St. Catherine's School
Waverley

2006

PRELIMINARY ASSESSMENT TASK 3
CLASS TEST

Mathematics

General Instructions

- Working time – 55 minutes
- Write using black or blue pen
- Board-approved calculators may be used.
- All necessary working should be shown in every question

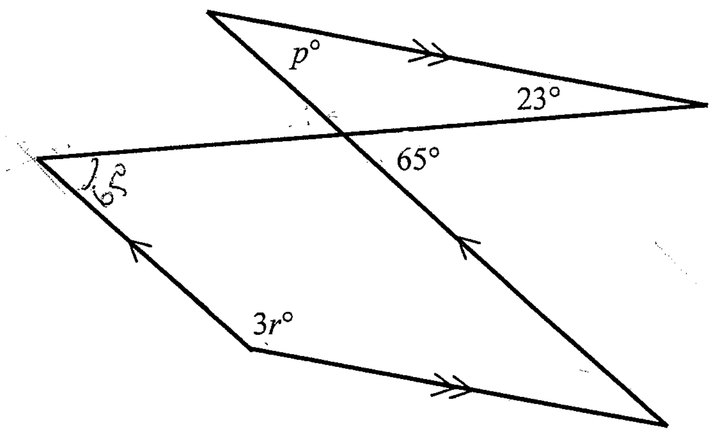
Total marks – 45

- Attempt Questions 1–3
- All questions are of equal value

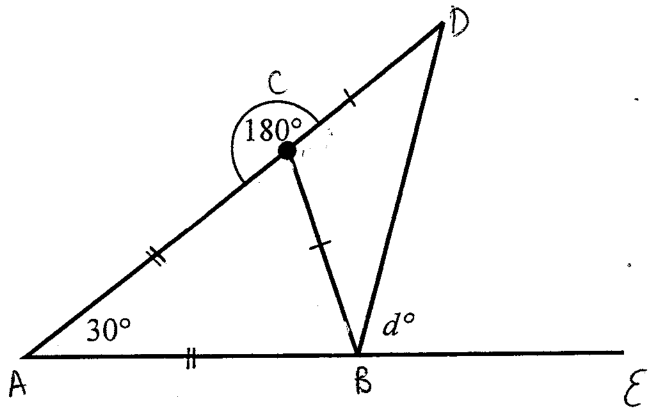
Question 1 **15 Marks**

a) Calculate the value of the pronumeral in each of the following diagrams:

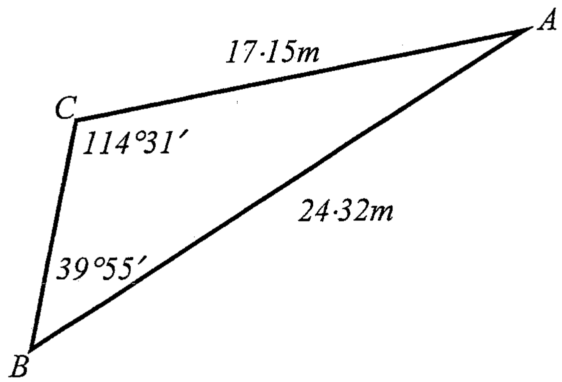
(i) **2**



(ii) **2**



b) Use the cosine rule to calculate the length of BC from the diagram below. **2**
Answer accurate to 1 decimal place.

