**Student Number:**__________________

**St. Catherine's School**
**Waverley**

**August 2008**

**TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION**

# Extension 1 Mathematics

*Time allowed:* *2 Hours + 5 mins Reading Time*

**INSTRUCTIONS**

- Write your STUDENT NUMBER on each page
- All questions are of equal value
- Marks for each part of a question are indicated
- All questions should be attempted on the separate paper provided
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- Standard integrals are printed at the end of the paper



**QUESTION 1** **(12 marks)** **Marks**

a) Solve the inequality $\dfrac{2x+5}{x-4} \le 1$ 3

b) Evaluate $\displaystyle\int_{-3}^{3} \frac{1}{9+x^2}\,dx$ 2

c) Evaluate $\displaystyle\lim_{x\to 0} \frac{\sin x}{5x}$ 2

d) Given that $\log_b\left(\dfrac{p}{q}\right)=3$ and $\log_b\left(\dfrac{q}{r}\right)=1\cdot 6$, evaluate $\log_b\left(\dfrac{p}{r}\right)$ 2

e) Evaluate $\displaystyle\int_{0}^{\frac{1}{2}} 2x\sqrt{1-2x}\,dx$ using the substitution $u=1-2x$ 3

**QUESTION 2 (12 marks) Start a *new* page.** **Marks**

-2

a) $\tan\theta = m$ and $\tan\phi = 3$ find the value of $m$ if $\theta - \phi = \dfrac{\pi}{4}$ **2**

b) Prove that, if $x^4 - x^3 + kx - 4$ has a factor of $(x+1)$, then it also has a factor $m = -\frac{1}{2}$ **2**
of $(x-2)$.

c) Prove that $\dfrac{2}{\cot x + \tan x} = \sin 2x$ **2**

d) Find the general solution of $\sqrt{3}\sin 2x = \cos 2x$ **2**

e) Consider the function $f(x) = \dfrac{\pi}{2} + 2\sin^{-1}\left(\dfrac{2x}{3}\right)$

(i) Find the domain and range of $f(x)$ **2**

(ii) Sketch the graph of $f(x)$ showing clearly its end points. **2**





**QUESTION 3 (12 marks) Start a *new* page.** **Marks**

a) Use mathematical induction to show that, for all positive integers $n \geq 1$, **3**

$$1^2 + 3^2 + 5^2 + \ldots\ldots + (2n-1)^2 = \frac{1}{3}n(2n-1)(2n+1)$$

b) Find the value of the term that is independent of $x$ in the expansion of **2**

$$\left(2x^2 + \frac{1}{x^3}\right)^{10}$$

c) $Q(x) = ax^2 + bx + c$

(i) State the sum of the roots of $Q(x) = 0$ **1**

(ii) When $Q(x)$ is divided by either $(x-m)$ or $(x-n)$ the remainder is the same. **2**
Prove that, if $m \neq n$, then $(m+n)$ is equal to the sum of the roots of $Q(x) = 0$

d) Consider the function $f(x) = \dfrac{x-2}{x-1}$.

(i) Prove that $f(x)$ is an increasing function for all values of $x$. **2**

(ii) Find the equation of the inverse function $f^{-1}(x)$ and deduce that $f(x)$ is symmetrical about the line $y = x$ **2**

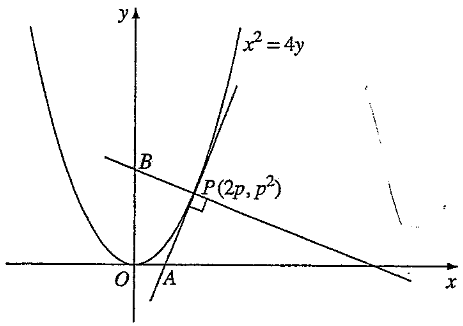
**QUESTION 4** (12 marks) Start a *new* page. **Marks**

a) (i) Prove that $\displaystyle\int_0^{\frac{\pi}{4}} \sin^2 x\, dx = \frac{\pi}{8} - \frac{1}{4}$ **2**

(ii) Prove that $\dfrac{d}{dx}(x\sin^2 x) - \sin^2 x = x\sin 2x$ **2**

(iii) Hence or otherwise, prove $\displaystyle\int_0^{\frac{\pi}{4}} x\sin 2x\, dx = \frac{1}{4}$ **2**

b) **6**




The diagram shows the graph of $x^2 = 4y$. The tangent to the parabola at $P(2p, p^2)$, $p > 0$, cuts the $x$ axis at $A$. The normal to the parabola at $P$ cuts the $y$ axis at $B$.

