MATHEMATICS

YEAR 11

ASSESSMENT TASK 1

2010

90 MINUTES

St Andrew's Cathedral School
Founded 1885

INSTRUCTIONS

- There are 4 sections, 15 marks per section

    TOTAL 60 MARKS

- Approved calculators may be used

- Attempt <u>all</u> questions, use a new booklet for each question

- Marks may be deducted for careless and/or messy work

- <u>Any question worth more than 1 mark requires working to be clearly shown to ensure the awarding of full marks.</u>

<u>Section A – 15 marks</u>

1. In a survey of residents in North Sydney 28% of those surveyed answered yes to the question:

   'Yes or no, did you switch off lights in your home during Earth hour?'

   If 28420 residents answered yes, how many residents in total were surveyed?　　(1 mark)

2. Given $A = \pi(R^2 - r^2)$, $R = 4.1$ and $r = 2.9$, find the value of A correct to 3 significant figures.

   　　(1 mark)

3. Simplify fully:

   $$2\sqrt{3} \times 3\sqrt{6}$$　　(1 mark)

4. Write $638 \div 10^7$ in scientific notation.　　(1 mark)

5. Solve $10 - \dfrac{y}{3} < 8$ and graph the solution on a number line.　　(2 marks)

6. Evaluate $\left|-5\right| + \left|3\right| \times \left|-4\right|$　　(1 mark)

7. Expand and simplify:　　$12(y - 3) - (y - 9)$　　(2 marks)

8. Given $\sqrt{54} - \sqrt{6} = \sqrt{a}$, find a.　　(2 marks)

9. Change $0.41\dot{6}$ from a recurring decimal to a fraction. (Giving the fraction in its simplest form)　　(2 marks)

10. Rationalise the denominator of $\dfrac{\sqrt{2}}{3\sqrt{2} + 2}$, giving your answer in simplest form.　　(2 marks)

(End of Section A)

Section B – 15 marks

1. Solve the equation $\dfrac{2x-1}{3} - \dfrac{x+5}{2} = 7$ (2 marks)

2. Solve $|x-6| \geq 3$ (2 marks)

3. Write $\dfrac{1}{\sqrt{x}}$ in index form. (1 mark)

4. Solve the following simultaneous equations:

$$x - 3y = 34$$
$$2x + y = 19$$

   (2 marks)

5. Simplify fully:

$$\frac{6x-9}{4x^2-9}$$

   (2 marks)

6. Expand and simplify:

$$(3k-7p)^2 - (k+6p)(k-2p)$$

   (3 marks)

7. a) Sketch on a number plane the function $y = |x| - 2$, clearly indicating its intercepts. (1 mark)

   b) State the domain and range of the function $y = |x| - 2$. (2 marks)

(End of Section B)

Section C - 15 marks

1. Factorise fully:

   a) $2x^2 + 7x - 15$ (1 mark)

   b) $8a^3 - 1000$ (2 marks)

2. Simplify fully $9^{2n+1} \div 3^{n-2}$ (2 marks)

3. Solve the equation $|3x - 5| = 7x + 1$. (3 marks)

4. a) Show algebraically that $f(x) = \dfrac{4}{x^2+1}$ is an even function. (1 mark)

   b) Explain geometrically the significance of a function being even. (1 mark)

5. a) Sketch on a number plane the function $y = -\sqrt{9-x^2}$, clearly indicating its intercepts. (1 mark)

   b) State the domain of the function $y = -\sqrt{9-x^2}$ (1 mark)

6. The function $f(x)$ is defined by $\begin{cases} f(x) = 2^{-x} & for \quad x \leq 0 \\ f(x) = x & for \quad x > 0 \end{cases}$

   Sketch the graph of $y = f(x)$ for $-3 \leq x \leq 3$ (3 marks)

(End of Section C)

<u>Section D – 15 marks</u>

1. Solve the following equations. (Giving irrational roots in their simplest surd form)

    a) $3x^2 = 5x$ (2 marks)

    b) $2x^2 + 8x + 3 = 0$ (2 marks)