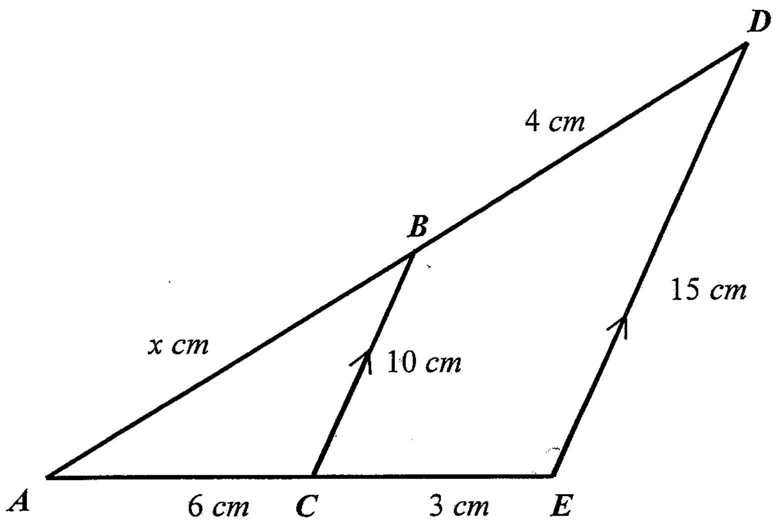


c) What is the size of each **exterior** angle for a regular octagon? **1**

d) Given that an n-sided polygon has an angle sum of $180(n-2)^{\circ}$, show that 127° cannot be the size of each interior angle for a **regular** polygon. **2**

e)

(i) Show that $\Delta ABC \,|||\, \Delta ADE$ **4**

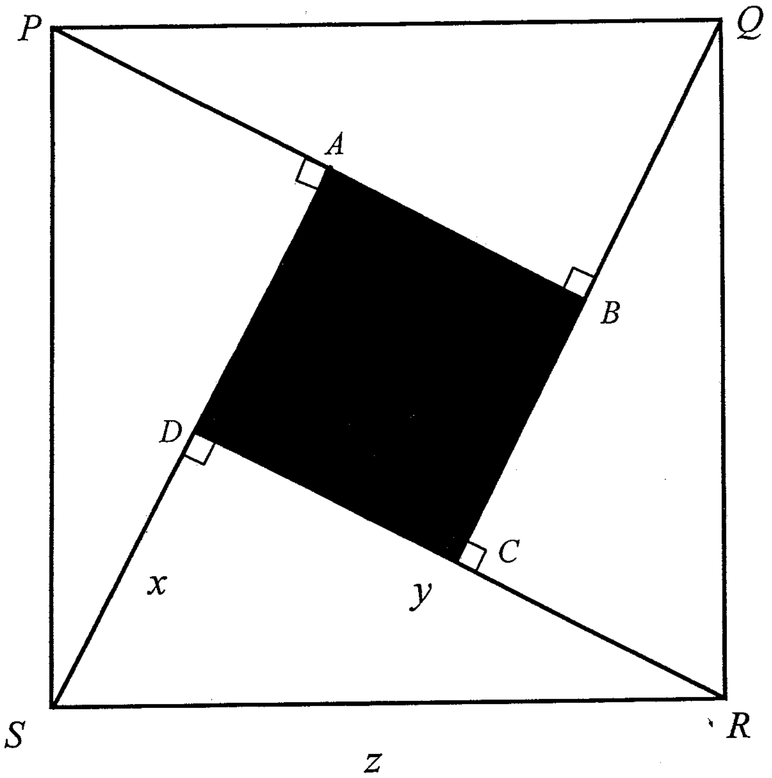


(ii) Find the value of x in the diagram above **2**

Question 2 begins on the next page

Question 2 15 Marks

a) *PQRS* is a large square made up of **four congruent right-angled triangles** and **one smaller square** as shown below.



For ΔSDR : $SR = z\ cm$, $DR = y\ cm$, and $DS = x\ cm$, with $y > x$

(i) Show that $DC = (y - x)\ cm$ **1**

(ii) In terms of the side lengths x , y and z, write expressions for the areas of:

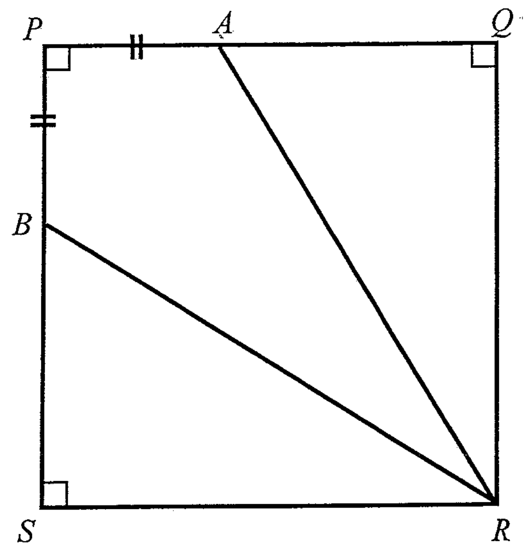
(α) the square *PQRS* **1**

(β) the square *ABCD* **1**

(γ) the four congruent triangles **1**

(iii) Use the area expressions from (ii) to show that $z^2 = x^2 + y^2$ **3**

b) In the diagram below, $PQRS$ is a square and $BP = AP$.