(i) Derive the equation of the tangent $AP$ **2**

(ii) Show that $B$ has coordinates $(0, p^2 + 2)$. **1**

(iii) Let $C$ be the midpoint of $AB$. Find the Cartesian equation of the locus of $C$. **3**





**QUESTION 5** (12 marks) Start a *new* page. **Marks**

a) The rate at which a drug is being expelled from the body at time $t$ hours is given by the equation $\dfrac{dM}{dt} = -k(M - 0.04)$ where $k$ is a constant and $M$ is measured in grams. **6**

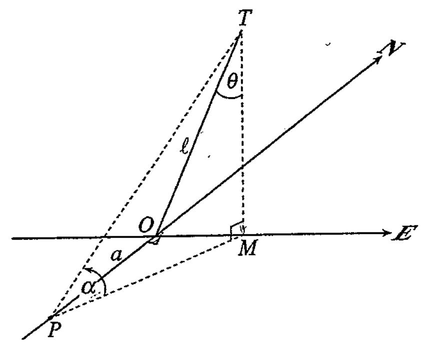
(i) Show that $M = 0.04 + M_0 e^{-kt}$, for some constant $M_0$, satisfies this equation.

(ii) Initially 4 grams was ingested. Find the value of $M_0$

(iii) After 10 hours, 1.6 grams was still present. Find the value of $k$

(iv) Show that the drug will never be entirely eliminated from the body.

b)




A pole, OT, of length $\ell$ m, stands on horizontal ground. The pole leans towards the east, making an angle of $\theta$ with the vertical. From $P$, $a$ m south of $O$ the elevation of $T$ is $\alpha$.

(i) Find expressions, in terms of $\ell$ and $\theta$, for $OM$ and $MT$

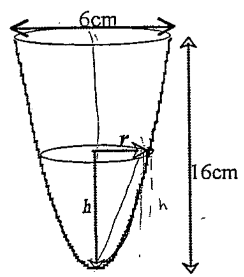
(ii) Prove that $PM = \ell\cos\theta\cot\alpha$.

(iii) Prove that $\ell^2 = \dfrac{a^2}{\cos^2\theta\cot^2\alpha - \sin^2\theta}$

(iv) Find the length of the pole, to the nearest m, if $a = 25$, $\theta = 20°$ and $\alpha = 24°$

**QUESTION 6** **(12 marks) Start a *new* page.** **Marks**

a) A wine glass is formed by rotating $y = ax^2$ around the $y$ axis.




The depth of liquid in the glass is $h$ and the radius at the top of the liquid is $r$.

(i) Find the value of $a$ **1**

(ii) Write an expression for $h$ in terms of $r$. **1**

(iii) Show that the volume of liquid in the glass when the depth is $h$ cm is $\frac{8\pi r^4}{9}$ **1**

(iv) Liquid is being added to the glass at the at a rate $3(15-h)$ ml per second. Find the rate at which the radius of the surface is increasing when $h = 10$cm. **3**

b) A particle moves in a straight line such that its displacement from a fixed point $O$ is given by ; **6**

$$x = \sqrt{3}\cos 3t - \sin 3t$$

(i) Show that $\ddot{x} = -n^2 x$

(ii) Express $x$ in the form $a\cos(nt+\alpha)$ and hence determine the period and amplitude of the motion.

(iii) Find the speed of the particle when it is 1m from $O$.

(iv) After how many seconds will the particle be 1m from $O$





**QUESTION 7** **(12 marks) Start a *new* page.** **Marks**

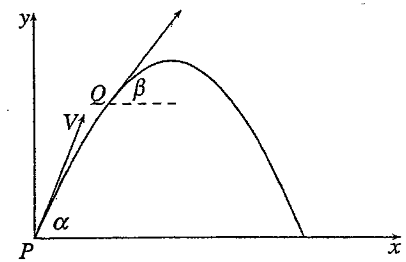
a) (i) Show that, in the binomial expansion of $\left(x - \frac{1}{x}\right)^{2n}$, the term independent of $x$ is $(-1)^n \, {}^{2n}C_n$ **2**

(ii) Show that $(1+x)^{2n}\left(1 - \frac{1}{x}\right)^{2n} \equiv \left(x - \frac{1}{x}\right)^{2n}$ $(1+x)^{2n}\left(\frac{x-1}{x^{2n}}\right)^{2n}$ **2**

(iii) Deduce that **2**

$$\left({}^{2n}C_0\right)^2 - \left({}^{2n}C_1\right)^2 + \left({}^{2n}C_2\right)^2 \ldots + \left({}^{2n}C_{2n}\right)^2 = (-1)^n \, {}^{2n}C_n$$

b) A particle is projected from a point $P$ on horizontal ground, with initial speed $V$ metres per second at an angle of elevation to the horizontal of $\alpha$.




