St Catherine's School

Year: 12
Subject: Extension I Mathematics
Time Allowed: 60 minutes
plus 5 min reading time things like more work v...
Date: March 2003 → 3D trig (takes v. long time to do)

Exam number: 1332686 Susy [illegible]

Directions to candidates:

- All questions are to be attempted.
- All questions are of equal value.
- All necessary **working** must be shown in every question.
- Full marks may not be awarded for careless or badly arranged work.
- Each question attempted should be started on a **new page**.
- Approved calculators and geometrical instruments are required.
- Hand up your questions in separate bundles. Include the exam paper with question 1

TEACHER'S USE ONLY Total Marks	
Q1	$\frac{10}{10}$
Q2	$\frac{10}{10}$
Q3	$\frac{10}{10}$
Q4	$\frac{10}{10}$
TOTAL	$\frac{40}{40} = 100\%$ Fantastic!

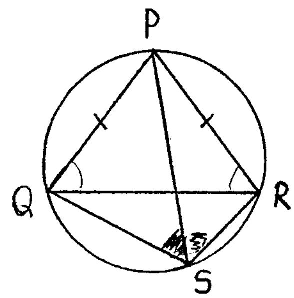
Question 1 (10 marks) please start new page

a) Show the equation $x^2 + x = 3$ has a root between $x = 1$ and $x = 2$.

Starting with $x_1 = 1$ use Newton's Method to show $x_2 = \frac{4}{3}$ (3)

b) P, Q and R are three points on the circumference of a circle such that chord PQ = chord PR. S is any point on the circle such that P and S are on the opposite sides of QR as shown.

Prove $\angle PSQ = \angle PSR$ (3)



d) Find the general solutions of the equation $\cos\theta - \sin 2\theta = 0$ (4)

Question 2 (10 marks) please start new page

a) If α, β and γ are the roots of $2x^3 - 5x - 3 = 0$, find the values of $\alpha + \beta + \gamma$, $\alpha\beta + \beta\gamma + \alpha\gamma$ and $\alpha\beta\gamma$. Hence find

i) $\alpha^2 + \beta^2 + \gamma^2$

ii) $(\alpha - 2)(\beta - 2)(\gamma - 2)$ (5)

b) Find all the angles where $0° \le \theta \le 360°$ for which $\sin\theta - \sqrt{3}\cos\theta = 1$ (5)

Question 3 (10 marks) please start new page

Consider the graph $y = \dfrac{x^2}{1-x^2}$

a) Write down the vertical asymptotes (2)

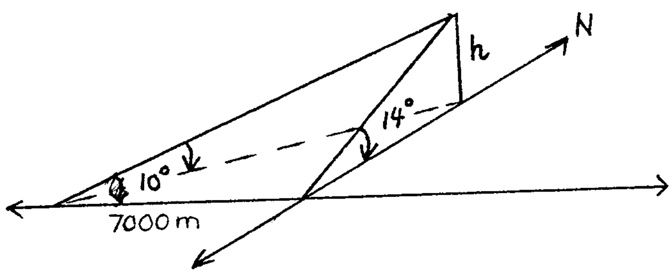
b) Find any stationary points and determine their nature (3)

c) Describe how the function behaves for both positive and negative large values of x. Does the curve approach this value from above or below ? (3)

d) Sketch a graph of this curve showing all relevant features (2)

Question 4 (10 marks) please start new page

a) The angle of elevation of the summit of a mountain due north is $14°$ and on walking 7000m due west is found to be $10°$. Find the height of the mountain to the nearest metre. (4)



b) Find the quotient and the remainder when $P(x) = x^3 + 7x^2 - bx - b$ is divided by $A(x) = x - 1$. If $A(x)$ is a factor of $P(x)$ find the value of b. (3)

c) Prove the identity $\cos 4\theta = 8\cos^4\theta - 8\cos^2\theta + 1$ (3)