Prove that $\Delta ARQ \equiv \Delta BRS$ **4**

c) Find the acute angle α, correct to the nearest minute, given that: **1**

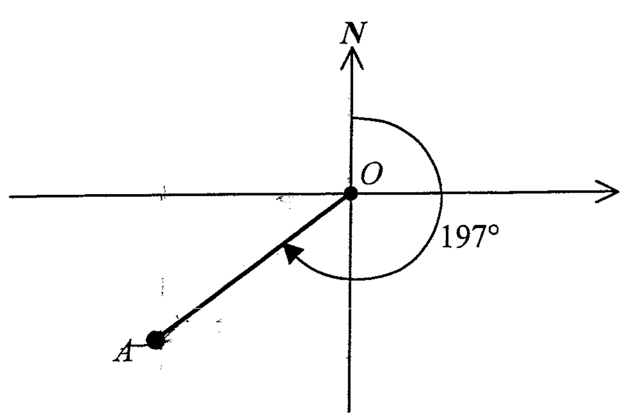
$$\tan\alpha = 2.36$$

d) Write down the **exact** values of each of the following:

(i) cos 120° **1**

(ii) cosec 315° **1**

e) Write down the bearing of ***O*** **from** ***A*** shown in the diagram below **1**



Question 3 **15 Marks**

a) If $\sin \beta = \frac{5}{13}$ and $90° < \beta < 180°$, find the values of $\cos \beta$ and $\tan \beta$. **2**

b) Solve the trigonometric equation below for $0° \le x \le 360°$ **2**

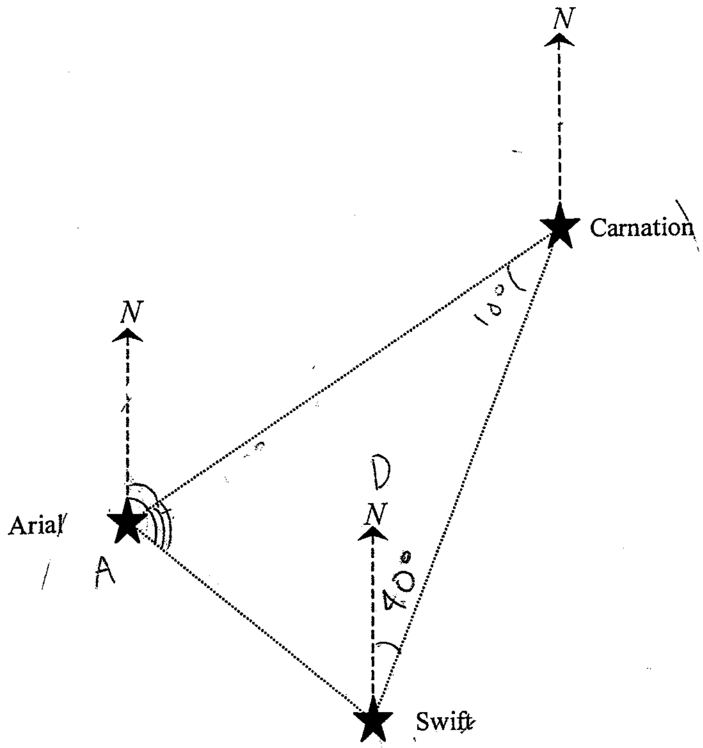
$$\sqrt{2}\sin x - 1 = 0$$

c) Simplify $\frac{-2 + 2\cos^2 \theta}{\sin^2 \theta}$ **2**

d) Show that $\frac{1}{\sin\theta} - \frac{1}{\sin^2\theta} = \frac{\sin\theta - 1}{(1+\cos\theta)(1-\cos\theta)}$ **3**

Question 3 continues on the next page

e) A operator roughly sketches the diagram below from a radar screen. It shows the relative positions in a yacht race of two yachts. There has been an emergency call from a third vessel, a container ship, Carnation.



