ST ANDREW'S CATHEDRAL SCHOOL
Founded 1885

MATHEMATICS

PRELIMINARY COURSE 2009

ASSESSMENT TASK 2

(35 marks)

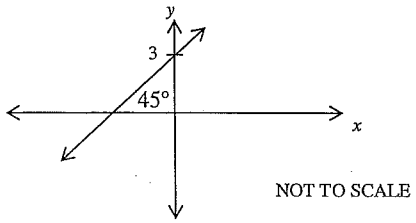
Time allowed: 50 minutes

Instructions

- Attempt **all** questions
- **All** necessary working must be shown in **all** questions
- Approved calculators and templates may be used

QUESTION 1 *(Start a new booklet)* **[7 marks]**

(a) Find the equation of the line shown below. [1]



(b) X (-3,8) is the midpoint of the interval PQ. P has coordinates (2,6). Find the coordinates of Q. [2]

(c) Fully simplify: $\dfrac{\sin\theta}{\cos(90-\theta)}$ [1]

(d) In ΔABC, side AB is 17cm long, side BC is 5cm long and $\angle BAC$ is 16°. Find the possible size(s) of $\angle BCA$ correct to the nearest minute. [3]

QUESTION 2 *(Start a new booklet)* **[6 marks]**

(a) Fully simplify: $\dfrac{\sin B}{\operatorname{cosec} B}+\dfrac{\cos B}{\sec B}$ [2]

(b) If $\cos\theta=\dfrac{5}{8}$ and $270°<\theta<360°$ find the value of $\tan\theta$ [2]

(c) Solve $\sqrt{3}\tan\theta=1$ for $0°\le\theta\le360°$ [2]

QUESTION 3 *(Start a new booklet)* **[7 marks]**

(a) A surveyor stands at a point (X) in a field. After taking some measurements he records the following in his log book: Post A is 78 m from X on a bearing of 112°T. Post B is 64 m from X on a bearing of 195°T

(i) Draw a diagram to illustrate this information [1]

(ii) What is the size of $\angle AXB$? [1]

(ii) Find the length, to the nearest metre, of fencing needed for the side AB [2]

(b) Solve $2\cos^2\theta = \cos\theta$ for $0° \le \theta \le 360°$ [3]

QUESTION 4 *(Start a new booklet)* **[7 marks]**

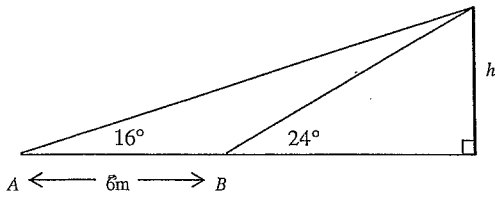
(a) Find the exact value of

(i) $\tan 120°$ [1]

(ii) $\sec(-30°)$ [1]

(b) Show that: $\operatorname{cosec}^2\theta(\sec^2\theta - 1) = \sec^2\theta$ [2]

(c) A flagpole is steadied using two guy ropes attached to the top of the pole. The first guy rope is secured at point A making an angle of 16° with the ground. The second guy rope is secured at point B making an angle of 24° with the ground. Point B is 6m closer to the flagpole than point A. Find the height, h, of the flag pole to the nearest centimetre. [3]



QUESTION 5 *(Start a new booklet)* **[8 marks]**

The points Q, R and S have the coordinates (1,5), (-3,2) and (0,0) respectively.

(a) Find the gradient of the line QR [1]

(b) Show that the equation of the line which passes through Q and R is given by $3x - 4y + 17 = 0$ [2]

(c) Find the point P where $3x - 4y + 17 = 0$ cuts the x-axis [1]

(d) Find the length of PR (Answer in simplest form) [2]

(e) Find the size of the acute angle PR makes with the x-axis [1]
(to the nearest minute)

(f) Find the area of triangle PRS (Answer to 1 decimal place) [1]

MARKING CRITERIA

QUESTION 1 [7 marks]

(a) 1 mark for answer

(b) 1 mark for one correct coordinate
2 marks for both correct coordinates

(c) 1 mark for answer

(d) 1 mark for sine rule $\frac{\sin\theta}{17}=\frac{\sin 16^\circ}{5}$
1 mark for acute angle
1 mark for obtuse angle

QUESTION 2 [6 marks]

(a) 1 mark for $\frac{\sin B}{\frac{1}{\sin B}}+\frac{\cos B}{\frac{1}{\cos B}}$
2 marks for answer

(b) 1 mark if $\tan\theta=\frac{\sqrt{39}}{5}$
2 marks for $\tan\theta=-\frac{\sqrt{39}}{5}$

(c) 1 mark for one angle
2 marks for both angles

QUESTION 3 [7 marks]

(a) (i) 1 mark for correct diagram

(ii) 1 mark for correct angle

(iii) 1 mark for correct substitution into cosine rule
2 marks for answer

(b) 1 mark for $\cos\theta(2\cos\theta-1)=0$
1 mark for $90^\circ, 270^\circ$
1 mark for $60^\circ, 300^\circ$

QUESTION 4 [7 marks]

(a) (i) 1 mark for answer

(ii) 1 mark for answer

(b) 1 mark for $\frac{1}{\sin^2\theta}\times\tan^2\theta$
2 marks for correct working

(c) 1 mark for correct substitution into sine rule
1 mark for a side
1 mark for $\sin 24^\circ=\frac{h}{BX}$

QUESTION 5

(a) 1 mark for correct answer

(b) 1 mark for $y-5=\frac{3}{4}(x-1)$
1 mark for showing result

(c) 1 mark for correct answer

(d) 1 mark for correct substitution into distance formula
2 marks for <u>EXACT</u> answer $\frac{10}{3}$

