St. Catherine's School

Year 10 Mathematics
Semester 1 Examination
May 2006
Mr Maitland's Class

Time allowed: *2 hours + 5 minutes reading time*

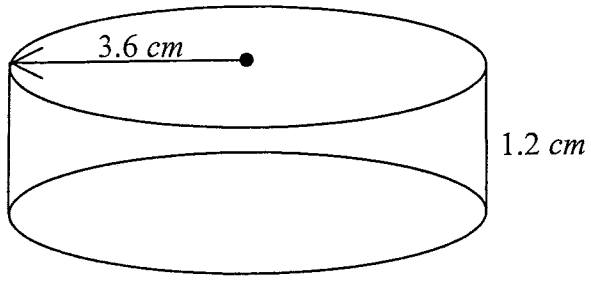
INSTRUCTIONS

- There are 3 sections in this paper.
- Complete all three sections
- Marks for each part of a question are indicated
- All questions should be attempted.
- All necessary working should be shown
- Start each section on a new page
- Approved scientific calculators and drawing templates may be used

SECTION	*1*	*2*	*3*	*TOTAL*
MARKS	/28	/55	/28	/111

Section 1 — *28 Marks*

1. Find the total surface area of the following closed cylinder, giving your answer correct to one decimal place. **3** MS 5.2.2



2. Write each of the following in index form: NS 5.3.1 & PAS 5.2.1

(a) $5\sqrt{h}$ **1**

(b) $\sqrt[3]{4}$ **1**

(c) $\dfrac{2}{3\sqrt{t}}$ **2**

3. Evaluate $100^{\frac{-2}{3}}$, correct to 2 significant figures **2** PAS 5.2.1

4. Write each of the following in surd form:

(a) $d^{\frac{2}{3}}$ **1**

(b) $(16g)^{-\frac{3}{4}}$ **2**

5. Expand and simplify $\left(3-5\sqrt{6}\right)^2$ **3**

6. Simplify the following:

NS 5.3.1

(a) $3\sqrt{96} - 2\sqrt{150} + \sqrt{24}$ **2**

(b) $2\sqrt{16m} - 4\sqrt{9m}$ **2**

(c) $\dfrac{-5\sqrt{8} \times 2\sqrt{90}}{10\sqrt{24}}$ **2**

7. Rationalise the denominator:

$$\frac{5\sqrt{2}}{2\sqrt{3}}$$ **2**

8. Mr Maitland loves designer shoes. However, they can be expensive. During the January sales, he found a pair of Marc Jacobs shoes that he has been wanting for a long time! **2**

NS 5.1.2

The original price was \$750, however Mr Maitland paid \$350.
Find the discount he received as a percentage of the original price.

9. A speed boat originally valued at \$220 000, depreciates at 5% p.a.

NS 5.2.2

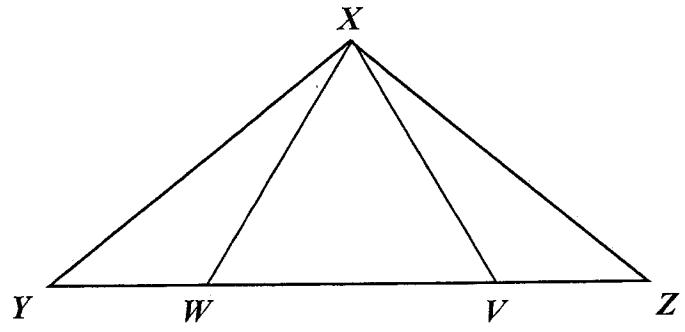
(a) Find the value of the speed boat after 5 years **2**

(b) Find the value of the speed boat after 5 years as a percentage of its original value. **1**

1. Factorise: **2** PAS 5.3.1

$$8g^2+14g+3$$

2. SGS 5.2.2



ΔXYZ is isosceles and $YW = VZ$.

