



St Catherine's
School
Waverley, Sydney

Student Number: _____

Year 11
Assessment Task 1
22/03/2007

Mathematics Extension I

Student Number

Time allowed:
55minutes

Reading time: NA

Course weighting:
25%

General Instructions

- Attempt ALL questions
- Write your Student NUMBER at the top of this page and on the writing paper used

Sections	Marks
Q1	/10
Q2	/12
Q3	/10
Q4	/13

Question 1 (10 marks)

Marks

- (a) Solve for x : $27^x = 9^{1-x}$ 2
- (b) Solve for n : $|3+2n| = n+1$ 3
- (c) Solve for x : $\frac{2x}{x-3} \leq 5$ 3
- (d) If $g(x) = \begin{cases} 3-2x & \text{if } x > 1 \\ 2x & \text{if } x \leq 1 \end{cases}$,
find $g(2) + g(-3)$. 2

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Question 2 (12 marks)

Marks

- (a) Find the domain & range and sketch each the following showing all the features:
- (i) $f(x) = \frac{3}{x+4}$ 3
- (ii) $y = \sqrt{25-x^2}$ 3
- (iii) $h(x) = |2x+1|$ 3
- (iv) $y = -4x^2 + 12x + 7$ 3

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Question 3 (10 marks)

Marks

(a) Sketch on separate axes and showing all their features:

(i) $y = \frac{x-1}{x^2-x-2}$

4

(b) Sketch the region defined by $y \geq x+1$, $y < x^2+x$

3

(c) Show that whether $f(x) = x^3 - x$ is an even, odd or even function.

3

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Question 4 (13 marks)

Marks

(a) Find the exact value of $\frac{\cos 45^\circ + \cos 60^\circ}{\sin 45^\circ}$

2

(b) Find the value of $\sin b = \cos(2b - 30)^\circ$

2

(c) The angle of elevation of the top A of a mountain AB from D is 13° . On advancing a distance of $1000m$ towards the mountain to C (over level ground), the angle elevation is 19° . If $AC = x$ metres and $AB = h$ metres.

(i) Draw a diagram to show the above information.

1

(ii) Show that $x = \frac{1000 \sin 13^\circ}{\sin 6^\circ}$

2

(iii) Hence, find the height h of the mountain to 3 significant figures.

2

(d) A hiker walks 10 km due west from P to Q and then proceeds 5 km in a direction 220° T to R . The distance between P and R is 13.8 km.

(i) Draw a diagram to represent the above information.

2

(ii) Find the bearing of the hiker from P to R .

2

✓ = 0.5 mark
 ✓ = 1 mark

Q1

(a) $27^x = 9^{1-x}$
 $(3^3)^x = (3^2)^{(1-x)}$ ✓
 $3^{3x} = 3^{2-2x}$ ✓
 $3x = 2-2x$ ✓
 $5x = 2$ ✓
 $x = \frac{2}{5}$ ✓

(b) $|3+2n| = n+1$

Case 1: $3+2n \geq 0$

$3+2n = n+1$
 $2n-n = 1-3$
 $n = -2$ ✓

Test $n = -2$

LHS = $|3+2(-2)|$
 $= |3-4|$
 $= |-1|$
 $= 1$

RHS = $-2+1$
 $= -1$

LHS \neq RHS
 $\therefore n = -2$ is not a solution

Case 2: $3+2n < 0$

$-(3+2n) = n+1$
 $-3-2n = n+1$
 $-3n = 4$
 $n = -\frac{4}{3}$ ✓

Test $n = -\frac{4}{3}$

LHS = $|3+2(-\frac{4}{3})|$
 $= |3-\frac{8}{3}|$
 $= |\frac{1}{3}|$ ✓
 $= \frac{1}{3}$

