St Catherine's School
Mathematics/Extension 1

June, 2002

Question1

a) Differentiate with respect to x:

i) $y = e^{1-3x}$ (1 mark)

ii) $y = \ln(x+1)$ (1 mark)

iii) $y = \ln\sqrt{x+1}$ (2 marks)

iv) $y = e^{x^2+2}$ (1 mark)

b) Find

i) $\int e^{-x}dx$ (1 mark)

ii) $\int \frac{x}{x^2+1}dx$ (2 marks)

iii) $\int \frac{x-1}{x}dx$ (2 marks)

c)

i) Differentiate $y = e^{x^2+4x+6}$

ii) Hence, find $\int (x+2)e^{x^2+4x+6}dx$ (3 marks)

Question 2

a) Consider the function $y = 2\cos 3x$

i) Find the amplitude (1 mark)

ii) Find the period (1mark)

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iii) Sketch $y = 2\cos 3x$, where $0 \le x \le 2\pi$ (2 marks)

iv) Find the value(s) of x for which $y = 2\cos 3x$, where $0 \le x \le 2\pi$ is a maximum. (2 marks)

b) Find the exact value of $\tan \frac{5\pi}{6}$ (1 mark)

c) Find the $\lim_{x \to 0} \frac{\sin 3x}{2x}$ (2 marks)

d) Consider the curve $y = \frac{\ln x}{x}$

i) Find $\frac{dy}{dx}$ (2 mark)

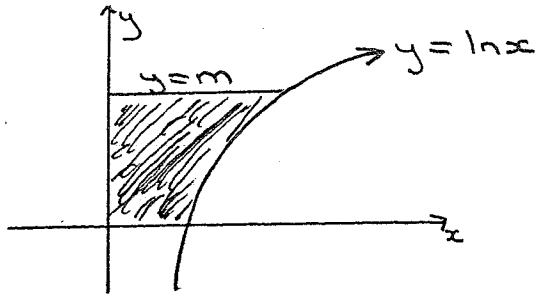
ii) Solve $\frac{dy}{dx} = 0$ (1 mark)

e) The area shaded is the area enclosed between $y = \ln x$, the x & y axes and the line y=m

i) Find the shaded area in terms of m.

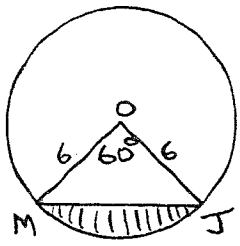
ii) If the area is 1 $unit^2$, find m.

(4 marks)



Question 3

a) In the diagram to the right



i) Find the length of arc MJ (1 mark)

ii) Find the exact area of sector MOJ. (11/2 marks)

iii) Find the exact area of ΔMOJ. (11/2 marks)

iv) Hence find the exact area of the shaded minor segment. (1 mark)

b) If $y = \dfrac{e^x - 1}{e^x + 1}$ find $\dfrac{dy}{dx}$ (3 marks)

c) The shaded is bounded by the curve $y = \dfrac{e^x + e^{-x}}{2}$, x=-1, x=1 and the x axis.

This area is rotated about the x axis. Find the volume.

(4 marks)

You've finished!!

June Assessment.

Maths / Ext 1

1. a) i) $y = e^{1-3x}$

$\frac{dy}{dx} = -3e^{1-3x}$ (1)

ii) $y = \ln(x+1)$

$\frac{dy}{dx} = \frac{1}{x+1}$ (1)

iii) $y = \ln\sqrt{x+1}$

$= \ln(x+1)^{\frac{1}{2}}$

$\frac{dy}{dx} = \frac{1}{\sqrt{x+1}} \cdot \frac{1}{2}(x+1)^{-\frac{1}{2}}$

$= \frac{1}{2(x+1)}$

OR $y = \ln\sqrt{x+1}$

$y = \frac{1}{2}\ln(x+1)$

$\frac{dy}{dx} = \frac{1}{2} \cdot \frac{1}{x+1}$

$= \frac{1}{2(x+1)}$ (2)

iv) $y = e^{x^2+2}$

$\frac{dy}{dx} = 2x\,e^{x^2+2}$ (1)

b) i) $\int e^{-x}\,dx$

$= -e^{-x} + c$ (1)

ii) $\int \frac{x}{x^2+1}\,dx$

$= \frac{1}{2}\ln(x^2+1) + c$ (2)

iii) $\int \frac{x-1}{x}\,dx$

$= \int \left(1 - \frac{1}{x}\right)dx$

$= x - \ln x + c$ (2)

c) i) $y = e^{x^2+4x+6}$

$\frac{dy}{dx} = (2x+4)e^{x^2+4x+6}$ (1)