(i) Prove that ΔXYW is congruent to ΔXZV, giving reasons for your answers. **4**

(ii) Prove that ΔXWV is isosceles, giving reasons for your answers. **2**

3. Factorise and simplify the following: PAS 5.3.1

(a) $\dfrac{d^2+4d+4}{d^2-d-6}$ **3**

(b) $\dfrac{a+2}{a^2+3a} \div \dfrac{a^2+2a}{a+3}$ **3**

(c) $\dfrac{6}{e^2-36} - \dfrac{5}{4e+24}$ **3**

4. Rationalise the denominator: **3**

$$\frac{6}{\sqrt{10}+2}$$

PAS 5.3.3

5. **(i)** Plot on a number plane the origin O and the points $A(-7,0)$, and $B(-9,3)$ **2**

(ii) Given that $M\left(\frac{-9}{2},6\right)$ is the midpoint of interval BC, find the coordinates of the point C and plot it on your number plane. **2**

(iii) Show that BC is perpendicular to BA. **2**

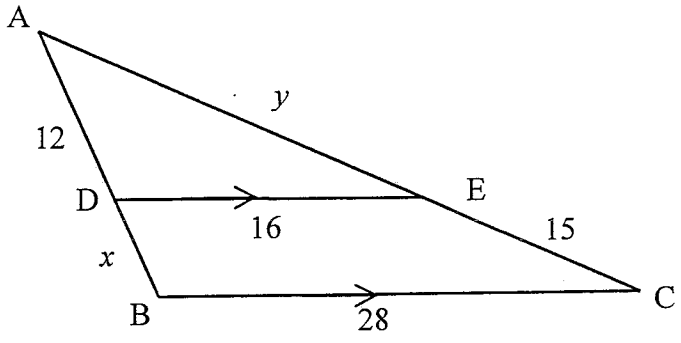
(iv) Show that the equation of the line BA in general form is $3x+2y+21=0$. **2**

(v) Does the point $L(-8,2)$ lie on the line BA? (Show working to prove that the point L does or does not lie on the line AB). **2**

(vi) Shade the region $3x+2y+21\leq 0$ on your number plane drawn in part (i) above **1**

SGS 5.2.2

6.



(a) Prove that ΔADE is similar to ΔABC, giving reasons for your answers. **3**

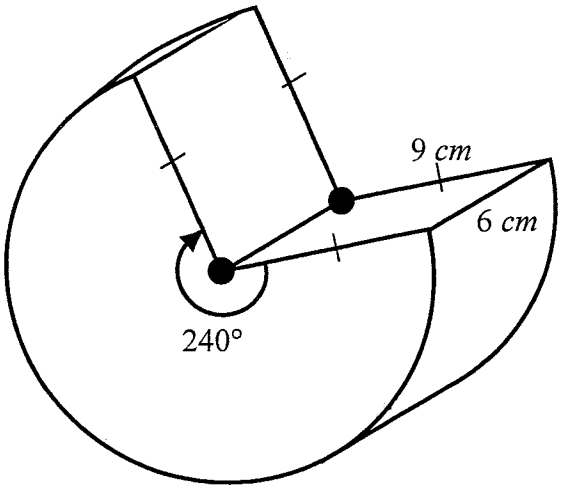
(b) Find the values of x and y, giving reasons for your answers. **4**

7. Solve the inequality $\frac{2-3x}{5}-2<x+7$ **3**

8. Find the values of x and y that satisfy both the equations: **3**

$$3x-2y=9$$
$$x+y=8$$

9. (i) Find the exact surface area of this solid. (ie leave in terms of π) **5**



(ii) Find the exact volume of the solid **3**

10. Faith is paddling her kayak and notices her friend Hope on the top of a cliff, 28 metres high. Hope quickly measures the angle of depression of Faith and records it as 25°.

(i) How far out from the base of the cliff is Faith? **2**

(ii) If Faith paddles in towards the cliff another 30 metres, what will Hope find to be the new angle of depression? **1**

Section 3 — *28 Marks*

1. Solve the following equations:

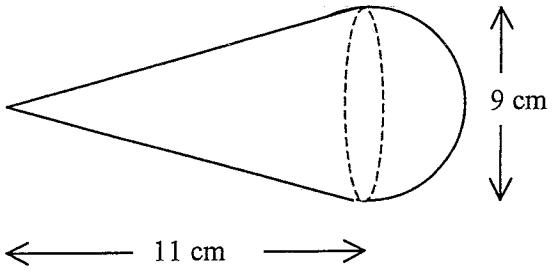
(a) $x^2 - 7x + 10 = 0$ **2**

(b) $x - \frac{4}{x} = 3$ **3**

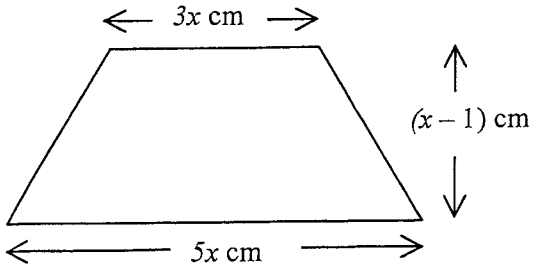
2. Solve the following equation, leaving your answer as a surd in simplest form:

$2x^2 - 4x - 3 = 0$ **3**

3. Find the volume of the following figure: (leave your answer in exact form) **4**



4. For what value(s) of x could the **area** of the trapezium below be 32 cm^2? **4**

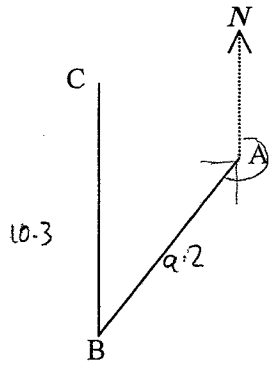


5. Find the value of x in the following:

(a) $\sin(2x° - 10°) = \cos 40°$ **2**

(b) $x = \dfrac{\sin(180° - \alpha)}{\sin(360° - \alpha)}$ **2**

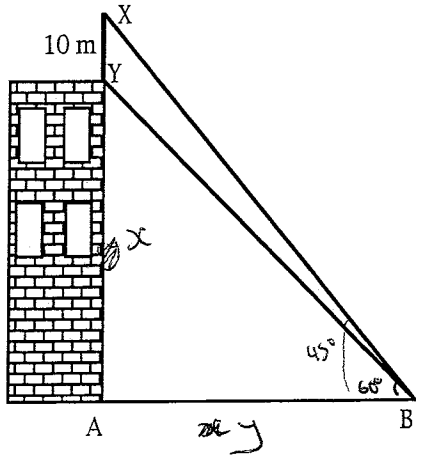
6. Verity sets off on a Duchess of Edinburgh hike from A on a bearing of 220°T, walking for 9.2 km to point B. She then heads directly north for 10.3 km, arriving at point C.



(i) Copy and complete the diagram above, showing all relevant information. **1**

(ii) How far is she from her starting point A? (to the nearest metre) **3**

7. XY is a 10 metre flagpole on top of a building x metres high.
If $\angle ABX = 60°$, $\angle ABY = 45°$, find the exact height of the building (leave your answer as a simple surd). **4**

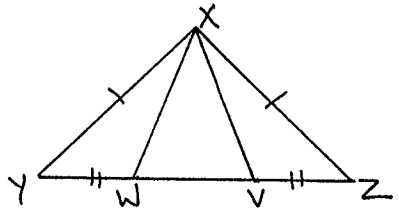


End of Examination

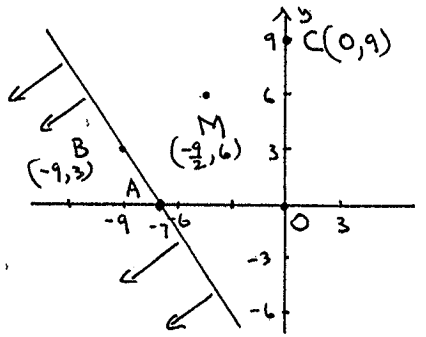
Qn	Solutions	Marks	Comments+Criteria
1	$SA = 2\pi r^2 + 2\pi rh$ $= 2\pi 3.6^2 + 2\pi . 3.6 . 1.2$ $= 81.4300... + 27.1433...$ $= 108.5734....$ $\doteq 108.6\ cm^2$	✓✓ ✓	1 one correct substitution 2 two correct subsn 3 correct value (IROE)
2	(a) $5\sqrt{h} = 5h^{\frac{1}{2}}$	✓	
	(b) $\sqrt[3]{4} = 4^{\frac{1}{3}}$	✓	
	(c) $\frac{2}{3\sqrt{t}} = \frac{2}{3}t^{-\frac{1}{2}}$ [$= 2.3^{-1}t^{-\frac{1}{2}}$]	✓✓	
3	$100^{-\frac{2}{3}} = \frac{1}{\sqrt[3]{100^2}} = 0.04641...$ $\doteq 0.046$ (2sf)	✓ ✓	
4	(a) $d^{\frac{2}{3}} = \sqrt[3]{d^2}$	✓	
	(b) $(16g)^{-\frac{5}{4}} = \frac{1}{8\sqrt[4]{g^3}}$	✓✓	
5	$(3-5\sqrt{6})^2 = 9 - 30\sqrt{6} + 150$ $= 159 - 30\sqrt{6}$	✓✓ ✓	
6	(a) $3\sqrt{96} - 2\sqrt{150} + \sqrt{24}$ $= 12\sqrt{6} - 10\sqrt{6} + 2\sqrt{6}$ $= 4\sqrt{6}$	✓ ✓	

