ST ANDREW'S CATHEDRAL SCHOOL
Founded 1885

MATHEMATICS

EXTENSION 1

PRELIMINARY COURSE

ASSESSMENT TASK 2 - 2009

(Weighting : 30%)

TIME ALLOWED : 50 minutes

INSTRUCTIONS

* Attempt all questions.
* All necessary working must be shown in all questions.
* Approved calculators and templates may be used.

y

Section A (17 Marks) — Marks

1. Solve $\frac{x}{x-5} \le 3$ (3)

2. The point $(13, -16)$ divides the interval AB externally in the ratio $3:5$. If A is the point $(4,-7)$, find the co-ordinates of point B. (3)

3. The acute angle between the lines $5x + 2y - 9 = 0$ and $y = kx + 7$ is 45°. Find the value(s) of k. (3)

4. (a) On the same axes sketch the graphs $y = 4 - x^2$ and $y = |x - 2|$ clearly indicating their intercepts. (2)

   (b) Hence find the values of x for which $x^2 + |x - 2| > 4$. (1)

5. Use $15^\circ = 60^\circ - 45^\circ$ to find, in its simplest form, the exact value of $\tan 15^\circ$. (2)

6. Solve $\cos 2\theta = \cos\theta - 1$ for $0^\circ \le \theta \le 360^\circ$ (3)

St. Andrew's Cathedral School, Sydney
Year 11, Mathematics Extension 1
Assessment Task 2, July 2009

Section B *Start a new booklet* (18 Marks)

Marks

1. Prove that $\cot 2\theta + \operatorname{cosec} 2\theta = \cot\theta$ (3)

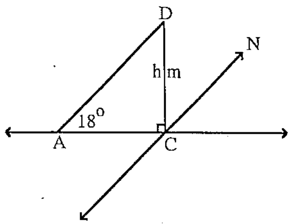
2. Use the "t results" to prove that $1 + \tan\theta \tan\frac{\theta}{2} = \sec\theta$. (2)

3. Given $\cos\theta = \frac{2}{5}$ and $270° < \theta < 360°$, find the exact value of $\tan\frac{\theta}{2}$. (3)

4. (a) Use the expansion of $A\cos(x-\alpha)$ to express $\sqrt{2}\cos x + \sqrt{7}\sin x$ in the form $A\cos(x-\alpha)$ where $A>0$ and α is acute.
(Give the value of α to the nearest minute.) (3)

   (b) Hence solve $\sqrt{2}\cos x + \sqrt{7}\sin x = \frac{3}{2}$ for $0° \le x \le 360°$. (2)

5. CD is a radio tower of height h metres.
A and B are two points 200 metres apart in the same plane as C, the foot of the tower.
From A, due west of the tower, the angle of elevation of D, the tower's summit is 18°.
From B, bearing 120°T from the tower, the angle of elevation of D is 24°.



   (a) Copy the diagram and complete it to represent all of the given information. (1)

   (b) Find expressions for AC and BC in terms of h. (1)

   (c) Find, correct to 1 decimal place, the height of the tower. (3)

Year 11 (Extension) Assessment Task 2 July 2009

Section A

1.

$\dfrac{x}{x-5} \leq 3$

(undefined at $x=5$)

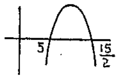
$\dfrac{x}{x-5} \times (x-5)^2 \leq 3 \times (x-5)^2$

$x(x-5) - 3(x-5)^2 \leq 0$

$(x-5)[x - 3(x-5)] \leq 0$

$(x-5)(-2x+15) \leq 0$

$x \leq 5, \quad x \geq \frac{15}{2}$



but $x \neq 5$

$\therefore \quad x < 5, \quad x \geq \frac{15}{2}$

3

2.

$\text{point} = \left[\dfrac{kx_2 + lx_1}{k+l}, \dfrac{ky_2 + ly_1}{k+l}\right]$

external division $k:l = -3:5$

$(13, -16) = \left(\dfrac{-3x_2 + 5 \times 4}{-3+5}, \dfrac{-3y_2 + 5 \times -7}{-3+5}\right)$

$\Rightarrow \quad 13 = \dfrac{-3x_2 + 20}{2}$

$26 = -3x_2 + 20$

$6 = -3x_2$

$x_2 = -2$

and $\quad -16 = \dfrac{-3y_2 - 35}{2}$

$-32 = -3y_2 - 35$

$3 = -3y_2$

$y_2 = -1$

$\therefore$ B is point $(-2, -1)$

3

M.Y.