End of Exam

EXTENSION 1 March Task 2003

SOLUTIONS

Question 1

a) $f(x) = x^2 + x - 3$

$f(1) = 1 + 1 - 3 = -1 < 0$

$f(2) = 4 + 2 - 3 = 3 > 0$

$\therefore$ a root in $1 < x < 2$

$f'(x) = 2x + 1$

$f'(1) = 3$

$$x_2 = x_1 - \frac{f(x_1)}{f'(x_1)}$$

$$= 1 + \frac{1}{3}$$

$$= \frac{4}{3}.$$

b) $\angle PQR = \angle PRQ$ ($PQ = PR$, base)

$\angle PSQ = \angle PRQ$ ($\angle$ on same arc)

$\angle PQR = \angle PSR$ ($\angle$ on same arc)

$\therefore \angle PSQ = \angle PSR$

c) $\cos\theta - 2\sin\theta\cos\theta = 0$

$\cos\theta(1 - 2\sin\theta) = 0$

$\cos\theta = 0$

$\theta = \frac{\pi}{2}, \pi + \frac{\pi}{2}, 2\pi + \frac{\pi}{2}, \ldots$

$-\pi + \frac{\pi}{2}, -2\pi + \frac{\pi}{2} \ldots$

$\theta = n\pi + \frac{\pi}{2}$

OR $n \in \mathbb{Z}$

$\theta = 2n\pi \pm \frac{\pi}{2}$

$\sin\theta = \frac{1}{2}$

$\theta = \pi/6, \pi - \pi/6, 2\pi + \frac{\pi}{6}$

$-\pi - \pi/6, -2\pi + \frac{\pi}{6}.$

$\theta = n\pi + (-1)^n \frac{\pi}{6}$

$n \in \mathbb{Z}$

Question 2

a) $\alpha + \beta + \gamma = -\frac{b}{a} = 0$

$\alpha\beta + \beta\gamma + \alpha\gamma = \frac{c}{a} = -\frac{5}{2}$

$\alpha\beta\gamma = -\frac{d}{a} = \frac{3}{2}$

i) $(\alpha + \beta + \gamma)^2 = \alpha^2 + \alpha\beta + \alpha\gamma + \alpha\beta + \beta^2 + \beta\gamma + \alpha\gamma + \gamma\beta + \gamma^2$

$= \alpha^2 + \beta^2 + \gamma^2 + 2(\alpha\beta + \gamma\beta + \alpha\gamma)$

$\therefore \alpha^2 + \beta^2 + \gamma^2 = (\alpha + \beta + \gamma)^2 - 2(\alpha\beta + \gamma\beta + \alpha\gamma)$

$= 0 - 2(-\frac{5}{2}) = 5$

ii) $(\alpha - 2)(\quad)(\gamma - 2)$

$= (\alpha\beta - 2\beta - 2\alpha + 4)(\gamma - 2)$

$= \alpha\beta\gamma - 2\beta\gamma - 2\alpha\gamma + 4\gamma - 2\alpha\beta + 4\beta + 4\alpha - 8$

$= \alpha\beta\gamma - 2(\alpha\beta + \alpha\gamma + \beta\gamma) + 4(\alpha + \beta + \gamma) - 8$