Qn	Solutions	Marks	Comments+Criteria
6	(b) $2\sqrt{16m} - 4\sqrt{9m}$ $= 8\sqrt{m} - 12\sqrt{m}$ $= -4\sqrt{m}$	✓ ✓	
	(c) $\frac{-5\sqrt{8} \times 2\sqrt{90}}{10\sqrt{24}}$ $= \frac{-10\sqrt{2} \cdot 6\sqrt{10}}{20\sqrt{6}}$ $= \frac{-3\sqrt{20}}{\sqrt{6}} = -\sqrt{30}$	✓ ✓	[OR $\frac{-10\sqrt{90}}{10\sqrt{3}} = -\sqrt{30}$] 1 for not rationalising
7	$\frac{5\sqrt{2}}{2\sqrt{3}} = \frac{5\sqrt{2}}{2\sqrt{3}} \cdot \frac{\sqrt{3}}{\sqrt{3}}$ $= \frac{5\sqrt{6}}{6}$	✓ ✓	
8	Discount % $= \frac{750-350}{750} \times 100$ $= 53\frac{1}{3}\%$	✓ ✓	
9	(a) $V = 220\,000\,(0.95)^5$ $= \$170\,231.8063$ $\doteq \$170\,231.81$	✓ ✓	
	(b) $(0.95)^5 = 0.773....$ $\doteq 77.3\%$ of original $\doteq 77.4\%$	 ✓	

Section 2

Qn	Solutions	Marks	Comments+Criteria
1	$8g^2+14g+3$ $=(4g+1)(2g+3)$		1 start on factn (any technique) 2 correct factn
2	(i) RTP: $\triangle XYW \equiv \triangle XZV$ Proof:  in $\triangle$'s XYW, XZV $XY = XZ$ (given $\triangle XYZ$ isos) S	✓	
	$X\hat{Y}W = X\hat{Z}V$ (base $\angle$s isos $\triangle$ =) A	✓	
	$YW = VZ$ (given) S	✓	
	$\therefore \triangle XYW \equiv \triangle XZV$ (SAS)	✓	
	(ii) $XW = XV$ (corres sides in $\equiv$ $\triangle$'s XYW, XZV)	✓	
	$\therefore \triangle XWV$ isosceles (2 sides =)	✓	
3	(a) $\frac{d^2+4d+4}{d^2-d-6} = \frac{(d+2)^2}{(d+2)(d-3)}$	✓	
	$= \frac{d+2}{d-3}$	✓ ✓	

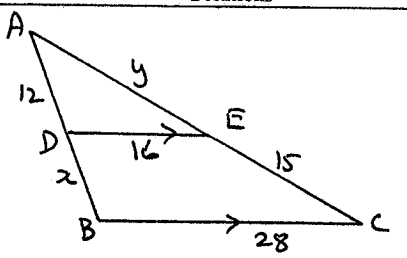
Section 2

Qn	Solutions	Marks	Comments+Criteria
3	(b) $\frac{a+2}{a^2+3a} \div \frac{a^2+2a}{a+3}$	•	
	$= \frac{(a+2)}{a(a+3)} \times \frac{(a+3)}{a(a+2)}$	✓ ✓	
	$= \frac{1}{a^2}$	✓	
	(c) $\frac{6}{e^2-36} - \frac{5}{4e+24}$		
	$= \frac{6}{(e+6)(e-6)} - \frac{5}{4(e+6)}$	✓	
	$= \frac{24 - 5(e-6)}{4(e+6)(e-6)}$	✓	
	$= \frac{54-5e}{4(e+6)(e-6)}$	✓	
4	$\frac{6}{\sqrt{10}+2} = \frac{6}{(\sqrt{10}+2)} \cdot \frac{(\sqrt{10}-2)}{(\sqrt{10}-2)}$	✓	
	$= \sqrt{10}-2$	✓ ✓	
5	(i)  	✓ ✓	A, B plots