The following details are known from other instruments:

- Carnation is on a bearing of 058°T from Arial
- Swift is on a bearing of 115°T from Arial at a distance of 25*km* from it.
- Carnation is on a bearing of 040°T from Swift.

(i) Calculate how far Carnation is from Arial. **3**

Give answer to the nearest *km*.

(ii) Calculate which yacht is closest to Carnation. **3**

END OF EXAM

Qn	Solutions	Marks	Comments+Criteria
1	(a)(i) $p = 65 - 23°$		Av: $\frac{27.7}{45}$.
	$= 42$	1	
	$3r + p = 180$		
	$3r = 180 - 42$		
	$= 138$		
	$r = 46$	1	
	(ii) $d = 30 + 37\frac{1}{2}$		
	$= 67\frac{1}{2}°$	2	
	(b) $BC^2 = 17.15^2 + 24.32^2 - 2.(17.15 \times 24.32) \times \cos A$		
	$A = 180 - (114°31' - 39°55')$		
	$= 25° 34'$		
	$BC^2 = 133.0882\ldots$		
	$BC = 11.5363\ldots$		
	$\doteqdot 11.5$ km	2	
	(c) Ext ∡ $= 360° \div 8$	1	
	$= 45°$		
	(d) let $\frac{180(n-2)}{n} = 127$		
	$127n = 180n - 360$		
	$53n = 360$		
	$n = 6.79\ldots$		
	ie n is not an integer ∴ no polygon with 127° as internal ∡	2	

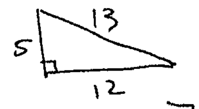
Qn	Solutions	Marks	Comments+Criteria
1	(e)(i) RTP: $\triangle ABC \parallel\!\mid \triangle ADE$		
	Proof: in $\triangle ABC$, ADE		
	$B\hat{A}C = D\hat{A}E$ (common) A	1	or by ratios + ∡C and E
	$A\hat{B}C = A\hat{D}E$ (corres ∡s on ∥ lines BC, DE) A	1	
	$A\hat{C}B = A\hat{E}D$ (∡ sum △) A	1	
	∴ $\triangle ABC \parallel\!\mid \triangle ADE$ (equiangular) QED	1	
	(ii) $\triangle ABC \parallel\!\mid \triangle ADE$		
	∴ $\frac{AB}{AD} = \frac{10}{15}$		
	ie $\frac{x}{x+4} = \frac{2}{3}$		
	$3x = 2x + 8$		
	$x = 8$	2	
2	(a)(i) $CR = x$ (≡△s)		
	∴ $DC = DR - CR$		
	$= y - x$ cm		
	(ii)(α) $Area_{PQRS} = z^2$ cm²		
	(β) $Area_{ABCD} = (y-x)^2$ cm²		
	(γ) $A_{\triangle} = \frac{1}{2}xy$ cm²		Also accepted $z^2 - (y-x)^2$
	(iii) $Area_{PQRS} = Area_{ABCD} + 4\, Area_{\triangle}$		
	$z^2 = (y-x)^2 + 4 \cdot \frac{1}{2}xy$		

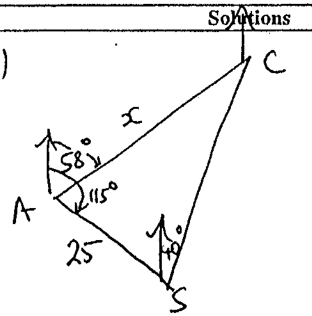
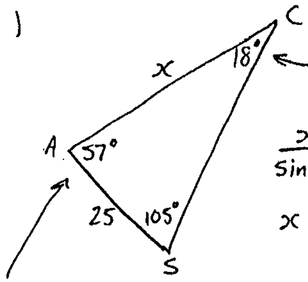
$= y^2 + x^2 - 2xy + 2xy$