2.   a) Factorise $x^2 - 6x + 5$ (1 mark)

    b) Hence simplify $\dfrac{1}{x^2 - 6x + 5} + \dfrac{1}{x - 5}$ (1 mark)

3. What is the equation of the vertical asymptote of the graph of the hyperbola $y = \dfrac{3}{x + 4} - 1$?   (1 mark)

4.   a) On the same number plane sketch the graphs of the functions $x + y = 2$ and $y = x^2 - 4$, clearly indicating their intercepts and points of intersection.   (3 marks)

    b) On the number p lane drawn in part a), shade the region for which
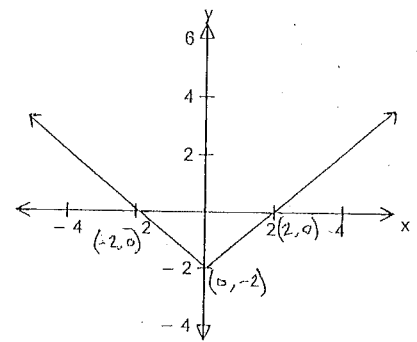$x + y \geq 2, y \leq x^2 - 4$ and $y > 0$ hold simultaneously.   (2 marks)

5. Given $g(x) = \begin{cases} x^2 - 3x & for & x \leq -3 & (i) \\ x^2 + x & for & -3 < x < 2 & (ii) \\ x^2 + 6x & for & x \geq 2 & (iii) \end{cases}$

    a)   Find $g(-2)$ (1 mark)

    b)   Find $g(a^2 + 2)$ (2 marks)

(End of Section D)

Solutions	Marking Criteria
Section A	
1. $\dfrac{28420}{28} \times 100 = 101500$	1. 1 mark for correct answer
2. $A = \pi(4.1^2 - 2.9^2) = 26.389\ldots = 26.4$	2. 1 mark for correct answer, must be 26.4.
3. $6\sqrt{18} = 18\sqrt{2}$	3. 1 mark for correct answer
4. 6.38×10^{-5}	4. 1 mark for correct answer
5. $-\dfrac{y}{3} < -2$ $\therefore y > 6$ 	5. 1 mark correct solution 1 mark if number line follows from solution
6. $5 + 3 \times 4 = 17$	6. 1 mark for correct answer
7. $12y - 36 - y + 9$ $= 11y - 27$	7. 1 mark correct expansions 1 mark if simplification follows
8. $\sqrt{54} - \sqrt{6} = 3\sqrt{6} - \sqrt{6}$ $= 2\sqrt{6}$ $= \sqrt{24} \therefore a = 24$	8. 1 mark for $\sqrt{a} = 2\sqrt{6}$ 1 mark if value of a follows
9. Let $x = 0.41\dot{6}$ $1000x = 416.\dot{1}\dot{6}$ $-10x = 4.\dot{1}\dot{6}$ $\overline{990x = 412}$ $x = \dfrac{412}{990} = \dfrac{206}{495}$	9. 1 mark for subtraction to give $990x = 412$ 1 mark for fraction in simplest form
10. $\dfrac{\sqrt{2}}{3\sqrt{2}+2} \times \dfrac{3\sqrt{2}-2}{3\sqrt{2}-2}$ $= \dfrac{6 - 2\sqrt{2}}{16 - 4}$ $= \dfrac{6 - 2\sqrt{2}}{14} = \dfrac{3 - \sqrt{2}}{7}$	10. 1 mark for $\dfrac{6 - 2\sqrt{2}}{14}$ 1 mark for correct simplification of answer

