St. Catherine's School

Year 10 Mathematics
Semester 2 Examination
October 2006

Time allowed: *2 hours + 5 minutes reading time*

INSTRUCTIONS

- There are 3 sections in this paper.
- Complete all three sections on the separate paper provided.
- Marks for each part of a question are indicated.
- All questions should be attempted.
- All necessary working should be shown
- Start each section on a new page
- Approved scientific calculators and drawing templates may be used

Section 1 *27 Marks*

1. Simplify each of the following algebraic expressions: PAS5.2.1

(a) $x^9 \div x^2 \times x^3$ 1

(b) $8y^7 \div 2y^0$ 1

(c) $(-3m^2)^3$ 1

2. Expand and simplify the following algebraic expressions: PAS 5.2.1

(a) $3(x+4)-2(x-5)$ 2

(b) $6-3(k-8)$ 2

3. Factorise the algebraic expression: PAS 5.2.1

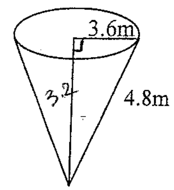
(a) $4m^2 - 32m$ 2

4. Solve the following quadratic equations: PAS 5.2.2

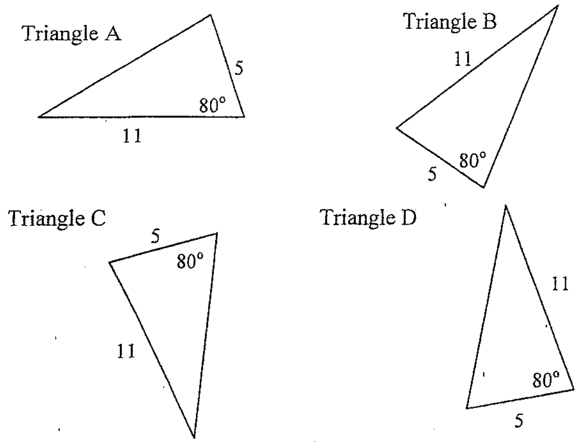
(a) $h^2 - 25 = 0$ 2

(b) $g^2 + 4g + 3 = 0$ 2

5. Calculate the volume of the cone below and leave your answer correct to 3 significant figures. MS 5.2.2 3



6. Determine which two of these triangles are congruent and state the congruence test you would use. 2 SGS 5.2.2



7. A die is rolled thirty times to obtain the following results: DS 5.1.1

6	5	1	3	3	4	2	6	1	2
3	4	4	5	3	5	6	1	1	3
5	2	2	4	1	6	6	3	3	2

(a) Construct a frequency distribution table including a cumulative frequency column and use it to determine the **mode, range and mean** of the above set of data. 5

(i) Draw a cumulative frequency histogram and polygon (ogive) for the above set of data and use it to **find the median.** 4

Section 2 ***54 Marks***

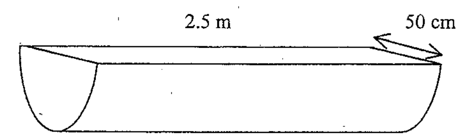
1. On the day Melissa turned 16 years old, her parents decided to invest \$2500 in a bank account where interest is compounded yearly at 4.5% p.a. 3 NS 5.2.2

What will the amount be if Melissa decides to withdraw the money at the age of 25?

2. A semicircular water trough is shown in the diagram. It is made of sheet metal with a length of 2.5 metres and a diameter of 50 centimetres. MS 5.2.2

(i) Find the volume of water when the trough is filled (in cubic centimetres to the nearest whole number). 2

(ii) Find the area of sheet metal required to make the trough (the nearest whole numbe 3



