YR11-E1 - ST CATHERINES SCHOOL - 2005 (JUNE ASSESSMENT)

TOPICS: QUADRATICS, LOCUS, SKETCHING REGIONS

## START A NEW PAGE

1. Sketch the following regions

(a) $xy \leq 1$

(b) $4 \leq x^2 + y^2 < 16$

4marks

2. Describe the roots of the following equations as unreal, real, distinct, equal, or rational:

(a) $5x^2 - 4x - 7 = 0$

(b) $4x^2 + 7x + 3 = 0$.

4m

3. Solve the equation for x: $4^{2x} - 5.4^x + 4 = 0$. 4m

4. Given the equation $x^2 - 3x + 5 = 0$, with roots α and β

Evaluate the following:

(a) $\alpha + \beta$

(b) $\alpha\beta$

(c) $\alpha^2 + \beta^2$

(d) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha}$

(e) $(\alpha - \beta)^2$, wh~~ere $\alpha > \beta$.~~

8m

4. Consider the equation $x^2 - mx + 3m = 0$.

Find the value of m such that the equation

(a) is positive definite

(b) has roots which are reciprocals of each other

(c) has one root equal to 2.

6 marks

5. Show that the expression $x^2 - px - 7 = 0$

has real and different roots for all real p.

3 marks

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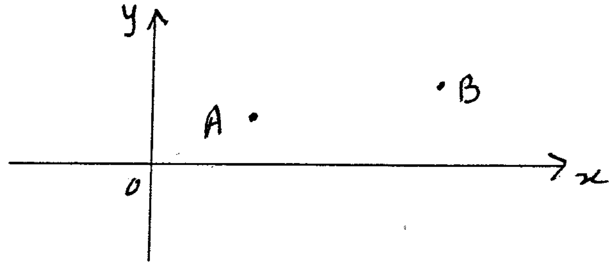
6. If $2x^2 - 7x - 4 \equiv a(x-b)(x-c)$ for all values of x, find a, b and c. 4m

7. Copy the diagram above and Sketch roughly the locus of points P(x,y) such that P is equidistant from the points A and B shown.




1m

8. State the equation of the locus of points P(x,y) which moves such that it is equidistant 6 units from the fixed point A(2,3).

2m

9. State the equation of the parabola with
(a) Vertex (0,0) and directrix y=-2
(b) Focus (-2,0) and directrix x=2
(Hint draw a sketch.) 4m

10. Given the parabola $(x-3)^2 = 8(y+1)$
State the
(a) Focal length
(b) Coordinates of the vertex
(c) Coordinates of the focus
(d) Equation of the directrix
(e) Length of the latus rectum
(f) sketch the parabola, showing above features. 6m

11. Derive the equation of the locus of points P(x,y) which moves such that the distance from P to A(2,3) is always twice the distance from P to B(-4, 7).
Show that the equation can be simplified to
$3x^2 + 36x + 3y^2 - 50y + 247 = 0$.

Hence show that this locus is a circle and state the centre and radius.

6m

**END**

Yr 11 - Ext 1 Maths Test 1, Term 2, 2005 Solutions

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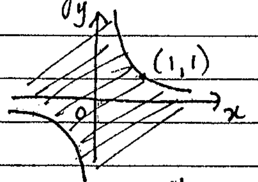
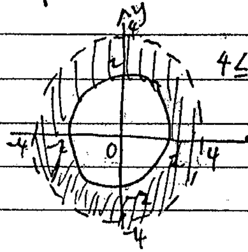
1. a) $xy \le 1$




(2)

b)




$4 \le x^2+y^2 < 16$

(2)

2. a) $5x^2-4x-7=0$

$\Delta = (-4)^2 - 4.5.(-7)$

$= 156$

$> 0$

$\therefore$ roots real, distinct (irrat.)

(2)

b) $4x^2+7x+3=0$

$\Delta = 7^2 - 4.4.3$

$= 1$

$> 0$ and a square

$\therefore$ roots real, distinct, rational

(2)

3. $4^{2x} - 5.4^x + 4 = 0$

Let $A = 4^x$.