Section B	
1. $2(2x-1) - 3(x+5) = 42$ $4x - 2 - 3x - 15 = 42$ $x - 17 = 42$ $x = 59$	1. 1 mark for $2(2x-1) - 3(x+5) = 42$ or $\dfrac{4x - 2 - 3x - 15}{6} = 7$ 1 mark if solution follows.
2. $x - 6 \geq 3$ $\quad$ or $\quad -x + 6 \geq 3$ $x \geq 9$ $\qquad\qquad -x \geq -3$ $\qquad\qquad\qquad\quad x \leq 3$	2. 1 mark for each solution (2 marks total).
3. $\dfrac{1}{x^{\frac{1}{2}}} = x^{-\frac{1}{2}}$	3. 1 mark correct answer.
4. $2x - 6y = 68$ $\underline{2x + y = 19}$ $-7y = 49$ $y = -7$ $x = 34 + 3y$ $x = 34 + 3 \times -7 \qquad \therefore y = -7, x = 13$ $x = 13$	4. 1 mark for value of x, 1 mark for value of y. Allow 1 mark for second solution if it follows correctly from error in first solution.
5. $\dfrac{3(2x-3)}{(2x-3)(2x+3)} = \dfrac{3}{2x+3}$	5. 1 mark difference of squares factorisation. 1 mark for correct answer.
6. $9k^2 - 42kp + 49p^2 - (k^2 - 2kp + 6kp - 12p^2)$ $= 9k^2 - 42kp + 49p^2 - k^2 + 2kp - 6kp + 12p^2$ $= 8k^2 - 46kp + 61p^2$	6. 1 mark for correctly expanding each binomial product. 1 mark if simplification follows (3 marks total).
7. a) 	7. a) 1 mark for correct graph with all intercepts marked clearly and accurately.
b) Domain – All real x values. Or $-\infty < x < \infty$. Range – $y \geq -2$	b) 1 mark correct domain, 1 mark correct range.

Section C

1. a) $\dfrac{(2x+10)(2x-3)}{2} = (x+5)(2x-3)$

   b) $8(a^3 - 125) = (a-5)(a^2 + 5a + 25)$

2. $3^{2(2n+1)} \div 3^{n-2}$
   $= 3^{4n+2} \div 3^{n-2}$
   $= 3^{4n+2-(n-2)}$
   $= 3^{3n+4}$

3. OPTION 1 –
   $3x - 5 = 7x + 1$
   $-6 = 4x$
   $x = -\dfrac{6}{4} = -\dfrac{3}{2}$
   Checking the solution
   $\left| 3 \times -\dfrac{3}{2} - 5 \right| = 7 \times -\dfrac{3}{2} + 1$
   $9\dfrac{1}{2} \neq -9\dfrac{1}{2}$
   Therefore this is not a solution.

   OPTION 2 –
   $-3x + 5 = 7x + 1$
   $4 = 10x$
   $x = \dfrac{4}{10} = \dfrac{2}{5}$
   Checking the solution
   $\left| 3 \times \dfrac{2}{5} - 5 \right| = 7 \times \dfrac{2}{5} + 1$
   $3\dfrac{4}{5} = 3\dfrac{4}{5}$
   Therefore this is a solution.

   ALTERNATIVE CHECK METHOD –

   Since $\left| 3x - 5 \right| \geq 0$
   Then $7x + 1 \geq 0$
   $x \geq -\dfrac{1}{7}$
   $\therefore x = \dfrac{2}{5}$ is the only solution.

1. a) 1 mark correct answer

   b) 1 mark for common factor, 1 mark difference of cubes.

2. 1 mark for $3^{4n+2} \div 3^{n-2}$.
   1 mark if simplification follows.

3. 1 mark for $x = \dfrac{2}{5}$.
   1 mark for $x = -\dfrac{3}{2}$
   1 mark for check resulting in $x = \dfrac{2}{5}$ being the only solution.
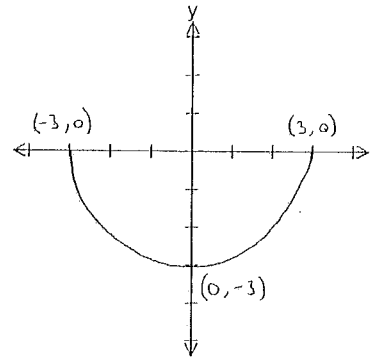
4. a) $f(-x) = \dfrac{4}{(-x)^2 + 1} = \dfrac{4}{x^2 + 1}$
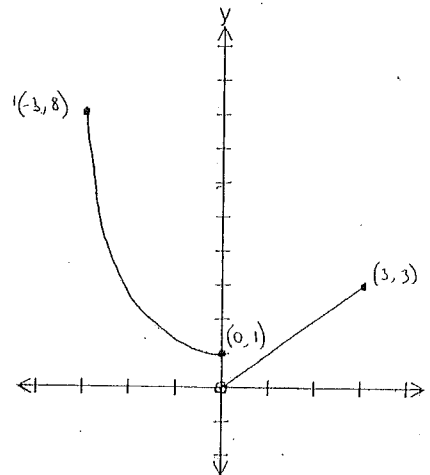 $\therefore f(x) = f(-x)$
   Proving the function is even.

