ST ANDREW'S CATHEDRAL SCHOOL

Founded 1885

MATHEMATICS

EXTENSION 1

PRELIMINARY COURSE

2012 SEMESTER ONE EXAMINATION

(Weighting: 30%)

Time Allowed

* Reading time – 5 minutes
* Working time – 60 minutes

Instructions

* Attempt all questions.
* All necessary working must be shown in all questions.
* Approved calculators and templates may be used.

Marks

1. Solve $\frac{x-8}{x+2} \leq 6$ **3**

2. (a) On the same axes sketch the graphs $y = x^2 + 2x$ and $y = |x+2|$ clearly indicating their intercepts and points of intersection. **3**

   (b) Hence find the values of x for which $x^2 < |x+2| - 2x$. **1**

3. Find in its simplest form the exact value of $\cos 160^\circ \cos 50^\circ - \sin 160^\circ \sin 50^\circ$. **2**

4. Use $105^\circ = 60^\circ + 45^\circ$, to find in its simplest form the exact value of $\tan 105^\circ$. **2**

5. Graph the solution of $\frac{1}{|6-x|} \geq \frac{1}{2}$ on a number line. **3**

6. Simplify fully $\frac{x^{-\frac{1}{2}} - 1}{x^{-\frac{1}{2}} - x^{-1}}$ **2**

7. Use the substitution $t = \tan\frac{\theta}{2}$, to show that $\frac{1}{\operatorname{cosec}\theta - \cot\theta} = \cot\frac{\theta}{2}$. **2**

8. (a) Prove that $\frac{1+\cos 2\theta}{\sin 2\theta} = \cot\theta$. **2**

   (b) Hence find in its simplest form the exact value of the exact value of $\cot 75^\circ$. **2**

9. On a number plane, shade the region for which $y > \sqrt{9-x^2}$. **2**

10. Solve $\sin 2\theta = 2\cos^2\theta$ for $0^\circ \leq \theta \leq 360^\circ$. **3**

11. Sketch the function $y = 1 - \frac{3}{x-2}$, clearly indicating its asymptotes and intercepts. **2**

12. (a) Simplify $1-2\sin^2\theta$ **1**

(b) Given $\sin 54^\circ = \frac{\sqrt{5}-1}{4}$, find in its simplest form the exact value of $\cos 108^\circ$. **2**

13. Given $\cos\theta = \frac{2}{7}$ and $270^\circ < \theta < 360^\circ$, find the exact value of $\tan\frac{\theta}{2}$. **3**

14. Simplify fully $\dfrac{(3-x)^2}{\sqrt{(3-x)^2}}$ **2**

15. (a) Solve $x(1-x^2) > 0$. **1**

(b) Hence solve $\sin x(1-\sin^2 x) > 0$ for $0^\circ \le \theta \le 360^\circ$. **2**

Year 11 Extension — Semester 1 Examination — May 2012

1.

$\frac{x-8}{x+2} \leq 6$

(undefined when $x=-2$)

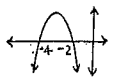
$\frac{x-8}{x+2} \times (x+2)^2 \leq 6(x+2)^2$

$(x-8)(x+2) - 6(x+2)^2 \leq 0$

$(x+2)[(x-8) - 6(x+2)] \leq 0$

$(x+2)(-5x-20) \leq 0$

$-5(x+2)(x+4) \leq 0$



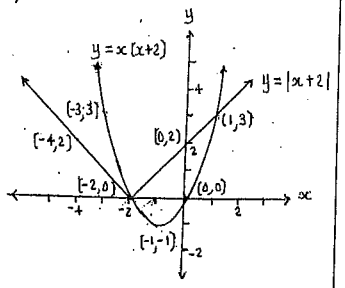
$x \leq -4, \quad x \geq -2$

but $x \neq -2$

$\therefore \quad x \leq -4, \quad x > -2$

3

2.a/



3

2.b/

$x^2 < |x+2| - 2x$

$x^2 + 2x < |x+2|$

$\Rightarrow \quad -2 < x < 1$

1

3.

$\cos 160^\circ \cos 50^\circ - \sin 160^\circ \sin 50^\circ$

$= \cos(160+50)^\circ$

$= \cos 210^\circ$

$= -\cos 30^\circ$

$= -\frac{\sqrt{3}}{2}$

2

4.

$\tan 105^\circ$

$= \tan(60^\circ + 45^\circ)$

$= \frac{\tan 60^\circ + \tan 45^\circ}{1 - \tan 60^\circ \tan 45^\circ}$

$= \frac{\sqrt{3}+1}{1-\sqrt{3}\times 1}$

$= \frac{\sqrt{3}+1}{1-\sqrt{3}} \times \frac{1+\sqrt{3}}{1+\sqrt{3}}$

$= \frac{3+2\sqrt{3}+1}{1-3}$

$= -(2+\sqrt{3})$

2

5.

$\frac{1}{|6-x|} \geq \frac{1}{2}$

(undefined when $x=6$)

$|6-x| \leq 2$

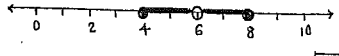
$-2 \leq 6-x \leq 2$

$-8 \leq -x \leq -4$

$8 \geq x \geq 4$

but $x \neq 6$

$\therefore \quad 4 \leq x < 6, \quad 6 < x \leq 8$



3

6.

