Name: .... Maths Class: ....

SYDNEY TECHNICAL HIGH SCHOOL

YEAR 11 PRELIMINARY COURSE

Extension 1 Mathematics

Assessment 2

July 2014

<u>TIME ALLOWED</u>: **75 minutes**

Instructions:

- ***Start each question on a new page.***
- Write your name and class at the top of this page, and on your answer booklet.
- Write in blue or black pen only.
- All necessary working must be shown. <u>Marks may not be awarded for careless or badly arranged work.</u>
- Marks indicated within each question are a guide only and may be varied at the time of marking.
- It is suggested that you spend no more than 7 minutes on Section A.
- Approved calculators may be used.

<u>SECTION A</u>: (5 Marks)

Answers to these multiple choice should be completed on the multiple choice answer sheet supplied with your answer booklet.

All questions are worth 1 mark

1	$\frac{d}{dx}\left(\frac{5}{\sqrt{x}}\right) =$ A. $\frac{5x\sqrt{x}}{2}$ B. $\frac{5}{2x\sqrt{x}}$ C. $\frac{-5x\sqrt{x}}{2}$ D. $\frac{-5}{2x\sqrt{x}}$
2	$\sin(-120^o) =$ A. $-\frac{1}{2}$ B. $\frac{1}{2}$ C. $\frac{-\sqrt{3}}{2}$ D. $\frac{\sqrt{3}}{2}$
3	The acute angle between the line x=3 and the line $x - \sqrt{3}\,y + 2 = 0$ is: A. 60^o B. 30^o C. 90^o D. 45^o
4	If the endpoints of a diameter of the circle $(x-2)^2 + (y+1)^2 = 25$ are A (-1, -5) and B(k, m) then the values of k and m are: A. $k = 5$ and $m = 3$ B. $k = 3$ and $m = 5$ C. $k = -4$ and $m = -9$ D. $k = -9$ and $m = -4$
5	Given that $\cos A = k$, $k > 0$, and $0^o \leq A \leq 90^o$, then $\sin 2A =$ A. $2\sqrt{1-k^2}$ B. $2\sqrt{1+k^2}$ C $2k\sqrt{1-k^2}$ D. $2k\sqrt{1+k^2}$

SECTION B

(START EACH QUESTION ON A NEW PAGE)

QUESTION 6: (10 Marks)

Marks

(a) Differentiate with respect to x:

(i) $y = (3x^2 - 5)^3$ **1**

(ii) $y = \dfrac{x^3 - x^2 + 1}{x}$ **1**

(iii) $y = (x+1)\sqrt{x+1}$ **2**

(b) (i) Find the slope of the tangent to the curve $y = x^3 - x^2 - x + 1$ at the point where $x = 1$. **2**

(ii) What does this imply about the x-axis and the curve at the point where $x = 1$? **1**

(c) The lines $3x + 4y - 2 = 0$ and $3x + 4y + k = 0$ are 3 units apart. **3**

Find the two values of k.

QUESTION 7: (9 Marks) *(Start on a new page)*

Marks

(a) (i) Show that $\dfrac{1}{x+h} - \dfrac{1}{x} = \dfrac{-h}{x(x+h)}$ **1**

(ii) Differentiate $f(x) = \dfrac{1}{x}$ using the method of First Principles. **3**

(b) Show that the equation of the tangent to the curve $y = \dfrac{x+2}{x-1}$ at the point where it crosses the x-axis is $x + 3y + 2 = 0$ **3**

(c) Find the point P which divides the interval joining R(4, 3) to S(2, -1) externally in the ratio 3: 5 **2**

QUESTION 8: (9 Marks) *(Start on a new page)*

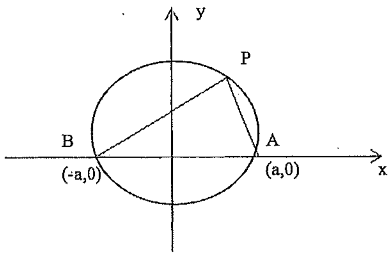
Marks

(a) Find $\lim_{h \to 0} \left\{ \frac{(5+h)^2 - 25}{h} \right\}$ **2**

(b) Give the equation of the perpendicular bisector of the line which joins the points A (3, - 2) and B (5, 2). **3**

Give your answer in general form.