   b) The graphs of even functions have line symmetry about the y-axis.

5. a)

   $(-3, 0)$ $(3, 0)$ $(0, -3)$

   b) $-3 \leq x \leq 3$

6.

   $(-3, 8)$ $(3, 3)$ $(0, 1)$

4. a) 1 mark for showing $f(x) = f(-x)$

   b) 1 mark correct explanation

5. a) 1 mark for correct graph with intercepts clearly marked.

   b) 1 mark correct answer

6. 1 mark exponential part with $(0,1)$ included.
   1 mark for straight line with $(0,0)$ not included.
   1 mark for endpoints

Section D

1. a) $3x^2 - 5x = 0$

$x(3x - 5) = 0$

$\therefore x = 0$

or

$3x - 5 = 0$

$3x = 5$

$x = \dfrac{5}{3} = 1\dfrac{2}{3}$

b) $x = \dfrac{-8 \pm \sqrt{8^2 - 4 \times 2 \times 3}}{2 \times 2}$

$x = \dfrac{-8 \pm \sqrt{40}}{4}$

$x = \dfrac{-8 \pm 2\sqrt{10}}{4}$

$x = \dfrac{-4 + \sqrt{10}}{2} \; or \; \dfrac{-4 - \sqrt{10}}{2}$

2. a) $(x - 5)(x - 1)$

b) $\dfrac{1}{(x-5)(x-1)} + \dfrac{1}{x-5} = \dfrac{1 + (x-1)}{(x-5)(x-1)}$

$= \dfrac{x}{(x-5)(x-1)}$

3. $x = -4$

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1. a) 1 mark for $x(3x - 5) = 0$

1 mark for $x = 0, \dfrac{5}{3}$

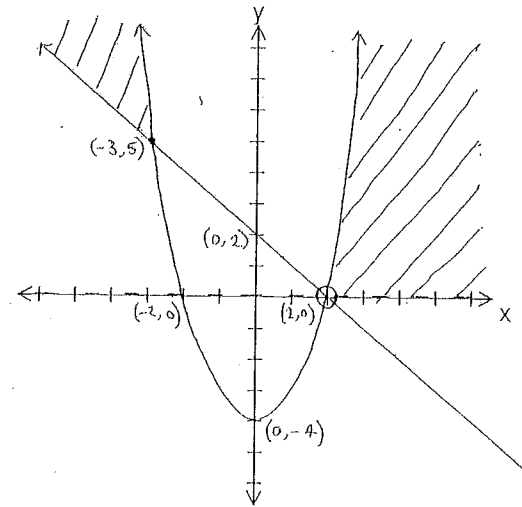**b)** 1 mark for $x = \dfrac{-8 \pm \sqrt{40}}{4}$.

1 mark if simplification follows.

2. a) 1 mark correct answer

b) 1 mark correct answer

3. 1 mark correct answer

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4. a) and b)

5. a) $(-2)^2 + (-2) = 4 - 2 = 2$

b) $(a^2 + 2)^2 + 6(a^2 + 2)$

$= a^4 + 4a^2 + 4 + 6a^2 + 12$

$= a^4 + 10a^2 + 16$

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4. a) 1 mark for each graph with intercepts marked clearly and accurately.

1 mark for both points of intersection.

b) 1 mark for broken line for $y > 0$.

1 mark for regions with open circle at $(2,0)$

5. a) 1 mark correct answer

b) 1 mark for correct substitution.
1 mark if simplification follows.