ST ANDREW'S CATHEDRAL SCHOOL

Founded 1885

MATHEMATICS

EXTENSION 1

PRELIMINARY COURSE

2012 ASSESSMENT TASK 2

(Weighting: 30%)

Time Allowed: 60 minutes

Instructions

* Attempt all questions.
* All necessary working must be shown in all questions.
* Approved calculators and templates may be used.

Marks

1. Solve $\frac{1}{|x+3|} \geq 2$ — 2

2. Solve $\frac{x+8}{5-2x} \geq 3$ — 3

3. (a) On the same axes sketch the graphs $y=(x+3)^2$ and $y=5-|x|$ clearly indicating their intercepts and points of intersection. — 3

   (b) Hence find the values of x for which $(x+3)^2+|x| \geq 5$. — 1

4. Given $\tan\frac{\theta}{2}=2\sqrt{5}$ and $90^\circ \leq \theta \leq 180^\circ$, find the exact value of $\operatorname{cosec}\theta$. — 2

5. Given $\tan(\alpha+\beta)=2$ and $\tan\alpha=\frac{4}{5}$, find the exact value of $\tan\beta$. — 2

6. Prove that $\frac{\cos 3\theta}{\sin\theta}+\frac{\sin 3\theta}{\cos\theta}=2\cot 2\theta$ — 2

7. A and B are the points (–1,3) and (p,q) respectively. The point R (7,–5) divides the interval AB externally in the ratio 4:9. Find the values of p and q. — 3

8. Find, to the nearest minute, the acute angle between the lines $5x+2y-9=0$ and $3x-4y+7=0$. — 3

9. Use the substitution $t=\tan\frac{\theta}{2}$, to simplify $\cot\frac{\theta}{2}(\sec\theta-1)$. — 2

10. Solve $\cos 2\theta=\sin\theta$ for $0^\circ \leq \theta \leq 360^\circ$. — 3

11. (a) Prove that $\cot 2\theta+\operatorname{cosec} 2\theta=\cot\theta$. — 3

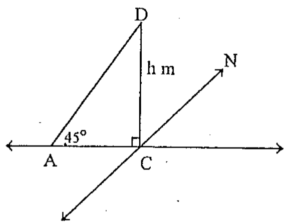
    (b) Hence find the exact value of $\cot 165^\circ$ — 1

Marks

12. (a) Use the expansion of $A\sin(x-\alpha)$ to express $\sqrt{7}\sin x - 3\cos x$ in the form $A\sin(x-\alpha)$ where $A>0$ and $0^\circ<\alpha<90^\circ$. (Give the value of α to the nearest minute.) **3**

(b) Hence solve the equation of $\sqrt{7}\sin x - 3\cos x = -2$ for $0^\circ \le x \le 360^\circ$. **2**

13. CD is a tower of height h metres. A and B are two points 400 metres apart, in the same plane as C, the foot of the tower. From A due west of the tower, the angle of elevation of D, the tower's summit, is 45°. From B, bearing 150°T from the tower, the angle of elevation of D is β°.



(a) Copy the diagram and complete it so it represents all of the given information. **1**

(b) Find an expression for BC. **1**

(c) Show that $h = \dfrac{400}{\sqrt{\operatorname{cosec}^2\beta^\circ + \cot\beta^\circ}}$. **3**

Year 11 Extension — Assessment Task 2 — July 2012

1.

$\frac{1}{|x+3|} \geq 2$

$\Rightarrow |x+3| \leq \frac{1}{2}$ where $x \neq -3$

$-\frac{1}{2} \leq x+3 \leq \frac{1}{2}$

$-\frac{7}{2} \leq x \leq -\frac{5}{2}$

$\therefore -\frac{7}{2} \leq x < -3, \quad -3 < x \leq -\frac{5}{2}$

[2]

2.