3. Simplify the following leaving your answer in exact form. MS 5.3.2

(a) $\cos 61^\circ + \sin 29^\circ$ 2

(b) $\cos\theta - \sin\left(90^\circ - \theta\right)$ 2

4. If $\cos\theta = -\dfrac{4}{5}$ and $\tan\theta > 0$, find the ratio of $\sin\theta$. 3 MS 5.3.2

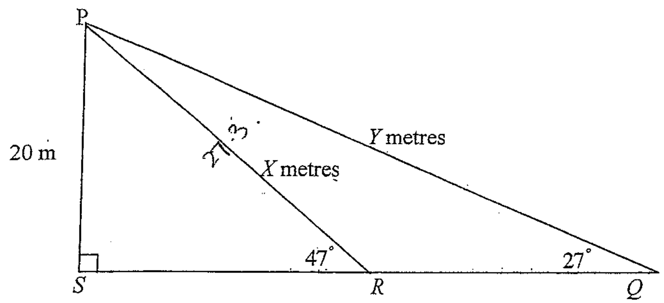
5. Find the exact value of: MS 5.3.2

(a) $\cos 315^\circ$ 1

(b) $\tan 120^\circ$ 1

6. Find all possible values of θ, for $0 \le \theta \le 360^\circ$, to the nearest degree if $\tan\theta = -\frac{5}{7}$. 3 MS 5.3.2

7. MS 5.2.3



In the diagram above, a ramp runs from R to P, a vertical height of 20 metres, and the ramp makes an angle of 47° with the horizontal.

(a) Show that $X = \dfrac{20}{\sin 47^\circ}$ and find the value of X correct to 1 decimal place. 2

(b) This ramp was found to be too steep and replaced by the ramp QP inclined at 27° to the horizontal. Find the value of Y and the difference in lengths of the ramps. 2

8. Solve the following linear inequalities: PAS 5.2.2

(a) $3m + 5 > -10$ 1

(b) $-2t - 6 \le -4$ 2

(c) $\dfrac{x-6}{7} > 3 - \dfrac{x}{2}$ 3

9. Solve the following two equations simultaneously PAS 5.2.2

(a) $y = \dfrac{x+2}{2}$ and $4y + x = -20$ 3

(b) $-3p + 2q = 2$ and $2p - 10q = -62$ 3

10. Jacinta's average mark for seven Maths class tests is 82%. What minimum mark does Jacinta need to obtain in her eighth test to improve her average to 90%? 3 DS 5.2.1

11. Joan and John are golfers. Each has played ten rounds of golf on the same course and their scores have been recorded below. DS 5.2.1

Joan's scores:	73	81	77	85	76	76	84	73	80	78
John's scores:	70	84	82	78	83	73	73	74	85	78

(i) Calculate the mean and standard deviation for each set of scores. 4

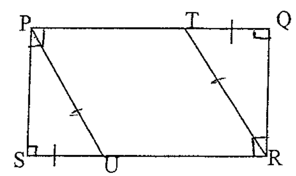
(ii) Considering your results from part (i), who is the more consistent golfer? Justify your answer.

(iii) Find the interquartile range for **Joan**'s scores and draw a box-and-whisker plot for her scores . 4

12. A letter is chosen at random from the word HIPPOPOTOMAS. Find these probabilities: NS 5.1.3

(i) P(letter O) 1
(ii) P(vowel) 1
(iii) P(letter P or I) 1

13. *PQRS* is a rectangle. *T* is a point on *PQ* and *U* is a point on *SR* such that $TQ = SU$. SGS 5.3.1



(i) Prove that ΔPSU is congruent to ΔRQT 3

(ii) Hence, prove that $PU = TR$, giving reasons for your answer. 1

Section 3 *37 Marks*

1. Expand and simplify the following: NS 5.3.1

$$\left(5-2\sqrt{7}\right)^2$$ 2

2. Rationalise the denominator: NS 5.3.1

$$\frac{7}{\sqrt{11}-3}$$ 2

3. Three points, A, B and C, are such that A is 72 m from B and 86 m from C. The bearing of B from A is 105° and of C from A is 214°. MS 5.3.2

(i) Show this information on a diagram. 2

(ii) Find the distance from B to C, to the nearest metre. 3

4. Solve the following quadratic equations: PAS 5.3.2

(a) By factorising: $2x^2+11x-6=0$ 2

~~(b) By completing the square:~~ $\cancel{4x^2-36x+75=0}$ 2

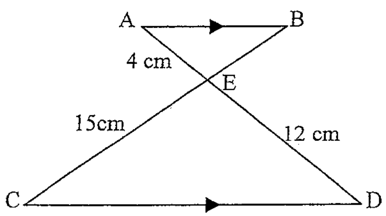
(c) Using the quadratic formula: $5x^2-4x-2=0$ 2

5. SGS 5.3.3

In the diagram below, AB is parallel to CD.