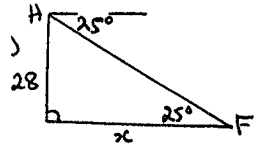
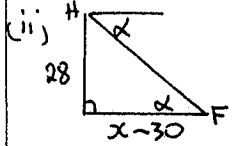
SECTION 2

Qn	Solutions	Marks	Comments+Criteria
5	(ii) if $M=(-\frac{9}{2},6)$ then C is $(0,9)$	✓✓	
	(iii) $m_{BC}=\frac{6}{9}=\frac{2}{3}$		
	$m_{BA}=\frac{-3}{2}$	✓	
	$m_{BC}\cdot m_{BA}=-1 \therefore BC\perp BA$	✓	
	(iv) $m_{BA}=-\frac{3}{2}$ through $(-7,0)$		1 test one point only
	$\therefore (y-0)=-\frac{3}{2}(x+7)$	✓	
	$2y=-3x-21$	✓	
	$3x+2y+21=0$ QED		
	(v) $L\equiv(-8,2)$		0 bald 'no'
	$3.-8+2.2+21$	✓	
	$=1\neq 0 \therefore$ not on line	✓	
	(vi) $3x+2y+21\leqslant 0$		
	test $(0,0)$,		
	$21\not\leqslant 0$ no	✓	correct shading
	$\therefore$ opposite side.		

Section 2

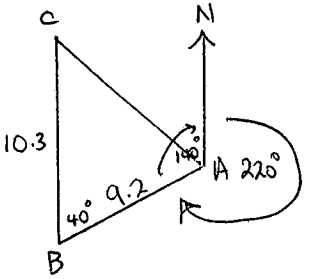
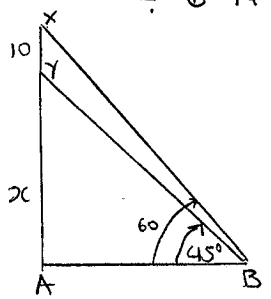
Qn	Solutions	Marks	Comments+Criteria
6	(a) 		
	RTP: $\triangle ADE \,///\, \triangle ABC$		
	Proof: in $\triangle ADE$, $\triangle ABC$		
	$\hat{A}=\hat{A}$ (common)	✓	
	$A\hat{D}E=A\hat{B}C$ (corres $\angle$s on // lines DE, BC)	✓	
	$A\hat{E}D=A\hat{C}B$ ($\angle$ sum $\triangle$)	✓	
	$\therefore \triangle ADE \,///\, \triangle ABC$ (AAA)		
	(b) as $\triangle ADE \,///\, \triangle ABC$		
	$\frac{12}{12+x}=\frac{16}{28}$		
	$84=4(12+x)$	✓	
	$12+x=21$	✓	
	$x=9$		
	$\frac{y}{y+15}=\frac{4}{7}$	✓	
	$7y=4y+60$		
	$3y=60$	✓	
	$y=20$		

Section 2

Qn	Solutions	Marks	Comments+Criteria
7	$\frac{2-3x}{5} - 2 < x+7$		
	$\frac{2-3x}{5} < x+9$	✓	
	$2-3x < 5x+45$	✓	
	$8x > -43$		
	$x > -\frac{43}{8}$	✓	
8	$3x-2y=9$, $x+y=8$, $8x+2y=16$	✓	
	$5x = 25$		
	$\therefore x=5 \quad y=3$	✓✓	
9	(i) $SA = 2(\pi.9^2).\frac{2}{3} + \frac{2}{3}(2.\pi.9).6$	✓✓	2 correct substⁿs
	$+ 2.9.6$		1 rectangles only
	$= 108\pi + 72\pi + 108$	✓✓	
	$= 180\pi + 108 \text{ cm}^2$	✓	
	(ii) $V = \frac{2}{3}.\pi.9^2.6 = 324\pi \text{ cm}^3$	✓✓	−1 for 1017.87602...
10	(i)  $\tan 25 = \frac{28}{x}$		
	$x = \frac{28}{\tan 25^\circ}$	✓	
	$= 60.046...$		
	$\div 60 \text{ m}$	✓	
	(ii)  $\tan\alpha = \frac{28}{30}$		
	$\alpha = \tan^{-1}\frac{28}{30}$		
	$= 43.025...$		
	$\div 43^\circ$	✓	