$= x^2 + y^2$

QED

Qn	Solutions	Marks	Comments+Criteria
2 (b)	RTP: $\triangle ARQ \equiv \triangle BRS$ Proof: in $\triangle$s ARQ, BRS $QR = SR$ (prop. of a square $PQRS$) S $\hat{AQR} = \hat{BSR}$ (given square) A $AQ = PQ - AP$ $= PS - PB$ ($AP = PB$, $PQ = PS$ given) $= BS$ $\therefore AQ = BS$ S $\therefore \triangle ARQ \equiv \triangle BRS$ (SAS)	1M	½ mark for each reason & data no c.o.e.
(c)	$\alpha = \tan^{-1} 2.36$ $= 67^\circ 2'$ [$10.42''$]		no part marks
(d) (i)	$\cos 120^\circ = -\cos 60^\circ$ $= -\frac{1}{2}$		½ mark for 1/2
(ii)	$\text{cosec}\, 315^\circ = \frac{1}{\sin 315}$ $= \frac{1}{-\sin 45}$ $= -\sqrt{2}$		½ mark for $\sqrt{2}$
(e)	017°T		

Qn	Solutions	Marks	Comments+Criteria
3 (a)	$\sin \beta = \frac{5}{13}$  in Q2 $\cos \beta = -\frac{12}{13}$ [$= -0.9230\ldots$] $\tan \beta = -\frac{5}{12}$ [$= -0.41\dot{6}$]	✓ ✓	accept decimal approx^{n}s $-0.923\ldots$ etc. -1 for positive exact values
(b)	$\sqrt{2} \sin x - 1 = 0$ $\sin x = \frac{1}{\sqrt{2}}$ ~~$x = 45^\circ, 135^\circ$~~	✓ ✓	1 for $x = 45^\circ$ and not 135° 1 for incorrect exact value but Q1, Q2 values. ½ for $\sin x = \frac{1}{\sqrt{2}}$
(c)	$\frac{-2 + 2\cos^2\theta}{\sin^2\theta} = \frac{2(\cos^2\theta - 1)}{\sin^2\theta}$ $= \frac{-2(1 - \cos^2\theta)}{\sin^2\theta}$ $= -2$	✓ ✓½ ✓½	$-\frac{1}{2}$ for 2
(d)	RTP: $\frac{1}{\sin\theta} - \frac{1}{\sin^2\theta} = \frac{\sin\theta - 1}{(1+\cos\theta)(1-\cos\theta)}$ RHS $= \frac{\sin\theta - 1}{(1+\cos\theta)(1-\cos\theta)}$ $= \frac{\sin\theta - 1}{1 - \cos^2\theta}$ $= \frac{\sin\theta - 1}{\sin^2\theta}$ $= \frac{1}{\sin\theta} - \frac{1}{\sin^2\theta}$ $=$ LHS QED	 ✓ ✓ ✓	OR LHS $= \frac{1}{\sin\theta} - \frac{1}{\sin^2\theta}$ $= \frac{\sin\theta}{\sin^2\theta} - \frac{1}{\sin^2\theta}$ $= \frac{\sin\theta - 1}{\sin^2\theta}$ $= \frac{\sin\theta - 1}{1 - \cos^2\theta}$ $= \frac{\sin\theta - 1}{(1+\cos\theta)(1-\cos\theta)}$ = RHS ½ for $\frac{\sin^2\theta - \sin\theta}{\sin^3\theta}$ only

Qn	Solutions	Marks	Comments+Criteria
3	(e) 		
	(i)  $\angle ACS = 180^\circ - 162^\circ = 18^\circ$	✓	1 use of sine rule with one incorrect angle
	$\angle CAS = 115^\circ - 58^\circ = 57^\circ$		
	$\frac{x}{\sin 105^\circ} = \frac{25}{\sin 18^\circ}$	✓	
	$x = \frac{25 \times \sin 105^\circ}{\sin 18^\circ}$		
	$= 78.145$ km		IROE
	$= 78$ km (to nearest km)	✓	
	(ii) $\frac{CS}{\sin 57^\circ} = \frac{25}{\sin 18^\circ}$	✓	
	$CS = \frac{25 \times \sin 57^\circ}{\sin 18^\circ}$		
	$= 67.84987$		IROE
	$= 68$ km (to nearest km)	✓	
	$\therefore$ Swift is closer than Arial	✓	