(a) Prove that ΔAEB is similar to ΔCED, giving reasons for your answers. 3

(b) Find the length of BE, giving reasons for your answer. 2



6. A box contains 6 red marbles and 4 blue marbles. Two marbles are drawn out in succession, without replacement. NS 5.3.2

(a) Draw a tree diagram that shows the possible outcomes from the above information including the probabilities on the branches. 2

(b) Use the tree diagram from part (a) to find the probability of:

(i) two blue marbles being drawn 1

(ii) a red marble and a blue marble being drawn in any order. 2

7. A baby's shoe box is similar to an adult's one. Their surface areas are in the ratio of 25:49. If the volume of the children's shoe box is $2400cm^3$, find the volume of the adult's one. 3 MS 5.3.1

8. **(i)** On a number plane, plot the points $A(2,3)$, $B(5,2)$ and $C(4,-1)$ 1 PAS 5.3.3

(ii) Find the midpoint E of AC. 1

(iii) If E is the midpoint of BD, find the coordinates of D. 2

(iv) Find the gradient m_1 of AB. 1

(v) Prove that AB is parallel to DC. 2

(vi) Find the equation of the line DC. 2

End of Examination

Section 1

(1) (a) $x^9 \div x^2 \times x^3$

$= x^7 \times x^3$ ✓ (1/2)

$= x^{10}$ ✓ (1/2)

(b) $\dfrac{8y^7}{2y^2}$

(1/2) $= 4y^7$ ✓ (1/2)

(c) $(-3m^2)^3$

$= -27m^6$ ✓ ✓ (1/2) (1/2)

(2) (a) $3(x+4) - 2(x-5)$

$= 3x + 12 - 2x + 10$ ✓ (1)

$= x + 22$ ✓ (1)

(b) $6 - 3(k-8)$

$= 6 - 3k + 24$ ✓ (1)

$= 30 - 3k$ ✓ (1)

(3) (a) $4m^2 - 32m$

$= 4m(m-8)$ ✓ ✓ ✓

(4) (a) $h^2 - 25 = 0$

$h^2 = 25$ ✓

$\sqrt{h^2} = \sqrt{25}$ ✓

$h = \pm 5$ ✓

(−0.5 marks if ± is not shown)

(b) $g^2 + 4g + 3 = 0$

$(g+3)(g+1) = 0$

$g+3=0 \quad | \quad g+1=0$

$g=-3 \quad | \quad g=-1$

(5) $V = \frac{1}{3}\pi r^2 h$ (1/2)

$= \frac{1}{3} \times \pi \times 3.6^2 \times \sqrt{10.08}$ (1/2)

$= 43.08 \ldots$ (1/2)

$= 43.1\ m^3$ (to 3 sig. figs) (1/2)

$h^2 + 3.6^2 = 4.8^2$

$h^2 = 4.8^2 - 3.6^2$ (1)

$h^2 = 10.08$

$\therefore h = \pm\sqrt{10.08}$

(6) Triangle A is congruent to triangle D.

The congruence test is SAS.

(Must get both answers to get 2/2)

(7) (a) 1 mark 1 mark

Score	Tally	Freq.	fx	cf
1	𝍸	5	5	5
2	𝍸	5	10	10
3	𝍸 II	7	21	17
4	IIII	4	16	21
5	IIII	4	20	25
6	𝍸	5	30	30
		Total f = 30	Total fx = 102	