Its equations of motion are $\ddot{x} = 0$, $\ddot{y} = -g$

(i) Derive expressions for its horizontal and vertical displacements from $P$ after $t$ seconds **2**

(ii) Determine the time of flight of the particle **2**

(iii) The particle reaches the point $Q$, as shown, where the direction of the flight makes an angle of $\beta$ with the horizontal. Show that the time taken to travel from $P$ to $Q$ is $\frac{VSin(\alpha-\beta)}{g\cos\beta}$ seconds **2**

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St. Catherine's School
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## August 2008

**TRIAL HIGHER SCHOOL CERTIFICATE EXAMINATION**

# Extension 1 Mathematics Solutions

*Time allowed:* *2 Hours + 5 mins Reading Time*

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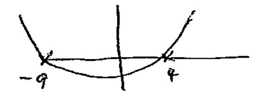
Course: MATHEMATICS EXTENSION 1

Marking Scheme for Task: HSC TRIAL EXAMINATION

Academic Year: 2007-8

| Solutions | Marks | Comments |
|---|---|---|
| Question 1: | | |
| a) $\frac{2x+5}{x-4} \le 1 \quad (x \ne 4)$ <br> $(x-4)(2x+5) \le (x-4)^2$ <br> $2x^2-3x-20 \le x^2-8x+16$ | | 1 for arriving this line |
| $x^2+5x-36 \le 0$ <br> $(x+9)(x-4) \le 0$   | | 1 for factorising & sketch |
| $-9 \le x \le 4$ <br> but $x \ne 4 \quad \therefore -9 \le x < 4$ * | 3 | 1 correct answer * (−0.5 if $-9 \le x \le 4$) |
| b) $\int_{-3}^{3} \frac{1}{9+x^2}\,dx = 2\int_0^3 \frac{1}{9+x^2}\,dx$ (even function) <br> $= 2 \cdot \frac{1}{3}\left[\tan^{-1}\frac{x}{3}\right]_0^3$ | | 1 correct primitive |
| $= \frac{2}{3}\left[\frac{\pi}{4}\right]$ <br> $= \frac{\pi}{6}$ | 2 | 1 correct answer |
| c) $\lim_{x\to 0} \frac{\sin x}{5x} = \frac{1}{5}\lim_{x\to 0}\frac{\sin x}{x}$ <br> $= \frac{1}{5}$ | 2 | 1 correct simplifying <br> ½ $\lim_{x\to 0}\frac{\sin x}{x} = 1$ <br> ½ correct answer |
| d) $\log_e\left(\frac{p}{r}\right) = \log\left(\frac{p}{q}\times\frac{q}{r}\right)$ line 1 <br> $= \log\frac{p}{q} + \log\frac{q}{r}$ line 2 <br> $= 3 + 1.6$ line 3 <br> $= 4.6$ | 2 | 1 mark for line 1 <br> 0.5 mark for line 2 <br> 0.5 line 3 |
| e) $\int_0^{1/2} 2x\sqrt{1-2x}\,dx$ <br> $u = 1-2x \quad \therefore 2x = 1-u$ <br> $du = -2dx \quad \therefore dx = -\frac{du}{2}$ <br> $x=0 \quad u=1$ <br> $x=\frac{1}{2} \quad u=0$ <br> → 1 | | |

| Solutions | Marks | Comments |
|---|---|---|

e) (continued)

$= -\int_1^0 (1-u)\sqrt{u}\,\frac{du}{2}$ line 1 — ½ mark for line 1

$= -\frac{1}{2}\int_1^0 \left(u^{1/2} - u^{3/2}\right) du$ line 2 — ½ mark for line 2

$= -\frac{1}{2}\left[\frac{2u^{3/2}}{3} - \frac{2u^{5/2}}{5}\right]_1^0$ line 3 — ½ mark for line 3