ii) $\int (x+2)e^{x^2+4x+6}\,dx$

$= \frac{1}{2}\int (2x+4)e^{x^2+4x+6}\,dx$

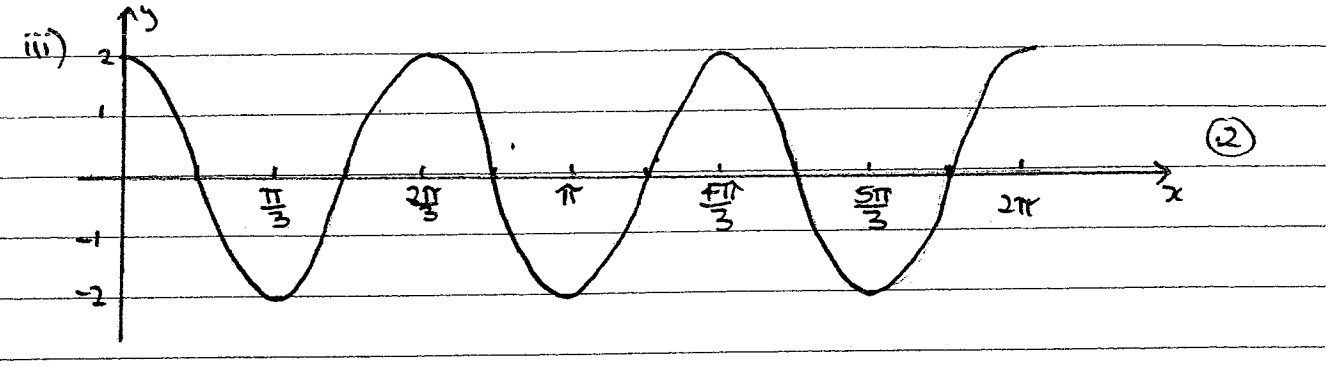
$= \frac{1}{2}e^{x^2+4x+6} + c$ (2)

(3)

2 a) i) Amp = 2 (1)

ii) Period = $\frac{2\pi}{3}$ (1)

iii)

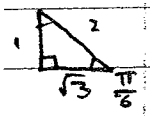


(2)

iv) $x = 0, \frac{2\pi}{3}, \frac{4\pi}{3}, 2\pi$ (2)

b) $\tan \frac{5\pi}{6} = \tan\left(\pi - \frac{\pi}{6}\right)$

$= -\tan \frac{\pi}{6}$



$= -\frac{1}{\sqrt{3}}$ (1)

c) $\lim_{x \to 0} \frac{\sin 3x}{2x} = \lim_{x \to 0} \frac{3}{2}\left(\frac{\sin 3x}{\frac{3}{2} \times 2x}\right)$

$= \lim_{x \to 0} \frac{3}{2}\left(\frac{\sin 3x}{3x}\right)$

$= \frac{3}{2} \lim_{x \to 0} \frac{\sin 3x}{3x}$

$= \frac{3}{2}$ (2).

d) $y = \frac{\ln x}{x}$

i) $\frac{dy}{dx} = \frac{x \cdot \frac{1}{x} - \ln x \cdot 1}{x^2}$

$= \frac{1 - \ln x}{x^2}$ (2)

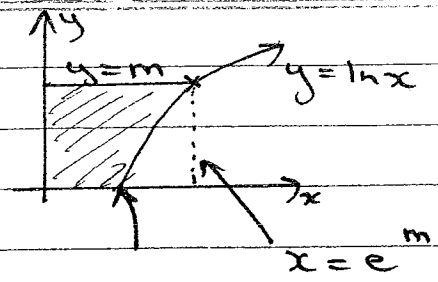
ii) $\frac{1 - \ln x}{x^2} = 0$

$\therefore 1 - \ln x = 0$

$\ln x = 1$

$x = e$ (1)

2 e)



$y = 0$

$\Rightarrow x = e^0$

$= 1$

i) Area $\Rightarrow$ $y = \ln x$

$\Rightarrow e^y = x$

$$\text{Area} = \int_0^m x \, dy$$

$$= \int_0^m e^y \, dy$$

$$= e^m - e^m$$

$$= e^m - 1$$

Area is $e^m - 1$ units^2

ii) If area = 1 units

then

$e^m - 1 = 1$

$e^m = 2$

$\therefore m = \ln 2$ (4).

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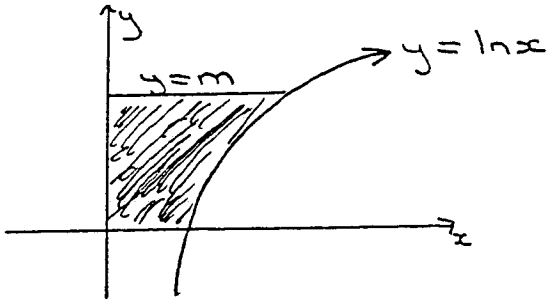

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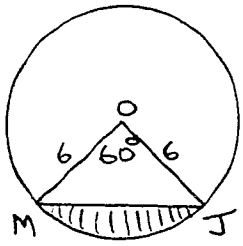

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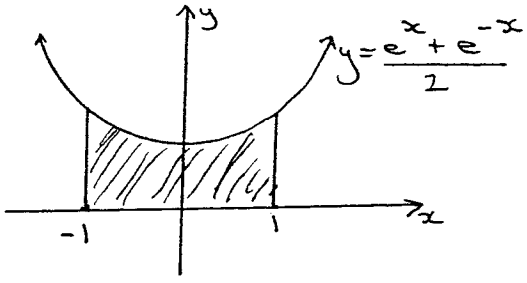

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