(c) (i) Show that the point P $(a\cos\theta, a\sin\theta)$ lies on the circle $x^2 + y^2 = a^2$ **1**



(ii) Find the gradients of the lines BP and AP **2**

(iii) Deduce that the line AP is at right angles to the line BP. **1**

(Use ONLY the information in parts (i) and (ii). You are NOT to use the circle geometry proof related to the angle in a semi-circle)

QUESTION 9: (9 Marks) *(Start on a new page)*

Marks

(a) Show that $\tan(x + 45^o) = \frac{\sin x + \cos x}{\cos x - \sin x}$ **2**

(b) (i) Find the equation of the normal to $y = x^3 - 2x^2 - 3x + 1$ at P(2, -5). **2**

(ii) Show that there is another point on the curve where the normal to the curve is parallel to the normal at P. **2**

Find the co-ordinates this second point.

(c) (i) Show that $\sin(A + B) + \sin(A - B) = 2\sin A\cos B$ **1**

(ii) Hence, find the value of $\sin 75^o + \sin 15^o$ **2**

QUESTION 10: (9 Marks) *(Start on a new page)*

Marks

(a) Find $\lim_{x \to \infty} \frac{2x^2+x}{3x^2-2}$ **1**

(b) (i) Show that the perpendicular distance of the point (4, 5) from the line $y = mx$ is: **1**

$$d = \frac{|4m-5|}{\sqrt{m^2+1}}$$

(ii) If y = mx is a tangent to the circle $(x-4)^2 + (y-5)^2 = 4$, explain why **2**

$$\frac{|4m-5|}{\sqrt{m^2+1}} = 2$$

(c) (i) Show that $\cos 3A = 4\cos^3 A - 3\cos A$ **2**

(ii) Hence solve $4\cos^3 A - 3\cos A = \frac{1}{2}$ for $0^o \leq A \leq 180^o$ **3**

QUESTION 11: (9 Marks) *(Start on a new page)*

Marks

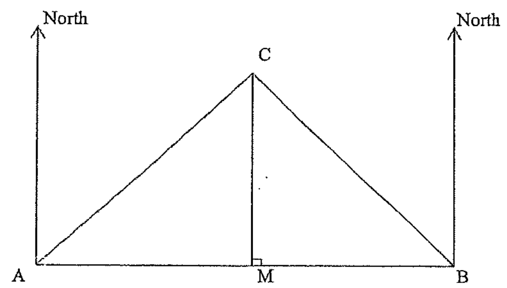
(a) (i) Express $\cos\theta + \sqrt{3}\sin\theta$ in the form $R\cos(\theta - \alpha)$ **2**

(ii) Hence solve the equation **2**

$$\cos\theta + \sqrt{3}\sin\theta = \sqrt{2} \qquad \text{for } 0^o \leq \theta \leq 360^o$$

(b) A surveyor stands at a point A and takes the bearing of a rock C, which he finds to be Nα^oE.
He walks due East 1000m to a second point B where he sees that C has a bearing of Nβ^oW.

He then moves to a third point, M, directly south of C.