mode = 3 ✓

range = 6 − 1

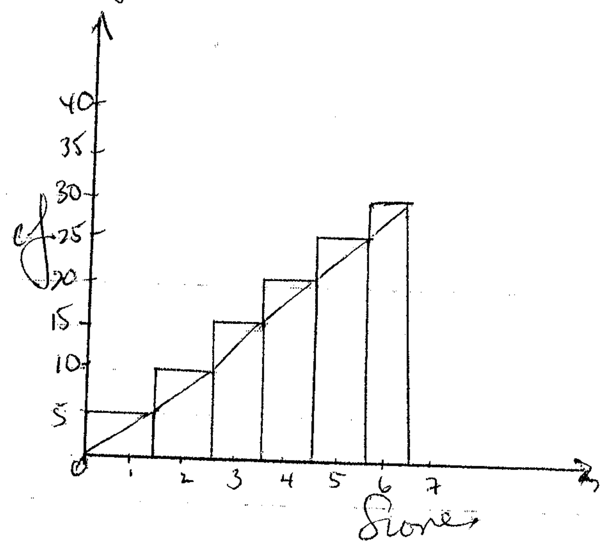
= 5 ✓

mean = $\dfrac{\text{Total } fx}{\text{Total } f}$

$= \dfrac{102}{30}$

$= 3.4$ ✓

median = 3 ✓



Section 2

(1) $A = P\left(1 + \frac{r}{100}\right)^n$

$= 2500\left(1 + \frac{4.5}{100}\right)^{9}$ ← 1 (1)

$= 3715.23785\ldots$ (1)

$= \$3715$

(2) (i) $V = \frac{\pi r^2 h}{2}$

$d = 50\text{cm}$

$\therefore r = 25\text{cm}$

and $h = 250\text{cm}$ (1/2)

$= \frac{\pi(25)^2 \times 250}{2}$ (1/2)

$= \frac{490873.8521\ldots}{2}$ (1/2)

$= 245437 \text{ cm}^3$ (1/2)

(ii) S.A = 2 semicircles + half curved part

$= 2\frac{\pi r^2}{2} + \frac{2\pi r}{2} \times h$

$= \pi r^2 + \pi r h$ (1)

$= \pi(25)^2 + \pi \times 25 \times 250$ (1/2)

$= 625\pi + 6250\pi$

$= 6875\pi$

$= 21598.449\ldots$ (1/2)

$= 21598 \text{ cm}^2$ (1)

(3) (a) $\cos 61^\circ + \sin 29^\circ$

$= \sin(90^\circ - 61^\circ) + \sin 29^\circ$ (1)

$= 2\sin 29^\circ$ (1)

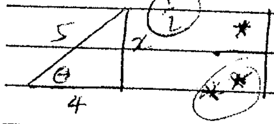
OR $2\cos 61^\circ$ (1/2 for decimal answers)

b) $\cos\theta - \sin(90^\circ - \theta)$

$= \cos\theta - \cos\theta$

$= 0$

(4) $\cos\theta = \frac{-4}{5}$



(1/2)

$\cos\theta$ (-ve) $\therefore$ 2nd + 3rd quad.

$\tan\theta$ (+ve) $\therefore$ 1st + 3rd quad.

$\therefore$ 3rd quad.

$5^2 = x^2 + 4^2$

$x^2 = 5^2 - 4^2$

$= 25 - 16$

$x^2 = 9$

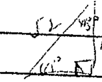
$x = \pm 3$ (1)

$\therefore \sin\theta = -\frac{3}{5}$ (1)

(5) (a) $\cos 315^\circ$

$= \cos(360^\circ - 315^\circ)$

$= \cos 45^\circ$ (1/2)



(1/2) for correct decimal value

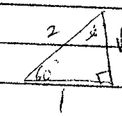
$= \frac{1}{\sqrt{2}}$ (1/2)

4th quad $\therefore$ +ve

(b) $\tan 120^\circ$

$= -\tan(180^\circ - 120^\circ)$

$= -\tan 60^\circ$ (1/2)



$= -\frac{\sqrt{3}}{1}$

$= -\sqrt{3}$ (1/2)

(c) $\tan\theta = \frac{-5}{7}$

$\theta = 35^\circ\ 32'\ 15.64''$ (½)

$= 36^\circ$ (nearest degree) (1)

Solution

$\theta = 180^\circ - 36^\circ,\ 360^\circ - 36^\circ$

$= 144^\circ$ (1), 324° (1)

(7) (a) $\sin 47^\circ = \frac{20}{X}$ (½)

$\therefore X = \frac{20}{\sin 47^\circ}$ (½)

$= 27.3463\ldots$ (½)