3.

$5x + 2y - 9 = 0$

$y = \dfrac{-5x + 9}{2}$

$\Rightarrow \quad m_1 = -\frac{5}{2}$

$y = kx + 7$

$\Rightarrow \quad m_2 = k$

$\tan\psi = \left|\dfrac{m_1 - m_2}{1 + m_1 m_2}\right|$

$\tan 45^\circ = \left|\dfrac{-\frac{5}{2} - k}{1 + -\frac{5}{2}k}\right|$

$1 = \left|\dfrac{-\frac{5}{2} - k}{1 - \frac{5k}{2}}\right|$

$\left|1 - \frac{5k}{2}\right| = \left|-\frac{5}{2} - k\right|$

$\Rightarrow \quad 1 - \frac{5k}{2} = -\frac{5}{2} - k$

$-\frac{3k}{2} = -\frac{7}{2}$

$k = \frac{7}{3}$

and $\quad 1 - \frac{5k}{2} = -\left(-\frac{5}{2} - k\right)$

$1 - \frac{5k}{2} = \frac{5}{2} + k$

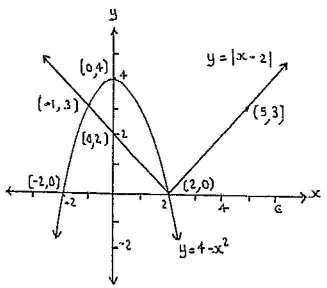
$-\frac{7k}{2} = \frac{3}{2}$

$k = -\frac{3}{7}$

$\therefore \quad k = \frac{7}{3}, -\frac{3}{7}$

3

4. a)



2

4. b)

$x^2 + |x-2| > 4$

$|x-2| > 4 - x^2$

$\therefore \quad x < -1, \quad x > 2$

1

5.

$\tan 15^\circ$

$= \tan(60^\circ - 45^\circ)$

$= \dfrac{\tan 60^\circ - \tan 45^\circ}{1 + \tan 60^\circ \tan 45^\circ}$

$= \dfrac{\sqrt{3} - 1}{1 + \sqrt{3} \times 1}$

$= \dfrac{\sqrt{3} - 1}{1 + \sqrt{3}} \times \dfrac{1 - \sqrt{3}}{1 - \sqrt{3}}$

$= \dfrac{\sqrt{3} - 3 - 1 + \sqrt{3}}{1 - 3}$

$= \dfrac{-4 + 2\sqrt{3}}{-2}$

$= 2 - \sqrt{3}$

2

6.

$\cos 2\theta = \cos\theta - 1$

$\cos^2\theta - \sin^2\theta = \cos\theta - 1$

$\cos^2\theta - (1 - \cos^2\theta) - \cos\theta + 1 = 0$

$2\cos^2\theta - \cos\theta = 0$

$\cos\theta(2\cos\theta - 1) = 0$

$\Rightarrow \quad \cos\theta = 0$

$\theta = 90^\circ, 270^\circ$

and $\quad 2\cos\theta - 1 = 0$

$\cos\theta = \frac{1}{2}$

$\theta = 60^\circ, 300^\circ$

$\therefore \quad \theta = 60^\circ, 90^\circ, 270^\circ, 300^\circ$

3

Section B

1.

$\text{L.H.S} = \cot 2\theta + \operatorname{cosec} 2\theta$

$= \dfrac{\cos 2\theta}{\sin 2\theta} + \dfrac{1}{\sin 2\theta}$

$= \dfrac{\cos^2\theta - \sin^2\theta + 1}{2\sin\theta\cos\theta}$

$= \dfrac{\cos^2\theta + \cos^2\theta}{2\sin\theta\cos\theta}$

$= \dfrac{2\cos^2\theta}{2\sin\theta\cos\theta}$

$= \dfrac{\cos\theta}{\sin\theta}$

$= \cot\theta$

$= \text{R.H.S}$

3

2.

let $\quad t = \tan\frac{\theta}{2}$

$\text{L.H.S} = 1 + \tan\theta\tan\frac{\theta}{2}$

$= 1 + \dfrac{2t}{1-t^2} \times t$

$= \dfrac{1 - t^2 + 2t^2}{1 - t^2}$

$= \dfrac{1 + t^2}{1 - t^2}$

$= \dfrac{1}{\cos\theta}$

$= \sec\theta$

$= \text{R.H.S.}$

2

3.