$\therefore A^2 - 5A + 4 = 0$

$(A-4)(A-1) = 0$

$A = 4$ or $1$

$\therefore 4^x = 4$ or $4^x = 1$

$x = 1$ or $0$

(4)

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4(i) $x^2 - 3x + 5 = 0$

a) $\alpha+\beta = \frac{-b}{a} = \frac{3}{1} = 3$ (1)

b) $\alpha\beta = \frac{c}{a} = \frac{5}{1} = 5$ (1)

c) $\alpha^2+\beta^2 = (\alpha+\beta)^2 - 2\alpha\beta$

$= 3^2 - 2(5)$

$= -1$

(2)

d) $\frac{\alpha}{\beta} + \frac{\beta}{\alpha} = \frac{\alpha^2+\beta^2}{\alpha\beta}$

$= \frac{-1}{5}$

(2)

e) $(\alpha-\beta)^2 = \alpha^2 - 2\alpha\beta + \beta^2$

$= -1 - 2(5)$

$= -11$

(2)

4(ii) $x^2 - mx + 3m = 0$

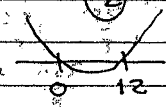
a) $\Delta < 0$ and $a > 0$

$\Delta = (-m)^2 - 4.1.3m$

$= m^2 - 12m$

$m^2 - 12m < 0$

$m(m-12) < 0$




$\therefore 0 < m < 12$

(2)

b) $\frac{c}{a} = 1$

$\frac{3m}{1} = 1$

$m = \frac{1}{3}$

(2)

c) $2^2 - m(2) + 3m = 0$

$4 + m = 0$

$m = -4$

(2)

5. $x^2 - px - 7 = 0$ Show $\Delta > 0$

$\Delta = (-p)^2 - 4.1.(-7)$

$= p^2 + 28$

$\geq 28$

$> 0$ for all $p$ $\therefore$ Real

(3)

Yr 11 - Ext 1 Test 1, Term 2, 2005 Solutions cont'd.

6. $2x^2 - 7x - 4 \equiv a(x-b)(x-c)$

$\equiv a(x^2 - bx - cx + bc)$

$\equiv ax^2 + x(-b-c)a + abc$

Equating coefficients:

$a = 2$, $a(-b-c) = -7$, $abc = -4$

$\therefore 2(b+c) = 7$, $2bc = -4$

$\therefore 2b = 7 - 2c$, $2bc = -4$

$\therefore (7-2c)c = -4$

$\therefore 7c - 2c^2 = -4$

$\therefore 2c^2 - 7c - 4 = 0$

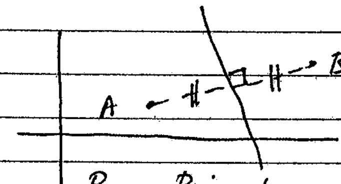
$(c-4)(2c+1) = 0$

$\therefore \{c = 4$ or $c = -\frac{1}{2}\}$

$\therefore \{b = -\frac{1}{2}$ or $b = 4\}$ & $a = 2$.

(4)

7.




Perp. Bisector

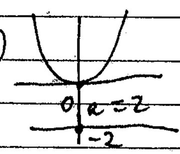
(1)

8. Circle $r = 6$ Centre $(2,3)$

$\therefore (x-2)^2 + (y-3)^2 = 36$

(2)

9. a)

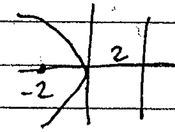



$x^2 = 4ay$

$x^2 = 8y$

(2)

b)

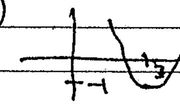



$y^2 = -4ax$

$y^2 = -8x$

(2)

10. $(x-3)^2 = 8(y+1)$



a) $a = 2$ (1)

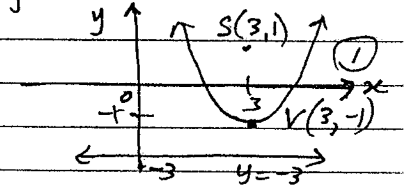
b) $(3,-1)$ (1)

c) $(3,1)$ (1)

d) $y = -3$ (1)

e) Length LR = 8 (1)

f)




(1)

11. $PA = 2PB$

$\sqrt{(x-2)^2+(y-3)^2} = 2\sqrt{(x+4)^2+(y-7)^2}$

$x^2-4x+4+y^2-6y+9$

$= 4(x^2+8x+16+y^2-14y+49)$

$x^2-4x+y^2-6y+13 = 4x^2+32x+4y^2-56y+196$ $(+64)$

$0 = 3x^2+36x+3y^2-50y+247$

$\therefore 0 = x^2+12x+y^2-\frac{50}{3}y+\frac{247}{3}$

$\therefore (x^2+12x+36)+y^2-\frac{50}{3}y+\left(\frac{25}{3}\right)^2$

$= -\frac{247}{3} + 36 + \frac{625}{9}$

$(x+6)^2 + \left(y-\frac{25}{3}\right)^2 = \frac{208}{9}$ or 23

$\therefore$ Centre $\left(-6, \frac{25}{3}\right)$ rad $= \frac{4\sqrt{13}}{3}$

YUK....

(6)

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