$= 27.3$ m (to 1 dec. pl.) (½)

(b) $\sin 27^\circ = \frac{20}{Y}$

$\therefore Y = \frac{20}{\sin 27^\circ}$ (½)

$= 44.05378$

$= 44.1$ m (to 1 dec. pl.) (½)

$\therefore$ The difference in lengths of the ramps is 16.8 m $(44.1 - 27.3)$ (1)

(8) (a) $3m + 5 > -10$

$3m > -15$ (½)

$m > -5$ (½)

(b) $-2t - 6 \leq -4$

$-2t \leq -4 + 6$

$\frac{-2t}{-2} \leq \frac{2}{-2}$ (1)

$\therefore t \geq -1$ (½)

(½)

(c) $\frac{x-6}{7} > 3 - \frac{x}{2}$

$\frac{2(x-6)}{14} > \frac{14\times 3 - 7x}{14}$ (1)

$2x - 12 > 42 - 7x$

$9x - 12 > 42$ (1)

$9x > 54$ (½)

$x > 6$ (½)

14. $y = \frac{x+2}{2}$ and $4y + x = -20$

$4y = -20 - x$

$y = \frac{-20 - x}{4}$ $\frac{1}{2}$

$\frac{x+2}{2} = \frac{-20-x}{4}$ $\frac{1}{2}$

$4(x+2) = 2(-20-x)$

$4x + 8 = -40 - 2x$ $\frac{1}{2}$

$4x + 2x = -40 - 8$

$6x = -48$

$x = -8$

$\therefore y = \frac{x+2}{2}$ $\frac{1}{2}$

$= \frac{-8+2}{2}$

$= \frac{-6}{2}$

$y = -3$

Solution is $x = -8$, $y = -3$ $\frac{1}{2}$ $\frac{1}{2}$

9 b) $-3p + 2q = 2$ — (1) $\times 2$

$2p - 10q = -62$ — (2) $\times 3$

$-6p + 4q = 4$ — (1)

$+ \; 6p - 30q = -186$ — (2) 1

$-26q = -182$

$\therefore q = 7$

$2p - 10q = -62$

$2p - 10 \times 7 = -62$ 1

$2p - 70 = -62$

$2p = -62 + 70$

$2p = 8$

$\therefore p = 4$

Solution is $q = 7$, $p = 4$ $\frac{1}{2}$ $\frac{1}{2}$

10) $\bar{x} = \frac{\text{Total of marks}}{7} = 82$ $\frac{1}{2}$

$\therefore$ Total of 7 tests $= 7 \times 82$

$= 574$ $\frac{1}{2}$

$\bar{x} = \frac{\text{total of 7 tests} + \text{Result of 8th test}}{8} = 90$ $\frac{1}{2}$

$\therefore \frac{574 + \text{Result of 8th test}}{8} = 90$ $\frac{1}{2}$

574 + result of test = 90 × 8 ($\frac{1}{2}$)

∴ Result of test = 720 − 574

= 146 % ($\frac{1}{2}$)

11 (i) Joan : $\bar{x} = 78$ (1)

$\sigma = 4.074...$ (1)

John : $\bar{x} = 78$ (1)

$\sigma = 5.0596...$ (1)

(ii) Joan is more consistent (1) since the standard deviation for her scores was lower than the standard deviation for John's scores. (1)

(iii) Joan's scores

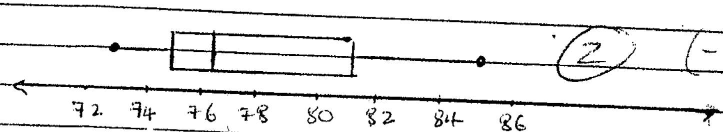
73 73 75 76 76 77 80 81 84 85

Q_1 ($\frac{1}{2}$) Q_2 Q_3 ($\frac{1}{2}$)

Interquartile range = $Q_3 - Q_1$

= 81 − 75

= 6 (1)



(2) ($-\frac{1}{2}$ for each error)

12 (i) $P(O) = \frac{3}{12}$

$= \frac{1}{4}$ (1)

(ii) $P(\text{vowel}) = \frac{5}{12}$ (1)