$\frac{x^{-\frac{1}{2}} - 1}{x^{\frac{1}{2}} - x^{-1}}$

$= \left(\frac{1}{\sqrt{x}} - 1\right) \div \left(\frac{1}{\sqrt{x}} - \frac{1}{x}\right)$

$= \frac{1-\sqrt{x}}{\sqrt{x}} \div \frac{\sqrt{x}-1}{x}$

$= \frac{1-\sqrt{x}}{\sqrt{x}} \times \frac{x}{\sqrt{x}-1}$

$= -\sqrt{x}$

2

7.

let $t = \tan\frac{\theta}{2}$

L.H.S

$= \frac{1}{\operatorname{cosec}\theta - \cot\theta}$

$= 1 \div \left(\frac{1}{\sin\theta} - \frac{1}{\tan\theta}\right)$

$= 1 \div \left(\frac{1+t^2}{2t} - \frac{1-t^2}{2t}\right)$

$= 1 \div \frac{2t^2}{2t}$

$= \frac{1}{t}$

$= \cot\frac{\theta}{2}$

$= $ R.H.S

2

8.a/

L.H.S

$= \frac{1+\cos 2\theta}{\sin 2\theta}$

$= \frac{1+(\cos^2\theta - \sin^2\theta)}{2\sin\theta\cos\theta}$

$= \frac{2\cos^2\theta}{2\sin\theta\cos\theta}$

$= \frac{\cos\theta}{\sin\theta}$

$= \cot\theta$

$= $ R.H.S

2

8.b/

$\cot 75^\circ$

$= \frac{1+\cos 150^\circ}{\sin 150^\circ}$

$= (1-\cos 30^\circ) \div \sin 30^\circ$

$= \left(1 - \frac{\sqrt{3}}{2}\right) \div \frac{1}{2}$

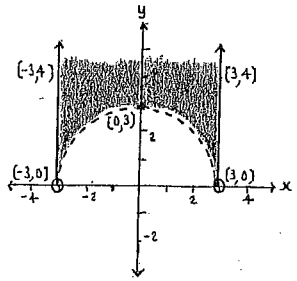
$= 2\left(1 - \frac{\sqrt{3}}{2}\right)$

$= 2 - \sqrt{3}$

2

9/

$y > \sqrt{9-x^2}$



2

10.

$\sin 2\theta = 2\cos^2\theta$

$2\sin\theta\cos\theta - 2\cos^2\theta = 0$

$2\cos\theta[\sin\theta - \cos\theta] = 0$

$\Rightarrow \quad 2\cos\theta = 0$

$\cos\theta = 0$

$\theta = 90^\circ, 270^\circ$

and $\sin\theta - \cos\theta = 0$

$\frac{\sin\theta}{\cos\theta} = \frac{\cos\theta}{\cos\theta}$

$\tan\theta = 1$

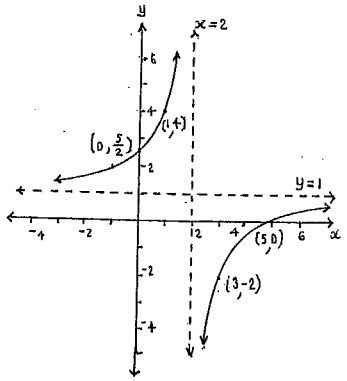
$\theta = 45^\circ, 225^\circ$

$\therefore \quad \theta = 45^\circ, 90^\circ, 225^\circ, 270^\circ$

3

11.

$y = 1 - \frac{3}{x-2}$



when $x=0$: $y = 1 - \frac{3}{-2} = \frac{5}{2}$

when $y=0$: $\frac{3}{x-2} = 1$, $x = 5$

2

12.a/

$1 - 2\sin^2\theta$

$= \cos 2\theta$

1

12.b/

$\cos 108^\circ$

$= 1 - 2\sin^2 54^\circ$

$= 1 - 2\left(\frac{\sqrt{5}-1}{4}\right)^2$

$= 1 - 2\left(\frac{5-2\sqrt{5}+1}{16}\right)$

$= 1 - \frac{3-\sqrt{5}}{4}$

$= \frac{1+\sqrt{5}}{4}$

2

13.

let $t = \tan\frac{\theta}{2}$

$\cos\theta = \frac{2}{7}$

$\Rightarrow \quad \frac{1-t^2}{1+t^2} = \frac{2}{7}$

$7 - 7t^2 = 2 + 2t^2$

$5 = 9t^2$

$t^2 = \frac{5}{9}$

$t = \pm\frac{\sqrt{5}}{3}$

since $270^\circ < \theta < 360^\circ$

$135^\circ < \frac{\theta}{2} < 180^\circ$

$-1 < \tan\frac{\theta}{2} < 0$

$\therefore \quad \tan\frac{\theta}{2} = -\frac{\sqrt{5}}{3}$

3

14.

$\frac{(3-x)^2}{\sqrt{(3-x)^2}}$

$= \frac{(3-x)^2}{|3-x|}$

$= \begin{cases} \frac{(3-x)^2}{3-x} & \text{if } 3-x>0 \\ \frac{(3-x)^2}{-(3-x)} & \text{if } 3-x<0 \end{cases}$

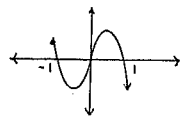
$= \begin{cases} 3-x & \text{if } x<3 \\ -(3-x) & \text{if } x>3 \end{cases}$

2

15.a/

$x(1-x^2) > 0$

$x(1-x)(1+x) > 0$



$\therefore \quad x < -1, \quad 0 < x < 1$

1

15.b/

$\sin x(1-\sin^2 x) > 0$

$\Rightarrow \quad \sin x < -1$

no solution

as $-1 \leq \sin x \leq 1$

and $0 < \sin x < 1$

$\therefore \quad 0^\circ < x < 90^\circ \quad 90^\circ < x < 180^\circ$

2

M.N.