(e) 1 mark for answer

(f) 1 mark for answer

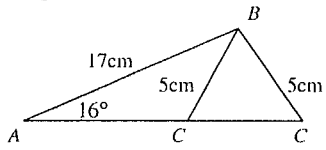
QUESTION 1 [7 marks]

(a) $y=x+3$ [1]

(b)
$$(-3,8)=\left(\frac{2+x}{2},\frac{6+y}{2}\right)$$
$$-3=\frac{2+x}{2}$$
$$-6=2+x$$
$$x=-8$$
$$8=\frac{6+y}{2}$$
$$16=6+y$$
$$y=10$$
$$Q(-8,10)$$
[2]

(c)
$$\frac{\sin\theta}{\cos(90-\theta)}$$
$$=\frac{\sin\theta}{\sin\theta}$$
$$=1$$
[1]

(d) Ambiguous case

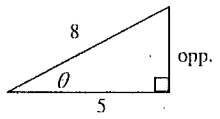


$$\frac{\sin\theta}{17}=\frac{\sin 16^\circ}{5}$$
$$\sin\theta=\frac{17\sin 16^\circ}{5}$$
$$\theta=69^\circ 35' or 110^\circ 25'$$
[3]

QUESTION 2 [6 marks]

(a)
$$\frac{\sin B}{\operatorname{cosec} B}+\frac{\cos B}{\sec B}$$
$$=\sin B\times\sin B+\cos B\times\cos B$$
$$=\sin^2 B+\cos^2 B$$
$$=1$$
[2]

(b)

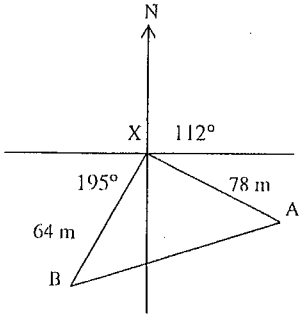


$$\cos\theta=\frac{5}{8},\ 270^\circ<\theta<360^\circ$$
$$opp.=\sqrt{8^2-5^2}$$
$$=\sqrt{39}$$
$$\tan\theta=-\frac{\sqrt{39}}{5}$$
[2]

(c)
$$\sqrt{3}\tan\theta=1$$
$$\tan\theta=\frac{1}{\sqrt{3}}$$
$$\theta=30^\circ, 210^\circ$$
[2]

QUESTION 3 [7 marks]

(a) (i)



[1]

(ii)

$$\angle AXB = 195° - 112°$$
$$= 83°$$

[1]

(iii)

$$a^2 = b^2 + c^2 - 2bcCosA$$
$$AB^2 = 78^2 + 64^2 - 2(78)(64)Cos83°$$
$$AB = 94.67447637...m$$
$$AB = 95m\,(nearest\ metre)$$

[2]

(b)

$$2\cos^2\theta = \cos\theta$$
$$2\cos^2\theta - \cos\theta = 0$$
$$\cos\theta(2\cos\theta - 1) = 0$$

$$\cos\theta = 0$$
$$\theta = 90°, 270°$$

$$2\cos\theta - 1 = 0$$
$$\cos\theta = \frac{1}{2}$$
$$\theta = 60°, 300°$$

$$\theta = 60°, 90°, 270°, 300°$$

[3]

QUESTION 4 [7 marks]

(a)

(i)

$$\tan 120° = -\sqrt{3}$$

[1]

(ii)

$$\sec(-30°) = \frac{2}{\sqrt{3}}$$

[1]

(b)

$$LHS = \text{cosec}^2\theta(\sec^2\theta - 1)$$
$$= \text{cosec}^2\theta \times \tan^2\theta$$
$$= \frac{1}{\sin^2\theta} \times \frac{\sin^2\theta}{\cos^2\theta}$$
$$= \frac{1}{\cos^2\theta}$$
$$= \sec^2\theta$$
$$= RHS$$

[2]

(c)

Let the top of the tower be X

$$\angle X = 24° - 16°\,(ext.\angle of\ triangle...)$$
$$= 8°$$

$$\frac{BX}{\sin 16°} = \frac{6}{\sin 8°}$$
$$BX = \frac{6\sin 16°}{\sin 8°}$$
$$= 11.88\,cm$$

$$\sin 24° = \frac{h}{BX}$$
$$h = BX\sin 24°$$
$$h = 4.83\,m$$

[3]

QUESTION 5 [8 marks]

(a)

$$m_{QR} = \frac{2-5}{-3-1}$$
$$= \frac{-3}{-4}$$
$$= \frac{3}{4}$$

[1]

(b)

$$y - y_1 = m(x - x_1)$$
$$y - 5 = \frac{3}{4}(x - 1)$$
$$4y - 20 = 3x - 3$$
$$3x - 4y + 17 = 0$$

[2]

(c)

$$3x - 4y + 17 = 0$$
$$3x - 4(0) + 17 = 0$$
$$3x = -17$$
$$x = \frac{-17}{3}$$

$$P\left(\frac{-17}{3}, 0\right)$$

[1]

(d)

$$d = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$$
$$d_{PR} = \sqrt{\left(-3 - \frac{-17}{3}\right)^2 + (2-0)^2}$$
$$= \sqrt{\left(-3 - \frac{-17}{3}\right)^2 + (2-0)^2}$$
$$= \sqrt{\frac{100}{9}}$$
$$= \frac{10}{3}\,units$$

[2]

(e)

$$m_{PR} = m_{QR}$$

$$\tan\theta = \frac{3}{4}$$
$$\theta = 36°52'$$

[1]

(f)

$$A = \frac{1}{2}ab\sin\theta$$
$$= \frac{1}{2}\left(\frac{10}{3}\right)\left(\frac{17}{3}\right)\sin 36°52'$$
$$= 5.7\,units^2$$

[1]