ✓ well answ. [2]

(iii) Exact Area $A = \int_0^{\ln 3} 2 + 3e^{-x} - e^x \, dx$ ✓

ie $A = [2x - 3e^{-x} - e^x]_0^{\ln 3} = (2\ln 3 - 3e^{-\ln 3} - e^{\ln 3}) - (0 - 4)$

$\therefore A = 2\ln 3 - 3[\frac{1}{3}] - 3 + 4$ [3]

$A = 2\ln 3$ ✓

many students could not correctly evaluate $-3e^{-\ln 2}$

c) 5R, 4B

(i) $\frac{5}{9}$ ✓ [1]

Ben Wins in 3 or less turns

(ii) $\frac{5}{9} + \frac{4}{9} \cdot \frac{5}{9} \cdot \frac{5}{9} + \frac{4}{9} \cdot \frac{5}{9} \cdot \frac{4}{9} \cdot \frac{5}{9} \cdot \frac{5}{9}$ ✓

(BL BL BW; BL BL BL BL BW)

$= \frac{5}{9} + (\frac{5}{9})^2 \cdot \frac{4}{9} + (\frac{5}{9})^3 (\frac{4}{9})^2$ ✓ [2]

Poorly answered.

note $r = \frac{5}{9} \cdot \frac{4}{9} = \frac{20}{81}$ $= \frac{505}{729} + \frac{2000}{59049} \approx 0.72659994 2$

Many student could not generate the seq.

(iii) Ben Wins $S_\infty = \frac{\frac{5}{9}}{1 - \frac{20}{81}} = \frac{5}{9} \cdot \frac{81}{61} = \frac{45}{61}$ ✓ [2]

$P(\text{Ben Wins}) = \frac{45}{61} \approx 0.737704918$ ✓

many student did not realise ~~the~~ Seq