$= -\frac{1}{2}\left[0 - \left(\frac{2}{3} - \frac{2}{5}\right)\right]$ — ½ mark for line 4

$= \frac{2}{15}$ line 4 — **3** — (−0.5 for incorrect limits)

**Question 2** a) $\tan\theta = m \quad \tan\phi = 3$

$\theta - \phi = \frac{\pi}{4}$

$\therefore \tan(\theta - \phi) = 1 \quad \left[\tan\frac{\pi}{4} = 1\right]$

$\frac{\tan\theta - \tan\phi}{1 + \tan\theta\tan\phi} = 1$ * — 1 mark for * or $\square$

$\therefore \frac{m-3}{1+3m} = 1$ $\square$

$m - 3 = 1 + 3m$ — 1 mark for solving 'm'

$-2 = m$ — **2**

b) $f(x) = x^4 - x^3 + kx - 4$

If $x+1$ is a factor $f(-1) = 0$

$\therefore 1 + 1 - k - 4 = 0 \implies k = -2$ ← 1 mark for arriving to this line

$\therefore f(x) = x^4 - x^3 - 2x - 4$

Now $f(2) = 16 - 8 - 4 - 4 = 0$

$\therefore (x-2)$ is also a factor. — **2** — 1 mark for showing $(x-2)$ is also a factor

$\theta = \tan^{-1} m$ and $\phi = \tan^{-1} 3$

$\tan^{-1} m - \tan^{-1} 3 = \frac{\pi}{4}$

$\tan^{-1} m = \frac{\pi}{4} + \tan^{-1} 3$

$m = \tan\left(\frac{\pi}{4} + \tan^{-1} 3\right)$

$m = -2$



| Solutions | Marks | Comments |
|---|---|---|

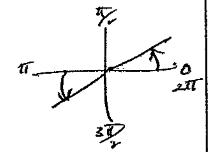
**Question 2** c) LHS $= \frac{2}{\cot x + \tan x} = \frac{2}{\frac{\cos x}{\sin x} + \frac{\sin x}{\cos x}}$ line 1 — ½ mark line 1

$= \frac{2\sin x\cos x}{\cos^2 x + \sin^2 x}$ line 2 — 1 mark line 2

$= \sin 2x$ line 3 — **2** — ½ mark for line 3

d) $\sqrt{3}\sin 2x = \cos 2x$

$\tan 2x = \frac{1}{\sqrt{3}}$ ✓✗ — "✓✗" means ½ marks




$2x = n\pi + \tan^{-1}\frac{1}{\sqrt{3}}$

$2x = n\pi + \frac{\pi}{6}$

$\therefore x = \frac{n\pi}{2} + \frac{\pi}{12} \quad n = 0, 1, 2, \ldots$ — **2**

e) $f(x) = \frac{\pi}{2} + 2\sin^{-1}\frac{2x}{3}$

Domain: $-1 \le \frac{2x}{3} \le 1$

$-3 \le 2x \le 3$

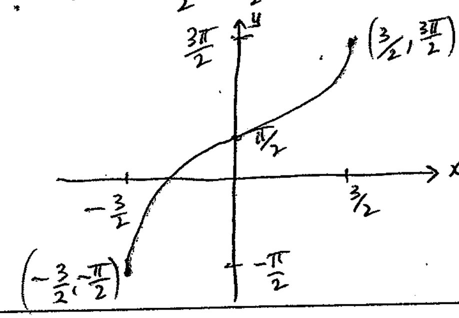
$-\frac{3}{2} \le x \le \frac{3}{2}$ ✓

Range: $-\frac{\pi}{2} \le \sin^{-1} x \le \frac{\pi}{2}$ — "✓" means 1 mark.

$-\pi \le 2\sin^{-1} x \le \pi \qquad y - \frac{\pi}{2}$

$-\frac{\pi}{2} \le \frac{\pi}{2} + 2\sin^{-1} x \le \frac{3\pi}{2}$

$\therefore -\frac{\pi}{2} \le \frac{\pi}{2} + 2\sin^{-1}\left(\frac{2x}{3}\right) \le \frac{3\pi}{2}$ ✓ — **2**




**2** — 1 for curve; ½ for clearly showing endpoints; ½ for correct y-intercept

| Solutions | Marks | Comments |
|---|---|---|

**Question 3** a) $1^2+3^2+5^2+\ldots+(2n-1)^2=\frac{1}{3}n(2n-1)(2n+1)$

① for $n=1$ LHS $=1^2=1$ RHS $=\frac{1}{3}\cdot 1\,(1)(3)=1$ — $\frac{1}{2}$ mark for ①