RHS = $n+1$
 $= -\frac{4}{3}+1$

$= -\frac{1}{3}$

LHS \neq RHS $\therefore n = -\frac{4}{3}$

\therefore There are no solutions

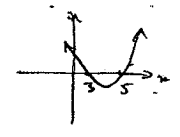
(c) $\frac{2x}{x-3} \leq 5$

$\frac{2x}{x-3} \leq 5$ $\times (x-3)^2$ ✓

$\frac{2x(x-3)^2}{(x-3)} \leq 5(x-3)^2$

$2x(x-3) \leq 5(x-3)^2$ ✓
 $2x^2 - 6x \leq 5(x^2 - 6x + 9)$
 $2x^2 - 6x - 5x^2 + 30x - 45 \leq 0$
 $-3x^2 + 24x - 45 \leq 0$
 -3

$x^2 - 8x + 15 \geq 0$ ✓
 $(x-5)(x-3) \geq 0$ ✓
 $x \geq 5$ ✓, $x < 3$ ✓



(d) $g(2) = 3-2(2)$
 $= 3-4$
 $= -1$

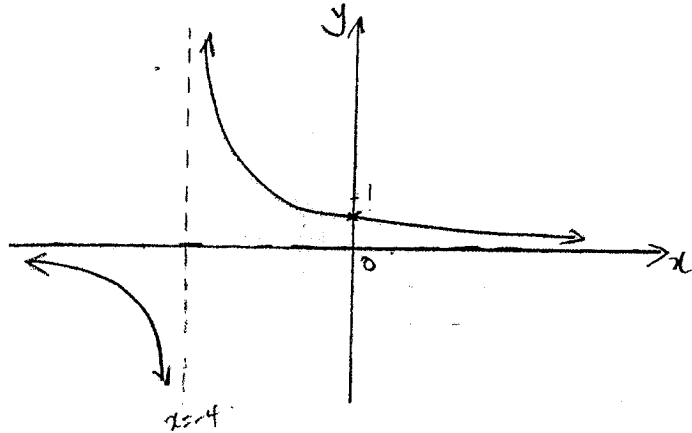
$g(-3) = 2(-3)$
 $= -6$ ✓

$\therefore g(2) + g(-3)$
 $= -1 + (-6)$ ✓
 $= -7$

Q2

$$f(x) = \frac{3}{x+4}$$

(a) D: $\forall \text{ real } x, x \neq -4$
R: $\forall \text{ real } y, y \neq 0$



$$x\text{-int} \rightarrow \text{let } y = 0$$

$$0 = \frac{3}{x+4}$$

$$0 = 3$$

no x-int.