$\frac{x+8}{5-2x} \geq 3$

(undefined when $x = \frac{5}{2}$)

$\frac{x+8}{5-2x} \times (5-2x)^2 \geq 3(5-2x)^2$

$(x+8)(5-2x) - 3(5-2x)^2 \geq 0$

$(5-2x)[x+8-3(5-2x)] \geq 0$

$(5-2x)(7x-7) \geq 0$

$7(x-1)(5-2x) \geq 0$

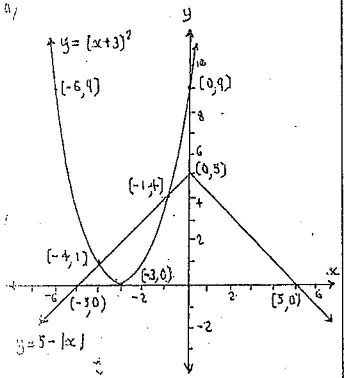
$1 \leq x \leq \frac{5}{2}$

but $x \neq \frac{5}{2}$

$\therefore 1 \leq x < \frac{5}{2}$

[3]

3. a)



[3]

3. b)

$(x+3)^2 + |x| \geq 5$

$(x+3)^2 \geq 5 - |x|$

$\therefore x \leq -4, \quad x \geq -1$

[1]

4.

$\tan\frac{\theta}{2} = 2\sqrt{5}$

let $t = \tan\frac{\theta}{2}$

$\text{cosec}\,\theta = \frac{1+t^2}{2t}$

$= \frac{1+(2\sqrt{5})^2}{2 \times 2\sqrt{5}}$

$= \frac{(1+4\times5)\times\sqrt{5}}{4\sqrt{5}\times\sqrt{5}}$

$= \frac{21\sqrt{5}}{20}$

[2]

5.

$\tan(\alpha+\beta) = 2$ and $\tan\alpha = \frac{4}{5}$

$\tan(\alpha+\beta) = \frac{\tan\alpha + \tan\beta}{1-\tan\alpha\tan\beta}$

$\Rightarrow 2 = \frac{\frac{4}{5}+\tan\beta}{1-\frac{4}{5}\tan\beta}$

$2 - \frac{8}{5}\tan\beta = \frac{4}{5} + \tan\beta$

$\frac{6}{5} = \frac{13}{5}\tan\beta$

$\therefore \tan\beta = \frac{6}{13}$

[2]

6.

L.H.S

$= \frac{\cos 3\theta}{\sin\theta} + \frac{\sin 3\theta}{\cos\theta}$

$= \frac{\cos 3\theta\cos\theta + \sin 3\theta\sin\theta}{\sin\theta\cos\theta}$

$= \frac{\cos(3\theta-\theta)}{\frac{1}{2}\times(2\sin\theta\cos\theta)}$

$= \frac{2\cos 2\theta}{\sin 2\theta}$

$= 2\cot 2\theta$

$=$ RHS

[2]

7.

$A(-1,3), \quad B(p,q), \quad R(7,-5)$

external division $\Rightarrow k:l = -4:9$

point $= \left(\frac{kx_2+lx_1}{k+l}, \frac{ky_2+ly_1}{k+l}\right)$

$(7,-5) = \left(\frac{-4p+9\times-1}{-4+9}, \frac{-4q+9\times3}{-4+9}\right)$

$\Rightarrow \frac{-4p-9}{5} = 7$

$-4p-9 = 35$

$-4p = 44$

$p = -11$

and $\frac{-4q+27}{5} = -5$

$-4q+27 = -25$

$-4q = -52$

$q = 13$

[3]

8.

for $5x+2y-9=0$

$y = \frac{-5x+9}{2}$

$\Rightarrow m_1 = -\frac{5}{2}$

for $3x-4y+7=0$

$y = \frac{3x+7}{4}$

$\Rightarrow m_2 = \frac{3}{4}$

$\tan\psi = \left|\frac{m_1-m_2}{1+m_1m_2}\right|$

$\tan\psi = \left|\frac{-\frac{5}{2}-\frac{3}{4}}{1+-\frac{5}{2}\times\frac{3}{4}}\right|$

$= \left|-\frac{13}{4} \div -\frac{7}{8}\right|$

$= \frac{26}{7}$

$\therefore \psi = 74°56'$

[3]

9.

let $t = \tan\frac{\theta}{2}$

$\cot\frac{\theta}{2}(\sec\theta - 1)$

$= \frac{1}{t}\left(\frac{1+t^2}{1-t^2} - 1\right)$

$= \frac{1}{t} \times \frac{1+t^2-(1-t^2)}{1-t^2}$

$= \frac{1}{t} \times \frac{2t^2}{1-t^2}$

$= \frac{2t}{1-t^2}$

$= \tan\theta$

[2]

10.