(i) Redraw the diagram above, and put on it all of the information contained in the question. **1**

(ii) Prove that from M the distance to the rock C is given by **4**

$$\frac{1000\cos\alpha\cos\beta}{\sin(\alpha+\beta)}$$

QUESTION 8:

(a) $\lim_{h\to 0}\left[\frac{25+10h+h^2-25}{h}\right]$

$= \lim_{h\to 0}(10+h)$

$= 10$

(b) $\text{Slope}_{AB} = 4/2$

$= 2$

Slope (perp) $= -1/2$

midpoint $= (4, 0)$

$y - 0 = -\frac{1}{2}(x-4)$

$2y = -x + 4$

$x + 2y - 4 = 0$

(c) (i) $x^2 + y^2 = a^2\cos^2\theta + a^2\sin^2\theta$

$= a^2$

(ii) $\text{Slope}_{AP} = \frac{a\sin\theta}{a\cos\theta - a}$

$= \frac{\sin\theta}{\cos\theta - 1}$

$\text{slope}_{BP} = \frac{a\sin\theta}{a\cos\theta + a}$

$= \frac{\sin\theta}{\cos\theta + 1}$

(iii) $m_{AP}\cdot m_{BP} = \frac{\sin^2\theta}{\cos^2\theta - 1}$

$= \frac{1-\cos^2\theta}{\cos^2\theta - 1}$ $\left(\text{or } \frac{\sin^2\theta}{-\sin^2\theta}\right)$

$= -1$

$\therefore \angle APB = 90^\circ$

QUESTION 9:

(a) $\tan(x+45^\circ) = \frac{\tan x + \tan 45}{1 - \tan x\tan 45}$

$= \frac{\tan x + 1}{1 - \tan x}$

$= \frac{\sin x/\cos x + 1}{1 - \sin x/\cos x}$

$= \frac{\sin x + \cos x}{\cos x - \sin x}$

(b) (i) $\frac{dy}{dx} = 3x^2 - 4x - 3$

At $x=2$ $\quad m_T = 1 \Rightarrow m_N = -1$

$\therefore$ Equation is $y + 5 = -1(x-2)$

$x + y + 3 = 0$

(ii) If normals are parallel, so are the tangents

$\therefore 3x^2 - 4x - 3 = 1$

$\therefore 3x^2 - 4x - 4 = 0$

$(3x+2)(x-2) = 0$

$\therefore x = 2$ or, $x = -2/3$

↑ found $\quad \therefore y = -8/27 - 8/9 + 2 + 1$

$= 49/27$

(c)(i) $\sin(A+B) + \sin(A-B)$

$= \sin(A)\cos B + \cos A\sin B + \sin A\cos B - \cos A\sin B$

$= 2\sin A\cos B$

(ii) Let $A = 45$ $B = 30$.

$\therefore \sin 75 + \sin 15 = 2\sin 45\cos 30$

$= 2\times \frac{1}{\sqrt{2}}\times\frac{\sqrt{3}}{2}$

$= \sqrt{3}/\sqrt{2}$ or, $\sqrt{6}/2$

YEAR 11 – SOLUTIONS – EXT 1 JUNE 2014

SECT. A. (1) D (2) C (3) A (4) A (5) C

SECTION B **QUESTION 6:**

(a) (i) $18x(3x^2-5)$

(ii) $2x - 1 - 1/x^2$

(iii) $\frac{dy}{dx} = \frac{d}{dx}(x+1)^{3/2}$

$= \frac{3}{2}(x+1)^{1/2}$ or $\frac{3\sqrt{x+1}}{2}$

(b) (i) $\frac{dy}{dx} = 3x^2 - 2x - 1$

At $x=1$ $\frac{dy}{dx} = 0$

(ii) the x-axis is a tangent.

(c) point on $3x+4y-2=0$ is $(0, +\frac{1}{2})$

$p = 3 = \left|\frac{0+2+k}{5}\right|$

$\therefore |k+2| = 15 \Rightarrow k = 13$ or $k = -17$

QUESTION 7:

(a) (i) $\frac{1}{x+h} - \frac{1}{x} = \frac{x-(x+h)}{x(x+h)}$

$= \frac{-h}{x(x+h)}$

(ii) $f'(x) = \lim_{h\to 0} \frac{f(x+h)-f(x)}{h}$

$= \lim_{h\to 0} \frac{1/(x+h) - 1/x}{h}$

$= \lim_{h\to 0} \frac{-h}{xh(x+h)}$

$= \lim_{h\to 0} \frac{-1}{x(x+h)}$

$= -1/x^2$

(b) $\frac{dy}{dx} = \frac{(x-1)1-(x+2)1}{(x-1)^2}$

$= -3/(x-1)^2$

At $y=0$, $x=-2$

$\therefore m_T = -3/9$

$\therefore m_T = -1/3$

Equation is: $y = -\frac{1}{3}(x+2)$

$x + 3y + 2 = 0$

(c) $k_1 : k_2$

$-3 : 5$

$(4,3)$ $(2,-1)$

P is $\left(\frac{-3\times 2 + 5\times 4}{2}, \frac{3+15}{2}\right)$

$= (7, 9)$

OR

QUESTION 10:

(a) $\lim_{x \to \infty} \frac{2x^2 + x}{3x^2 - 2} = \lim_{x \to \infty} \frac{2 + 1/x}{3 - 2/x^2}$

$= 2/3$

(b)

(i) $y = mx \Rightarrow mx - y = 0$

$d = \left| \frac{4m + 5(-1) + 0}{\sqrt{m^2+1}} \right|$

$= \frac{|4m - 5|}{\sqrt{m^2+1}}$

(ii) y = mx is a tangent to the circle means the distance from the point of intersection to the centre of the circle is a radius, AND is the shortest distance of the line from the origin ie d (above) = 2.

(b) (i) $\cos 3A = \cos(2A + A)$

$= \cos 2A \cos A - \sin 2A \sin A$

$= (2\cos^2 A - 1)\cos A - 2\sin^2 A \cos A$

$= 2\cos^3 A - \cos A - 2(1 - \cos^2 A)\cos A$

$= 4\cos^3 A - 3\cos A$

(ii) $4\cos^3 A - 3\cos A = 1/2$.

means $\cos 3A = 1/2$ $\quad 0^\circ \leq 3A \leq 540^\circ$

$\therefore 3A = 60^\circ, 300^\circ, 420^\circ$

$\therefore A = 20^\circ, 100^\circ, 140^\circ$

(a) (i) $R = 2$ $\quad \cos\theta + \sqrt{3}\sin\theta = 2\left(\frac{1}{2}\cos\theta + \frac{\sqrt{3}}{2}\sin\theta\right)$

$= 2\cos(\theta - \alpha)$

where $\cos\alpha = 1/2$

$\Rightarrow \alpha = 60^\circ$

$\therefore$ Exp^N $= 2\cos(\alpha - 60^\circ)$

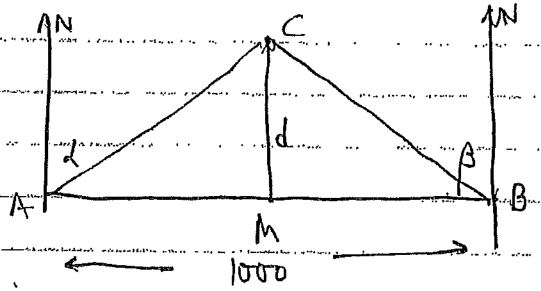
(ii) $2\cos(\theta - 60^\circ) = \sqrt{2}$

$\therefore \cos(\theta - 60^\circ) = 1/\sqrt{2}$.

$\therefore \theta - 60^\circ = 45^\circ, 315^\circ$

$\therefore \theta = 105^\circ, ~~375^\circ~~, 15^\circ$

(b)



(ii) $\angle CAM = (90 - \alpha)^\circ$ $\quad \angle CBM = (90 - \beta)^\circ$

In $\triangle CBM$, $\quad d/CB = \sin(90 - \beta)$

$= \cos\beta$

In $\triangle ABC$ $\quad \frac{CB}{\sin(90 - \alpha)} = \frac{1000}{\sin(\alpha + \beta)} \leftarrow (1)$

$CB = \frac{1000\cos(90 - \alpha)}{\sin(\alpha + \beta)}$

$\therefore d = \frac{1000\cos\alpha\cos\beta}{\sin(\alpha + \beta)}$