$$y\text{-int}$$

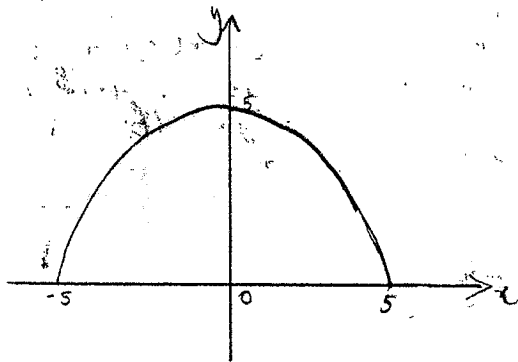
$$\text{let } x = 0$$

$$y = \frac{3}{4}$$

(b) $y = \sqrt{25 - x^2}$

D: $-5 \leq x \leq 5$

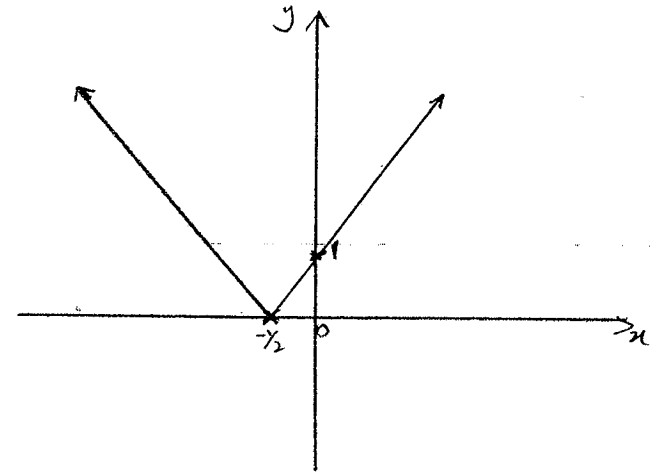
R: $0 \leq y \leq 5$



(iii) $h(x) = |2x+1|$

D: $\forall \text{ real } x$

R: $y \geq 0$



(iv) $y = -4x^2 + 12x + 7$

* x-int $\rightarrow \text{let } y = 0$

$$-4x^2 + 12x + 7 = 0$$

$$-(4x^2 - 12x - 7) = 0$$

$$-(2x-7)(2x+1) = 0$$

$$x = \frac{7}{2}, x = -\frac{1}{2}$$

* vertex $(\frac{3}{2}, 16)$

$$y = -4\left(-\frac{3}{2}\right)^2 + 12\left(\frac{3}{2}\right) + 7$$

$$y = -9 + 18 + 7$$

$$y = 16$$

* y-int $\rightarrow \text{let } x = 0$

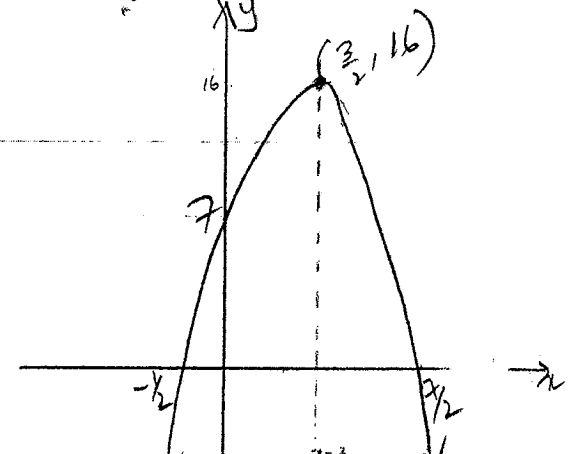
$$y = 7$$

* axis of symmetry

$$x = \frac{-b}{2a}$$

$$x = \frac{-12}{-8}$$

$$x = \frac{3}{2}$$



Q3

(a) $y = \frac{x-1}{x^2-x-2}$

$y = \frac{x-1}{(x+1)(x-2)}$

* x-int - let $y=0$
 $x-1=0$
 $x=1$

* y-int - let $x=0$
 $y = \frac{1}{2}$

* Vertical asymptote
 $x = -1, x = 2$

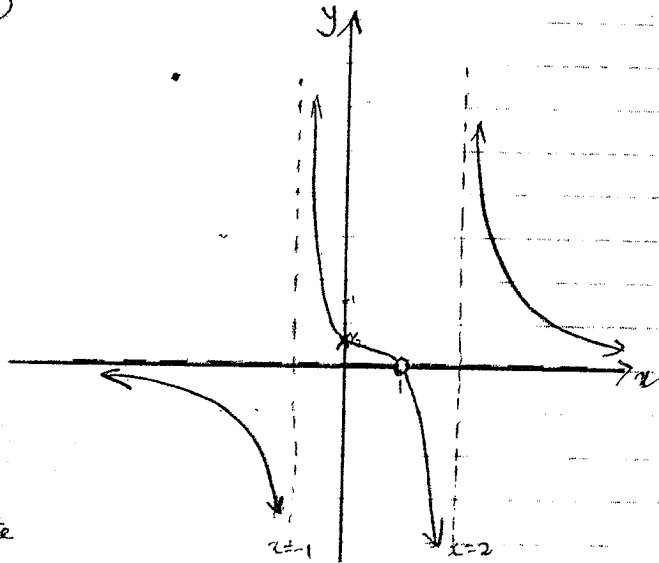
* Horizontal asymptote

$\lim_{x \rightarrow \infty} \frac{\frac{x}{x^2} - \frac{1}{x^2}}{\frac{x^2}{x^2} - \frac{x}{x^2} - \frac{2}{x^2}}$