$\cos 2\theta = \sin\theta$

$1 - 2\sin^2\theta = \sin\theta$

$2\sin^2\theta + \sin\theta - 1 = 0$

$(\sin\theta+1)(2\sin\theta-1) = 0$

$\Rightarrow \sin\theta = -1$

$\theta = 270°$

and $\sin\theta = \frac{1}{2}$

$\theta = 30°, 150°$

$\therefore \theta = 30°, 150°, 270°$

[3]

11. a)

L.H.S

$= \cot 2\theta + \text{cosec}\,2\theta$

$= \frac{\cos 2\theta}{\sin 2\theta} + \frac{1}{\sin 2\theta}$

$= \frac{\cos 2\theta + 1}{\sin 2\theta}$

$= \frac{\cos^2\theta - \sin^2\theta + 1}{2\sin\theta\cos\theta}$

$= \frac{\cos^2\theta + \cos^2\theta}{2\sin\theta\cos\theta}$

$= \frac{2\cos^2\theta}{2\sin\theta\cos\theta}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$

$=$ R.H.S

[3]

11. b)

$\cot 165°$

$= \cot 330° + \text{cosec}\,330°$

$= -\sqrt{3} - 2$

$= -(2+\sqrt{3})$

[1]

12. a)

$\sqrt{7}\sin x - 3\cos x$

$= A\sin(x-\alpha)$

$= A(\sin x\cos\alpha - \cos x\sin\alpha)$

$\Rightarrow \sqrt{7} = A\cos\alpha$ — ①

and $3 = A\sin\alpha$ — ②

①² + ②²

$(\sqrt{7})^2 + 3^2 = (A\cos\alpha)^2 + (A\sin\alpha)^2$

$7 + 9 = A^2(\cos^2\alpha + \sin^2\alpha)$

$16 = A^2$

$A = 4 \quad (\text{as } A > 0)$

② ÷ ①

$\frac{3}{\sqrt{7}} = \frac{A\sin\alpha}{A\cos\alpha}$

$\frac{3}{\sqrt{7}} = \tan\alpha$

$\alpha = 48°35' \quad (0° < \alpha < 90°)$

$\therefore \sqrt{7}\sin x - 3\cos x = 4\sin(x - 48°35')$

[3]

12. b)

$\sqrt{7}\sin x - 3\cos x = -2$

$4\sin(x-48°35') = -2$

$\sin(x-48°35') = -\frac{1}{2}$

since $0° \leq x \leq 360°$

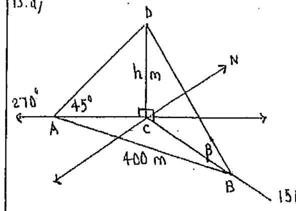
then $-48°35' \leq x - 48°35' \leq 311°25'$

$x - 48°35' = 210°, (330° - 360°)$

$x = 258°35', \quad 18°35'$

[2]

13. a)



$\angle ACB = 120°$

[1]

13. b)

in ΔBCD

$\tan\beta = \frac{h}{BC}$

$\cot\beta = \frac{BC}{h}$

$BC = h\cot\beta$

[1]

13. c)

in ΔACD

$\tan 45° = \frac{h}{AC}$

$1 = \frac{h}{AC}$

$AC = h$

in ΔACB

$c^2 = a^2 + b^2 - 2ab\cos C$

$400^2 = (h\cot\beta)^2 + h^2 - 2h\cot\beta \times h\cos 120°$

$400^2 = h^2[\cot^2\beta + 1 - 2\cot\beta \times -\frac{1}{2}]$

$400^2 = h^2[\text{cosec}^2\beta + \cot\beta]$

$h^2 = \frac{400^2}{\text{cosec}^2\beta + \cot\beta}$

$h = \frac{400}{\sqrt{\text{cosec}^2\beta + \cot\beta}}$

[3]

M.N.