$\therefore$ true for $n=1$ — $\frac{1}{2}$ mark for ②

② assume true for $n=k$

ie. $1^2+3^2+5^2+\ldots+(2k-1)^2=\frac{1}{3}k(2k-1)(2k+1)$

③ Aim to prove true for $n=k+1$ if true for $n=k$

ie. ATP. $\underbrace{1^2+3^2+5^2+\cdots+(2k-1)^2}+(2k+1)^2=\frac{1}{3}(k+1)(2k+1)(2k+3)$ — 2 marks for ③

LHS $=\frac{1}{3}k(2k-1)(2k+1)+(2k+1)^2$

$=(2k+1)\left[\frac{1}{3}k(2k-1)+(2k+1)\right]$

$=(2k+1)\left[\frac{2k^2-k}{3}+2k+1\right]$

$=(2k+1)\left(\frac{2k^2-k+6k+3}{3}\right)$

$=\frac{1}{3}(2k+1)(2k^2+5k+3)$

$=\frac{1}{3}(2k+1)(2k+3)(k+1)=$ RHS

$\therefore$ true for $n=k+1$ if true for $n=k$

Since true for $n=1$ then by theory of mathematical induction true for all $n\geq 1$ — **3**

b) $\left(2x+\frac{1}{x^3}\right)^{10}$ $T_{k+1}={}^{10}C_k(2x^2)^{10-k}\left(\frac{1}{x^3}\right)^k$

$x^{20-2k-3k}=x^0 \Rightarrow 20-5k=0 \therefore k=4$ ✓ — ✓ means 1 mark

$T_5={}^{10}C_4(2x^2)^6\left(\frac{1}{x^3}\right)^4$ ✓✗ $=210\cdot 64\cdot x^{12}\cdot\frac{1}{x^{12}}$ — ✓✗ means 0.5 mark

$=13440$ ✓✗ or ${}^{10}C_4\,2^6$ — **2**



| Solutions | Marks | Comments |
|---|---|---|

**Question 3;** c) $Q(x)=ax^2+bx+c$

(i) sum of roots $=-\frac{b}{a}$ — **1**

(ii) remainder when $Q(x)$ divided by $(x-m)$

$Q(m)=am^2+bm+c$ ✓✗

remainder when $Q(x)$ divided by $(x-n)$ ✓✗

$Q(n)=an^2+bn+c$

now $am^2+bm+c=an^2+bn+c$

$am^2-an^2=-bm+bn$ ✓✗

$a(m-n)(m+n)=-b(m-n)$ ✓✗

$\therefore m+n=-\frac{b}{a}=$ sum of roots from part (i) — **2**

d) $f(x)=\frac{x-2}{x-1}$ — $f'(x)$ correct (1 mark)

(i) $f'(x)=\frac{(x-1)-(x-2)}{(x-1)^2}$ — $f'(x)\geq 0$ ($\frac{1}{2}$ mark)

$=\frac{1}{(x-1)^2}\geq 0$ for all $x$, $x\neq 1$ — $\frac{1}{2}$ mark for $\frac{1}{(x-1)^2}\geq 0$

$\therefore f(x)$ is increasing for all $x$. — **2**

(ii) let $y=\frac{x-2}{x-1}$

$\therefore x=\frac{y-2}{y-1}$ is inverse

$xy-x=y-2$

$xy-y=x-2$

$y(x-1)=x-2$

$\therefore y=\frac{x-2}{x-1}$ is inverse function — 1 for finding inverse

$\therefore f(x)$ is its own inverse — 1 mark.