(iii) $P(\text{letter P or I}) = \frac{3}{12} + \frac{1}{12}$ ($\frac{1}{2}$)

$= \frac{4}{12}$

$= \frac{1}{3}$ ($\frac{1}{2}$)

13 (i) In Δ PSU and RQT, we have –

PS = QR ($\frac{1}{2}$) (opp. sides of a rectangle) ($\frac{1}{2}$)

∠PSU = ∠TQR = 90° ($\frac{1}{2}$) (∠s in a rectangle) ($\frac{1}{2}$)

SU = TQ (given) ($\frac{1}{2}$)

∴ Δ PSU ≡ Δ RQT (SAS) ($\frac{1}{2}$)

(ii) PU = TR (corresponding sides of 2 congruent Δs PSU & RQT)

(1)

Section 3.

(1) $\left(5-2\sqrt{7}\right)^2$

$= 5^2 - 2\times5\times2\sqrt{7} + \left(2\sqrt{7}\right)^2$

$= 25 - 20\sqrt{7} + 4\times7$

$= 53 - 20\sqrt{7}$

(2) $\dfrac{7}{\sqrt{11}-3}$

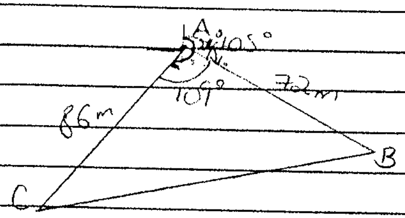
$= \dfrac{7}{\sqrt{11}-3} \times \dfrac{\sqrt{11}+3}{\sqrt{11}+3}$

$= \dfrac{7\left(\sqrt{11}+3\right)}{\left(\sqrt{11}-3\right)\left(\sqrt{11}+3\right)}$

$= \dfrac{7\sqrt{11}+3\times7}{11-9}$

$= \dfrac{7\sqrt{11}+21}{2}$

(3)



$\angle CAB = 214° - 105°$

$= 109°$

Using the cosine rule

$BC^2 = 72^2 + 86^2 - 2\times72\times86\cos109°$

$BC = \sqrt{72^2+86^2-2\times72\times86\cos109°}$

$= 128.8869 \ldots$

$= 129$ m (to the nearest m)

(4) (a) $2x^2+11x-6=0$

$(2x-1)(x+6)=0$

$2x-1=0 \qquad x+6=0$

$2x=1 \qquad x=-6$

$x=\dfrac{1}{2}$

~~(b)~~ $\cancel{4x^2-36x+75=0}$

$\cancel{x^2-9x+\dfrac{75}{4}=0}$

$\cancel{x^2-9x+\left(\dfrac{9}{2}\right)^2 = -\dfrac{75}{4}+\left(\dfrac{9}{2}\right)^2}$

(c) $5x^2-4x-2=0$

$x = \dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$

$= \dfrac{--4\pm\sqrt{(-4)^2-4\times5\times(-2)}}{2\times5}$

$= \dfrac{4\pm\sqrt{16+40}}{10}$

$= \dfrac{4\pm\sqrt{56}}{10}$

$= \dfrac{4\pm\sqrt{14\times4}}{10}$

$= \dfrac{4\pm2\sqrt{14}}{10}$

$= \dfrac{2\left(2\pm\sqrt{14}\right)}{10} = \dfrac{2\pm\sqrt{14}}{5}$

(5.) (a) In Δs AEB and CED, we have:

$\angle ABE = \angle ECD$ (alt $\angle s$ b/w 2 $\parallel$ lines AB & CD)

$\angle AEB = \angle CED$ (vert. opp. $\angle s$)

$\therefore \Delta AEB \parallel\mid \Delta CED$ (AA)

b) $\frac{AE}{ED} = \frac{BE}{EC}$ (corresponding sides of 2 similar Δs)