Qn	Solutions	Marks	Comments+Criteria
	SECTION 3		
1	(a) $x^2 - 7x + 10 = 0$		
	$(x-5)(x-2) = 0$	✓	
	$x = 2, 5$	✓	
	(b) $x - \frac{4}{x} = 3$		
	$x^2 - 3x - 4 = 0$	✓	
	$(x-4)(x+1) = 0$	✓	
	$x = 4, -1$	✓	
2	$2x^2 - 4x - 3 = 0$		
	$x = \frac{4 \pm \sqrt{4^2 + 4.2.3}}{4}$	✓	1 correct substⁿ
			2 unsimplif'd surd
	$= \frac{4 \pm \sqrt{40}}{4} = 4 \pm \frac{2\sqrt{10}}{4}$	✓	3 simple surd
	$= \frac{2 \pm \sqrt{10}}{2}$	✓	
3	$V_c = \frac{1}{3}\pi(4\frac{1}{2})^2.11$		
	$= \frac{297\pi}{4}$	✓	
	$V_{\frac{1}{2}sp} = \frac{1}{2}.\frac{4}{3}\pi(4\frac{1}{2})^3$	✓✓	
	$= \frac{243\pi}{4}$		
	$V_{total} = \left(\frac{297+243}{4}\right)\pi$	✓	−1 for decimal approx without exact value
	$= 135\pi \text{ cm}^3$		

Qn	Solutions	Marks	Comments+Criteria
4.	$A_{trap} = \frac{1}{2}(3x+5x)(x-1)$		
	$= 4x(x-1)$	✓	
	let $4x(x-1) = 32$		
	$\therefore x^2 - x = 8$	✓	
	$x^2 - x - 8 = 0$		
	$x = \frac{1 \pm \sqrt{1+32}}{2}$		
	$= \frac{1 \pm \sqrt{33}}{2}$	✓	
	$\therefore x$ could be $\frac{1+\sqrt{33}}{2}$	✓	
	as $x > 0$ and $\frac{1-\sqrt{33}}{2} < 0$		
5. (a)	$\sin(2x^\circ - 10^\circ) = \cos 40^\circ$		
	$\therefore 2x - 10 = 90 - 40$	✓	
	$2x = 60$		
	$x = 30$	✓	
(b)	$x = \frac{\sin(180-\alpha)}{\sin(360-\alpha)}$	✓	
	$= \frac{\sin\alpha}{-\sin\alpha} = -1$	✓	

Qn	Solutions	Marks	Comments+Criteria
6 (i)		✓	
(ii)	in $\triangle ABC$ $A\hat{B}C = 40^\circ$		
	$\therefore CA^2 = 10.3^2 + 9.2^2 - 2 \cdot 10.3 \cdot 9.2 \cos 40^\circ$	✓	1 correct subs^n
	$= 45.549\ldots$	✓	2 correct square
	$CA = 6.7490\ldots$	✓	3 correct distan
	$\doteq 6.749$ km		1 ROE
7.			
	in $\triangle ABY$ $AB = AY = x$		
	$\because A\hat{B}Y = 45^\circ$		
	$\therefore \tan 60^\circ = \frac{10+x}{x}$	✓	
	$\therefore \sqrt{3} = \frac{10+x}{x}$	✓	
	$x\sqrt{3} = 10 + x$		
	$x(\sqrt{3}-1) = 10$	✓	
	$x = \frac{10}{(\sqrt{3}-1)} \cdot \frac{(\sqrt{3}+1)}{(\sqrt{3}+1)}$		
	$= 5(\sqrt{3}+1)$ m	✓	