$\therefore f(x)$ is symmetrical about $y=x$ — **2**

Course:



Marking Scheme for Task: Academic Year: 2007-8

| Solutions | Marks | Comments |
|---|---|---|
| **Question 4:** a) (i) $\int_0^{\pi/4} \sin^2 x\,dx = \int_0^{\pi/4} \frac{1}{2}(1-\cos 2x)\,dx$ ✓ $= \frac{1}{2}\left[x - \frac{1}{2}\sin 2x\right]_0^{\pi/4}$ ✓ $= \frac{1}{2}\left[\frac{\pi}{4} - \frac{1}{2}\right]$ ✓ $= \frac{\pi}{8} - \frac{1}{4}$ | 2 | |
| (ii) $\frac{d}{dx}(x\sin^2 x) - \sin^2 x$ $= \sin^2 x + x.2\sin x\cos x - \sin^2 x$ ✓✓ $= x\sin 2x$ ✓ | 2 | |
| (iii) $\int_0^{\pi/4} x\sin 2x\,dx$ $= \left[x\sin^2 x\right]_0^{\pi/4} - \int_0^{\pi/4}\sin^2 x\,dx$ ✓ $= \frac{\pi}{8} - \left(\frac{\pi}{8} - \frac{1}{4}\right)$ from part (i) ✓ $= \frac{1}{4}$ | 2 | |
| b) (i) $P(2p, p^2)$ $y = \frac{x^2}{4}$ $y' = \frac{x}{2}$ ✓ at $P$ $y' = p$ ✓ $y - p^2 = p(x-2p)$ $y - p^2 = px - 2p^2$ ✓ $y - px + p^2 = 0$ ✓ is tangent AP. | 2 | |



Course:

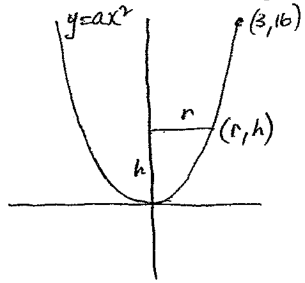


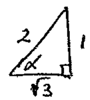
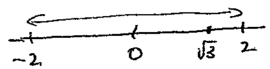
Marking Scheme for Task: Academic Year: 2007-8

| Solutions | Marks | Comments |
|---|---|---|
| b) (ii) normal: $y - p^2 = -\frac{1}{p}(x-2p)$ ✓ $py - p^3 = -x + 2p$ ✓ let $x=0$ $py - p^3 = 2p$ ✓ $py = 2p + p^3$ $y = 2 + p^2$ ✓ $\therefore B(0, 2+p^2)$ | 2 | |
| (iii) $A(p,0)$ $B(0,p^2+2)$ $\therefore C\left(\frac{p}{2}, \frac{p^2+2}{2}\right)$ ✓ ① $x = \frac{p}{2}$ ② $y = \frac{p^2+2}{2}$ from ① $p = 2x$ ✓ $\therefore$ in ② $y = \frac{4x^2+2}{2}$ ✓ $\therefore y = 2x^2+1$ ✓ is locus of C | 2 | |
| **Question 5:** a) (i) $M = 0.04 + M_0 e^{-kt}$ — ① $\frac{dM}{dt} = -kM_0 e^{-kt}$ but from ① $M_0 e^{-kt} = M - 0.04$ $\therefore \frac{dM}{dt} = -k(M - 0.04)$ | 2 | |
| (ii) when $t=0$ $M=4$ $\therefore 4 = 0.04 + M_0 \Rightarrow M_0 = 3.96$ | 1 | |
| (iii) when $t=10$ $M=1.6$ $\therefore M = 1.6 = 0.04 + 3.96e^{-10k}$ $1.56 = 3.96e^{-10k}$ $e^{-10k} = \frac{1.56}{3.96}$ | 2 | Full marks if wrong $M_0$ used |

| Solutions | Marks | Comments |
|---|---|---|
| Question 5 (continued) | | |
| $\therefore -10k = \ln\left(\frac{1.56}{3.96}\right)$ | | |
| $k = \frac{\ln\frac{1.56}{3.96}}{-10}$ | | |
| $= 0.0932$ (4 d.p.) | | |
| (iv) $M = 0.04 + 3.96e^{-0.093t}$ | | |
| as $t \to \infty$ $e^{-0.093t} \to 0$ | | |
| $\therefore M \to 0.04$ $\therefore M \neq 0$ | 1 | |
| thus never eliminated. | | |
| b) (i) $OM = l\sin\theta$ $MT = l\cos\theta$ | 1 | ½ each |
| (ii) $\tan\alpha = \frac{MT}{PM}$ | | ½ for ratio |
| $\therefore PM = \frac{MT}{\tan\alpha}$ | | |
| $= l\cos\theta \cdot \frac{1}{\tan\alpha}$ | | |
| $= l\cos\theta\cot\alpha$ | 2 | |
| (iii) in $\triangle POM$ $PM^2 - OM^2 = a^2$ (pythagoras) | | |
| $\therefore l^2\cos^2\theta\cot^2\alpha - l^2\sin^2\theta = a^2$ | | |
| $l^2(\cos^2\theta\cot^2\alpha - \sin^2\theta) = a^2$ | | |
| $\therefore l^2 = \frac{a^2}{\cos^2\theta\cot^2\alpha - \sin^2\theta}$ | 2 | |
| (iv) $l^2 = \frac{25^2}{\cos^2 20\cot^2 24 - \sin^2 20}$ | 1 | |
| $\therefore l \doteqdot 12$ | | |