$\lim_{x \rightarrow \infty} \frac{\frac{1}{x} - \frac{1}{x^2}}{1 - \frac{1}{x} - \frac{2}{x^2}}$

$y = 0$

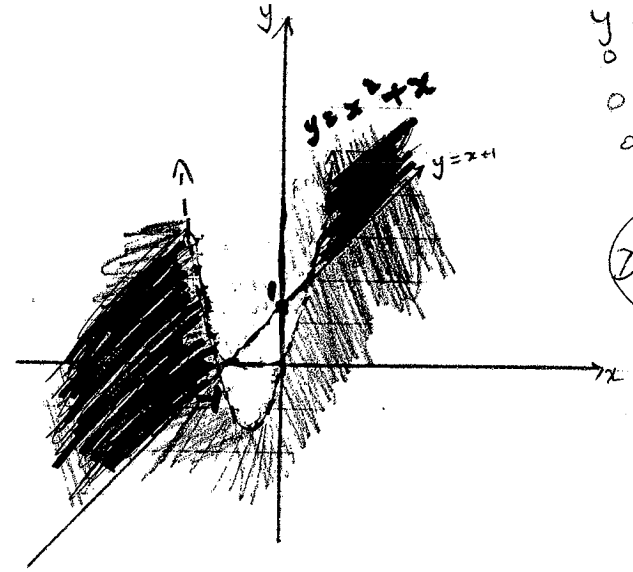
Sign of y



Q3

(b) $y \geq x+1$ & $y < x^2+x$

$y \geq x+1$
 07/07
 07/1X



Test (0,0)

$y < x^2+x$
 $0 < (0)^2 + (0)$
 $0 < 0.25 + 1$
 $0 < \frac{5}{4}$

Darkest region is the solution

(c) $f(x) = x^3 - x$

EVEN: $f(x) = f(-x)$

$f(x) = x^3 - x$

$f(-x) = (-x)^3 - (-x)$
 $= -x^3 + x$

$f(x) \neq f(-x) \therefore$ not even

ODD: $f(-x) = -f(x)$

$f(-x) = -x^3 + x$

$-f(x) = -(x^3 - x)$
 $= -x^3 + x$

$f(-x) = -f(x)$

$\therefore f(x) = x^3 - x$ is an odd function

Q4

$$(a) \frac{\cos 45^\circ + \cos 60^\circ}{\sin 45^\circ}$$

$$= \frac{\frac{1}{\sqrt{2}} + \frac{1}{2}}{\frac{1}{\sqrt{2}}} \quad \checkmark$$

$$= \frac{\frac{2 + \sqrt{2}}{2\sqrt{2}}}{\frac{1}{\sqrt{2}}} \quad \checkmark$$

$$= \frac{2 + \sqrt{2}}{2\sqrt{2}} \times \frac{\sqrt{2}}{1} \quad \checkmark$$

$$= \frac{2 + \sqrt{2}}{2} \text{ or } 1 + \frac{\sqrt{2}}{2}$$

$$(b) \sin b = \cos(2b - 30^\circ)$$

* Using the result:

$$\sin \theta = \cos(90^\circ - \theta) \quad \checkmark$$

$$90 - b = 2b - 30 \quad \checkmark$$

$$b = 90^\circ - (2b - 30^\circ) \quad \checkmark$$

$$b = 90^\circ - 2b + 30^\circ \quad \checkmark$$

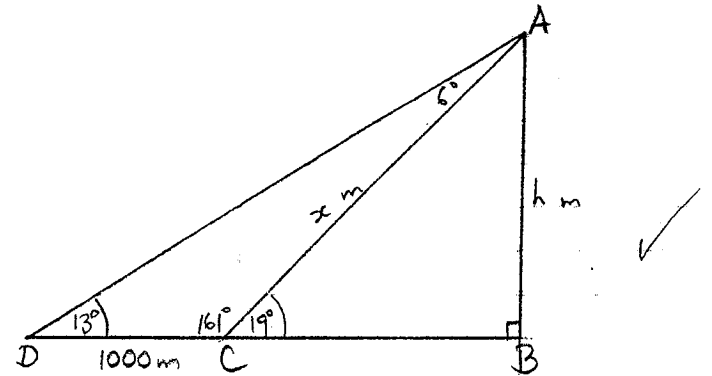
$$b = 120^\circ - 2b \quad \checkmark$$

$$\therefore 3b = 120^\circ \quad \checkmark$$

$$b = 40^\circ \quad \checkmark$$

Q4 (c)