$= \frac{3}{2} - 2(-\frac{5}{2}) + 0 - 8$

$= -\frac{3}{2}$

b) $\sin\theta - \sqrt{3}\cos\theta = 1$

choose $r\sin(\theta - \alpha)$

$= r\sin\theta\cos\alpha - r\cos\theta\sin\alpha$

$r = \sqrt{a^2 + b^2}$

$= \sqrt{4}$

$= 2$

$\tan\alpha = \frac{\sqrt{3}}{1}$

$= 60°$

$\therefore \sin\theta - \sqrt{3}\cos\theta = 1$

$2\sin(\theta - 60)° = 1$

$\sin(\theta - 60)° = \frac{1}{2}$

let $A = \theta - 60°$ $\quad 0 \leq \theta \leq 300°$

$-60° \leq A \leq 300°$

$A = 30°, 150° = \theta - 60°$

$\theta = 90°, 210°$

③ $y = \dfrac{x^2}{1-x^2}$

a) $0 = 1 - x^2 \implies x = \pm 1$

b) $y' = \dfrac{vu' - uv'}{v^2}$

$= \dfrac{(1-x^2)\,2x - x^2(-2x)}{(1-x^2)^2}$

$= \dfrac{2x - 2x^3 + 2x^3}{(1-x^2)^2}$

$= \dfrac{2x}{(1-x^2)^2}$

$y' = 0 \implies x = 0$

x	less	0	greater
y'	$-ve$	0	$+ve$

$\therefore (0,0)$ a minimum

c) $f(-x) = \dfrac{(-x)^2}{1-(-x)^2} = \dfrac{x^2}{1-x^2} = f(x)$ EVEN

$\displaystyle\lim_{x\to\infty} \frac{x^2}{1-x^2}$

$= \displaystyle\lim_{x\to\infty} \frac{\frac{x^2}{x^2}}{\frac{1}{x^2} - \frac{x^2}{x^2}}$

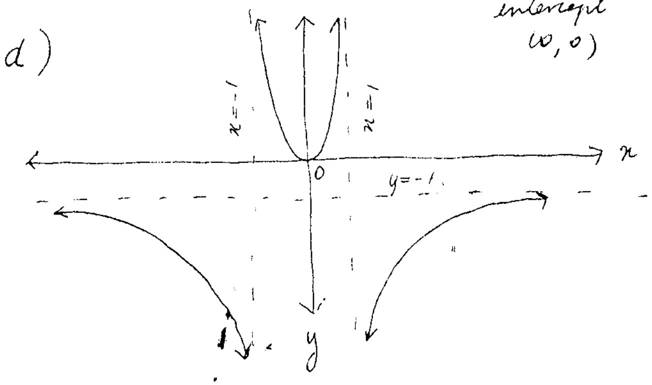
$= \displaystyle\lim_{x\to\infty} \frac{1}{\frac{1}{x^2} - 1}$

$= \dfrac{1}{-1}$

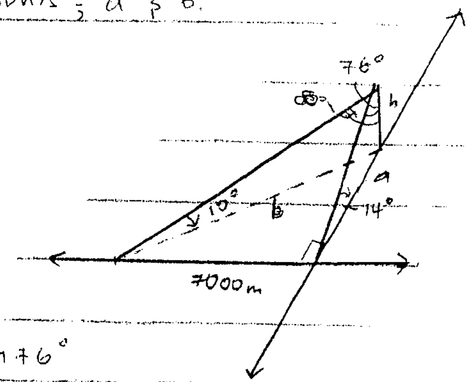
(EVEN!)

the curve will approach -1 from below

d)



④ *please refer to diagram for position of unknowns; a & b.



$\tan 76° = \dfrac{a}{h}$

$\therefore a = h\tan 76°$

$\tan 80° = \dfrac{b}{h}$

$\therefore b = h\tan 80°$

By pyth. theorem,

$7000^2 + a^2 = b^2$

$7000^2 + h^2\tan^2 76° = h^2\tan^2 80°$

$7000^2 = h^2\tan^2 80° - h^2\tan^2 76°$

$= h^2(\tan^2 80° - \tan^2 76°)$

$\therefore h^2 = \dfrac{7000^2}{\tan^2 80° - \tan^2 76°}$

$\therefore h = \dfrac{7000}{\sqrt{\tan^2 80° - \tan^2 76°}}$

$= 1745.8001..$

$\therefore$ height of the mountain $=$

1746 m (to the nearest m)

b)

$$
\begin{array}{r}
x^2 + 8x + (8-b) \\
x - 1\,\overline{)\,x^3 + 7x^2 - bx - b} \\
x^3 - x^2 \\
\hline
8x^2 - bx \\
8x^2 - 8x \\
\hline
(8-b)x - b \\
(8-b)x - (8-b) \\
\hline
8 - 2b
\end{array}
$$

If $A(x)$ is a factor then $8-2b$ equals 0

ie $8 - 2b = 0$

$b = 4$

c) $\cos 4\theta = \cos(2\theta + 2\theta)$

$= 2\cos^2 2\theta - 1$

$= 2(\cos(\theta+\theta))^2 - 1$

$= 2(2\cos^2\theta - 1)^2 - 1$

$= 2[4\cos^4\theta - 4\cos^2\theta + 1] - 1$

$= 8\cos^4\theta - 8\cos^2\theta + 2 - 1$

$= 8\cos^4\theta - 8\cos^2\theta + 1$

END.