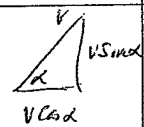
| Solutions | Marks | Comments |
|---|---|---|
| Question 6 $y = ax^2$ | | |
|   | | |
| (i) curve passes through $(3,16)$ $\therefore 16 = 9a$, $a = \frac{16}{9}$ | 1 | |
| (ii) curve passes through $(r,h)$ $\therefore h = ar^2$, $h = \frac{16}{9}r^2$ | 1 | |
| (iii) $V = \pi\int_a^b x^2\,dy$ note $y = \frac{16}{9}x^2$ $\therefore x^2 = \frac{9y}{16}$ | | |
| $= \pi\int_0^h \frac{9y}{16}\,dy$ | | |
| $= \pi\left[\frac{9y^2}{32}\right]_0^h$ | | |
| $= \frac{9\pi h^2}{32}$ but $h = \frac{16}{9}r^2$ $\therefore h^2 = \frac{16^2}{9^2}r^4$ | | |
| $\therefore V = \frac{9\pi}{32} \times \frac{16^2 r^4}{9^2}$ | 2 | |
| $= \frac{8\pi r^4}{9}$ | | |
| (iv) $\frac{dV}{dt} = 3(15-h)$ find $\frac{dr}{dt}$ | | ½ for $\frac{dV}{dt}$ |
| $\frac{dr}{dt} = \frac{dV}{dt} \times \frac{dr}{dV}$ $V = \frac{8\pi r^4}{9}$ | | |
| $\therefore \frac{dV}{dr} = \frac{32\pi r^3}{9}$ | 2 | |
| $= \frac{32}{9} \times \pi \times \left(\frac{3}{4}\sqrt{h}\right)^3$ | from (ii) | |
| $= \frac{3\pi\, 10\sqrt{10}}{2}$ | | |
| $= 15\sqrt{10}\,\pi$ when $h = 10$ | | |
| $= 15 \times \frac{1}{15\sqrt{10}\,\pi}$ | | |
| $= \frac{1}{\sqrt{10}\,\pi} \times \frac{\sqrt{10}}{\sqrt{10}}$ | | |
| $= \frac{\sqrt{10}}{10\pi}$ cm/s | | |

| Solutions | Marks | Comments |
|---|---|---|
| **Question 6 b) (i)** $x = \sqrt{3}\cos 3t - \sin 3t$<br>$\dot{x} = -3\sqrt{3}\sin 3t - 3\cos 3t$<br>$\ddot{x} = -9\sqrt{3}\cos 3t + 9\sin 3t$<br>$= -9(\sqrt{3}\cos 3t - \sin 3t)$<br>$\therefore \ddot{x} = -9x$ (S.H.M.) | 1 | |
| (ii) $x = 2\cos\left(3t + \frac{\pi}{6}\right)$ <br>amplitude $= 2$<br>period $= \frac{2\pi}{n} = \frac{2\pi}{3}$<br>Note: when $t=0$ $x=\sqrt{3}$<br>$x=0$ $\dot{x}=0$ $\therefore$ centre origin<br> | 2 | |
| (iii) when $x=1$ $\cos\left(3t+\frac{\pi}{6}\right) = \frac{1}{2}$<br>$3t+\frac{\pi}{6} = \frac{\pi}{3}, \frac{5\pi}{3}, \frac{7\pi}{3}, \ldots$<br>$3t = \frac{\pi}{6}, \frac{9\pi}{6}, \ldots$<br>$t = \frac{\pi}{18}, \frac{\pi}{2}, \ldots$<br>now speed $= \lvert -3\sqrt{3}\sin 3t - 3\cos 3t \rvert$<br>when $t = \frac{\pi}{18}$ $= \left\lvert -3\sqrt{3}\cdot\frac{1}{2} - \frac{3\sqrt{3}}{2} \right\rvert$<br>$= \left\lvert -\frac{3\sqrt{3}}{2} - \frac{3\sqrt{3}}{2} \right\rvert$<br>$= 3\sqrt{3}$ m/s | 2 | or use $V^2 = n^2(a^2 - x^2)$ |
| (iv) Particle is at $x=1$ when $t = \frac{\pi}{18}$ secs<br>then $\frac{\pi}{2}$ secs | 1 | |