$\frac{4}{12} = \frac{BE}{15}$

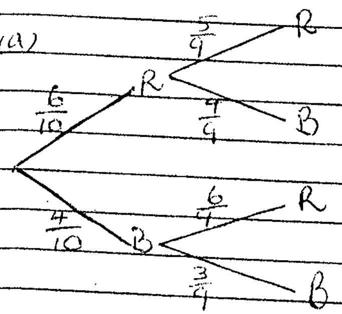
$12BE = 4 \times 15$

$12BE = 60$

$\therefore BE = \frac{60}{12}$

$= 5$ cm (Corresp. sides of 2 similar Δs)

(6) (a)



(b) (i) $P(BB) = \frac{4}{10} \times \frac{3}{9}$

$= \frac{12}{90}$

$= \frac{2}{15}$

(ii) $P(RB \text{ OR } BR)$

$= \left(\frac{6}{10} \times \frac{4}{9}\right) + \left(\frac{4}{10} \times \frac{6}{9}\right)$

$= \frac{4}{15} + \frac{4}{15}$

$= \frac{8}{15}$

(9) Baby : Adult

Ratio of S.A. 25 : 49

$5^2 : 7^2$

$\therefore$ Ratio of matching sides $= 5 : 7$

$\therefore$ Ratio of volume $= 5^3 : 7^3$

$= 125 : 343$

Let the volume of the adult's shoe box be V

$2400 : V = 125 : 343$

$\frac{2400}{V} = \frac{125}{343}$

$125V = 2400 \times 343$

$125V = 823\,200$

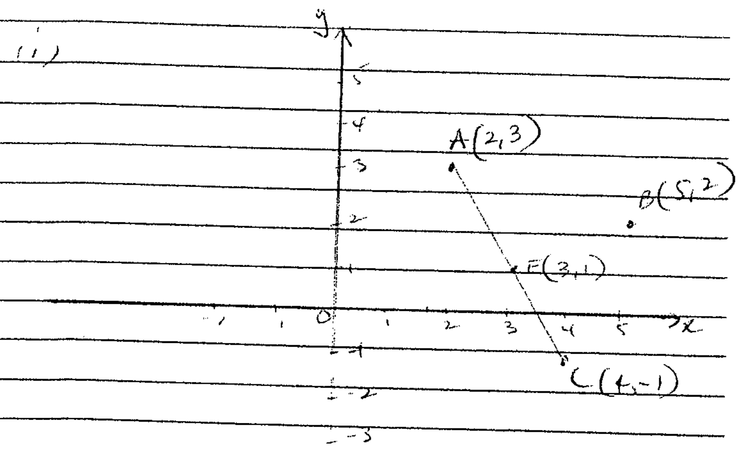
$\therefore V = \frac{823\,200}{125}$

$= 6585.60 \text{ cm}^3$

$\therefore$ The volume of the adult's shoe box is $54\,880 \text{ cm}^3$

(8)

(i)



(ii) midpoint of AC, $E = \left(\frac{x_1+x_2}{2}, \frac{y_1+y_2}{2}\right)$

$$= \left(\frac{2+4}{2}, \frac{3+-1}{2}\right)$$

$$= (3, 1)$$

(iii) $D(x_1, y_1)$ $\quad E(3,1)$ midpoint $\quad B(5,2)$

$3 = \frac{x_1+5}{2}$ $\qquad 1 = \frac{y_1+2}{2}$

$6 = x_1+5$ $\qquad 2 = y_1+2$

$\therefore x_1 = 6-5$ $\qquad \therefore y_1 = 0$

$x_1 = 1$

$$\therefore D(1, 0)$$

(iv) $m_{AB} = \frac{y_2-y_1}{x_2-x_1}$

$$= \frac{2-3}{5-2} = \frac{-1}{3}$$

(v) For AB to be parallel to DC

$m_{AB} = m_{DC}$ $\qquad D(1,0)$, $C(4,-1)$

$m_{AB} = \frac{-1}{3}$ and $m_{DC} = \frac{-1-0}{4-1}$

$$= \frac{-1}{3}$$

$m_{AB} = m_{DC} = -\frac{1}{3}$ $\quad \therefore AB \parallel CD$

(vi) Equation of DC

$y - y_1 = m(x - x_1)$

$y - 0 = -\frac{1}{3}(x - 1)$

$y = \frac{-1}{3}x + \frac{1}{3}$

$3y = -x + 1$

$x + 3y - 1 = 0$