let $\quad t = \tan\frac{\theta}{2}$

$\cos\theta = \frac{2}{5}$

$\dfrac{1 - t^2}{1 + t^2} = \dfrac{2}{5}$

$5 - 5t^2 = 2 + 2t^2$

$3 = 7t^2$

$t^2 = \frac{3}{7}$

since $\quad 270^\circ < \theta < 360^\circ$

$135^\circ < \frac{\theta}{2} < 180^\circ$

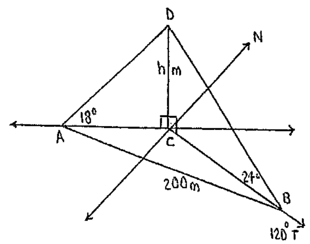
$-1 < \tan\frac{\theta}{2} < 0$

hence $\quad t = -\sqrt{\frac{3}{7}}$

$\tan\frac{\theta}{2} = -\sqrt{\frac{3}{7}}$

3

5. a)



1

5. b)

in ΔACD

$\tan 18^\circ = \dfrac{h}{AC}$

$\cot 18^\circ = \dfrac{AC}{h}$

$AC = h\cot 18^\circ$

in ΔBCD

$\tan 24^\circ = \dfrac{h}{BC}$

$\cot 24^\circ = \dfrac{BC}{h}$

$BC = h\cot 24^\circ$

1

5. c)

$\angle ACB = 270^\circ - 120^\circ$

$= 150^\circ$

in $\Delta ACB \quad c^2 = a^2 + b^2 - 2ab\cos C$

$200^2 = (h\cot 24^\circ)^2 + (\cot 18^\circ)^2 - 2 \times h\cot 24^\circ \times h\cot 18^\circ \times \cos 150^\circ$

$200^2 = h^2(\cot^2 24^\circ + \cot^2 18^\circ - 2\cot 24^\circ\cot 18^\circ\cos 150^\circ)$

$h^2 = \dfrac{200^2}{\cot^2 24^\circ + \cot^2 18^\circ - 2\cot 24^\circ\cot 18^\circ\cos 150^\circ}$

$h = \dfrac{200}{\sqrt{\cot^2 24^\circ + \cot^2 18^\circ - 2\cot 24^\circ\cot 18^\circ\cos 150^\circ}}$

$h = 38.4 \text{ m}$ (c.t. 1 dec. pl.)

3

4. a)

$\sqrt{2}\cos x + \sqrt{7}\sin x = A\cos(x - \alpha)$

$\sqrt{2}\cos x + \sqrt{7}\sin x = A(\cos x\cos\alpha + \sin x\sin\alpha)$

$\Rightarrow \quad A\cos\alpha = \sqrt{2} \quad \text{—} ①$

$A\sin\alpha = \sqrt{7} \quad \text{—} ②$

$①^2 + ②^2 \quad (A\cos\alpha)^2 + (A\sin\alpha)^2 = (\sqrt{2})^2 + (\sqrt{7})^2$

$A^2(\cos^2\alpha + \sin^2\alpha) = 2 + 7$

$A^2 = 9$

$A = 3 \quad (\text{as } A > 0)$

$② \div ① \quad \dfrac{A\sin\alpha}{A\cos\alpha} = \dfrac{\sqrt{7}}{\sqrt{2}}$

$\tan\alpha = \sqrt{\frac{7}{2}}$

$\alpha = 61^\circ 52'$ (α is acute)

$\therefore \quad \sqrt{2}\cos x + \sqrt{7}\sin x = 3\cos(x - 61^\circ 52')$

3

4. b)

$\sqrt{2}\cos x + \sqrt{7}\sin x = \frac{3}{2}$

$\Rightarrow \quad 3\cos(x - 61^\circ 52') = \frac{3}{2}$

$\cos(x - 61^\circ 52') = \frac{1}{2}$

since $\quad 0^\circ \leq x \leq 360^\circ$

$-61^\circ 52' \leq x - 61^\circ 52' \leq 298^\circ 8'$

$\therefore \quad x - 61^\circ 52' = 60^\circ, -60^\circ$

$x = 121^\circ 52', \quad 1^\circ 52'$

2