| Solutions | Marks | Comments |
|---|---|---|
| **Question 7 (i)** $\left(x - \frac{1}{x}\right)^{2n}$ $T_{k+1} = {}^{2n}C_k\, x^{2n-k}\left(-\frac{1}{x}\right)^k$<br>$= {}^{2n}C_k\, x^{2n-k-k}\cdot(-1)^k$<br>for term independent of $x$ $2n-2k=0$<br>$\therefore k=n$<br>$\therefore T_{n+1} = {}^{2n}C_n(-1)^n$ | 2 | |
| (ii) $(1+x)^{2n}\left(1-\frac{1}{x}\right)^{2n} = \left[(1+x)\left(1-\frac{1}{x}\right)\right]^{2n}$<br>$= \left(1 - \frac{1}{x} + x - 1\right)^{2n}$<br>$= \left(x - \frac{1}{x}\right)^{2n}$ | 2 | |
| (iii) examine terms independent of $x$<br>LHS $= \left[\binom{2n}{0} + \binom{2n}{1}x + \binom{2n}{2}x^2 + \cdots + \binom{2n}{2n}x^{2n}\right]\left[\binom{2n}{0} - \binom{2n}{1}\left(\frac{1}{x}\right) + \binom{2n}{2}\left(\frac{1}{x^2}\right) - \cdots \binom{2n}{2n}\left(\frac{1}{x^{2n}}\right)\right]$<br>terms independent of $x$ are<br>$\binom{2n}{0}\binom{2n}{0} - \binom{2n}{1}\binom{2n}{1} + \binom{2n}{2}\binom{2n}{2} - \cdots \binom{2n}{2n}\binom{2n}{2n}$<br>RHS term independent of $x$ is $(-1)^n\,{}^{2n}C_n$ from (i)<br>$\therefore \binom{2n}{0}^2 - \binom{2n}{1}^2 + \binom{2n}{2}^2 - \cdots + \binom{2n}{2n}^2 = (-1)^n\,{}^{2n}C_n$. | 2 | must mention independent of $x$ terms for 1 mark |

| Solutions | Marks | Comments |
|---|---|---|
| Q7 b-(i) $\ddot{x} = 0$  $\ddot{y} = -g$ | | |




$\dot{x} = c$  $\dot{y} = -gt + c$

when $t=0$ $\dot{x} = V\cos\alpha$  when $t=0$ $\dot{y} = V\sin\alpha$

$\therefore c = V\cos\alpha$

$\dot{x} = V\cos\alpha$  $\therefore \dot{y} = -gt + V\sin\alpha$

(Comments: 2)

$x = Vt\cos\alpha + c$  $y = -\frac{gt^2}{2} + Vt\sin\alpha + c$

when $t=0$ $x=0 \therefore c=0$  when $t=0$ $y=0 \therefore c=0$

$\therefore x = Vt\cos\alpha$  $y = Vt\sin\alpha - \frac{1}{2}gt^2$  (Marks: 2)

(ii) at time of flight $y=0$

$\therefore Vt\sin\alpha - \frac{1}{2}gt^2 = 0$

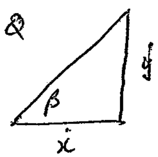
$2Vt\sin\alpha - gt^2 = 0$

$t(2V\sin\alpha - gt) = 0$

$\therefore t = 0, \frac{2V\sin\alpha}{g}$

$\therefore$ time of flight $= \frac{2V\sin\alpha}{g}$  (Marks: 2)

(iii) at Q




$\tan\beta = \frac{\dot{y}}{\dot{x}}$

$\frac{\sin\beta}{\cos\beta} = \frac{V\sin\alpha - gt}{V\cos\alpha}$

$V\sin\beta\cos\alpha = V\sin\alpha\cos\beta - gt\cos\beta$

$gt\cos\beta = V(\sin\alpha\cos\beta - \cos\alpha\sin\beta)$

$\therefore t = \frac{V\sin(\alpha-\beta)}{g\cos\beta}$ secs  (Marks: 2)