(i)



$$(ii) \angle DCA = 180^\circ - 19^\circ \text{ (supp. } \angle\text{s) or (straight line)} \\ = 161^\circ \quad \checkmark$$

$$\angle DAC = 180^\circ - (161^\circ + 13^\circ) \text{ (} \angle \text{sum of a } \Delta\text{)} \\ = 6^\circ \quad \checkmark$$

$$\frac{1000}{\sin 6^\circ} = \frac{x}{\sin 13^\circ} \quad \checkmark$$

$$x \sin 6^\circ = 1000 \sin 13^\circ \quad \checkmark$$

$$\therefore x = \frac{1000 \sin 13^\circ}{\sin 6^\circ} \quad \checkmark$$

$$(iii) \sin 19^\circ = \frac{h}{x} \quad \checkmark$$

$$\sin 19^\circ = \frac{h}{\frac{1000 \sin 13^\circ}{\sin 6^\circ}} \quad \checkmark$$

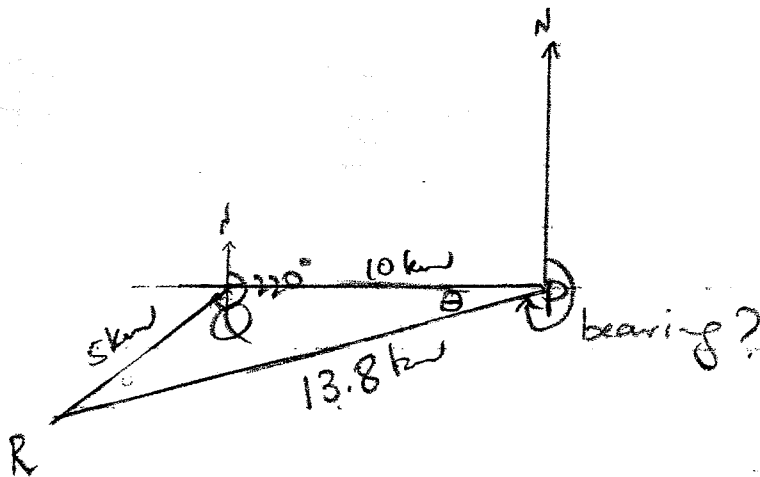
$$\therefore h = \frac{1000 \sin 13^\circ}{\sin 6^\circ} \times \sin 19^\circ \quad \checkmark$$

$$= 700.64073 \dots \quad \checkmark$$

$$= 700 \text{ m (to 3 sig. fig.)} \quad \checkmark$$

Q4

(d)



$$\begin{aligned}5^2 &= 10^2 + 13.8^2 - 2 \times 10 \times 13.8 \cos \theta \\25 &= 100 + 190.44 - 276 \cos \theta \\25 &= 290.44 - 276 \cos \theta\end{aligned}$$

$$276 \cos \theta = 290.44 - 25$$

$$\cos \theta = \frac{265.44}{276}$$

$$\therefore \theta = 15^\circ 54' 1.64'' \checkmark$$

$$\therefore \theta = 16^\circ \text{ (nearest degree)}$$

OR $\frac{\sin \frac{130^\circ}{5}}{\sin \theta} = \frac{13.8}{\sin 130^\circ}$

$$\therefore \sin \theta = \frac{5 \sin 130^\circ}{13.8}$$

$$\therefore \theta = 16^\circ$$

\therefore The bearing of R from P is $270^\circ - 16^\circ$
 $= 254^\circ \text{ T} \checkmark$