St Catherine's School

Year: 11
Subject: Extension 1 Course
Time allowed: 55 minutes

Date: February 2004

Name: Winnie Yao

Maths Teacher: Dr. Bose

Directions to candidates:

- All questions are to be attempted.
- Marks may be deducted for careless or badly arranged work
- All necessary **working** must be shown in every question.
- The answers are to be written on blank examination paper that is provided.
- Approved calculators and geometrical instruments are required.
- Hand in your work in **2 bundles**:
  Section A, and Section B separately

TEACHER'S USE ONLY
Total Marks

Topics	Total
Arithmetic, Algebra & Equations	25/26
Functions and Relations	12½/14
TOTALS	37½/40

SECTION A

Marks

1. If $\left(3\sqrt{12} - 2\sqrt{5}\right)^2 = p + q\sqrt{15}$, find p and q. 2

2. Rationalise the denominator 2

$$\frac{3\sqrt{5}}{2 + \sqrt{5}}$$

3. Convert $0.\dot{2}\dot{4}$ to a fraction in its simplest form. Show all working. 2

4. Solve for x: 1

$$27^{2x-1} = 3^{4-x}$$

4. Find all values of x for which

(a) $6 + 5x - x^2 \geq 0$ 2.5

(b) $-1 \leq 4 - x < 3$ 2

(c) $\dfrac{2x-3}{x-2} \leq 1$ 3

5. Simplify 3.5

$$\frac{x^3 - x^2 - 9x + 9}{2x^2 - 5x - 3} \div \frac{2x^2 - x - 1}{4x^2 - 1}$$

6. Solve for x and y: 3

xy = 1 and x - 3y = 2

7. Is the function $f(x) = \dfrac{x}{x^2 + 1}$ even, odd or neither? 2

8. Find and confirm all values of x that satisfy 3

$$|x + 1| = 2x - 1$$

SECTION B

Marks

9. (a) Sketch $y = |2x - 3|$ showing x and y intercepts — 2

(b) Hence or otherwise solve $|2x - 3| < 5$ for x. — 2

10. Sketch $y = f(x)$ for the function below. — 3

$$f(x) = \begin{cases} -1 & for\ x < -2 \\ \sqrt{4 - x^2} & for\ -2 \le x \le 2 \\ x - 2 & for\ x > 2 \end{cases}$$

11. (a) Sketch the parabola $y = 3 + 2x - x^2$ showing x and y intercepts. — 2

(b) State the domain and range of the above function. — 2

(c) Using the above or otherwise, state the domain and range of — 3

$$y = \frac{1}{\sqrt{3 + 2x - x^2}}$$

YEAR 11 EXTENSION 1 — FEB '04.

SOLUTIONS

1) $(3\sqrt{12} - 2\sqrt{5})^2$

$= 9\times 12 - 2\times 3\sqrt{12}\times 2\sqrt{5} + 4\times 5$

$= 108 - 12\times\sqrt{60} + 20$

$= 128 - 24\sqrt{15}$ 2

$\therefore \underline{\underline{p = 128 \quad q = -24}}$

2) $\dfrac{3\sqrt{5}}{2+\sqrt{5}} \times \dfrac{2-\sqrt{5}}{2-\sqrt{5}}$

$= \dfrac{6\sqrt{5} - 15}{4-5}$

$= \dfrac{6\sqrt{5} - 15}{-1}$

$= \underline{\underline{15 - 6\sqrt{5}}}$ 2

3) Let $x = 0.242424\ldots$ Q.4

$100x = 24.242424\ldots$

$100x - x = 24$

$99x = 24$

$x = \dfrac{24}{99}$

$x = \dfrac{8}{33}$ 2

$\therefore 0.\dot{2}\dot{4} = \underline{\underline{\dfrac{8}{33}}}$

$3^{3(2x+1)} = 3^{4+x}$

$3(2x+1) = 4+x$

$6x+3 = 4+x$

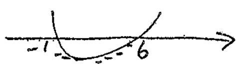
$7x = 7$

$x = 1$ 1

4) (a) $6 + 5x - x^2 \geq 0$

$x^2 - 5x - 6 \leq 0$

$(x-6)(x+1) \leq 0$



2.5

$\underline{\underline{-1 \leq x \leq 6}}$

4) (b) $-1 \leq 4 - x < 3$

$-5 \leq -x < -1$

$5 \geq x > 1$

$\underline{\underline{1 < x \leq 5}}$ 2

(c) $\dfrac{2x-3}{x-2} \leq 1 \qquad x \neq 2$

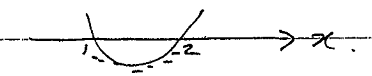
$(x-2)^2 \times \dfrac{(2x-3)}{x-2} \leq (x-2)^2$

$(2x-3)(x-2) - (x-2)^2 \leq 0$

$(x-2)[2x-3-(x-2)] \leq 0$

$(x-2)(2x-3-x+2) \leq 0$ 3

$(x-2)(x-1) \leq 0$



$\underline{1 \leq x < 2}$

5) $\dfrac{x^3 - x^2 - 9x + 9}{2x^2 - 5x - 3} \div \dfrac{2x^2 - x - 1}{4x^2 - 1}$

$= \dfrac{x^2(x-1) - 9(x-1)}{(2x+1)(x-3)} \times \dfrac{(2x-1)(2x+1)}{(2x+1)(x-1)}$

$= \dfrac{(x^2-9)\cancel{(x-1)}}{(2x+1)(x-3)} \times \dfrac{(2x-1)\cancel{(2x+1)}}{\cancel{(2x+1)}\cancel{(x-1)}}$

$= \dfrac{\cancel{(x-3)}(x+3)(2x-1)}{(2x+1)\cancel{(x-3)}}$ $3\frac{1}{2}$

$= \dfrac{(x+3)(2x-1)}{2x+1}, \quad x \neq 1, -\frac{1}{2}, 3$

(6) $xy = 1 \qquad x - 3y = 2.$

$$y = \frac{1}{x}$$

$$x - 3\left(\frac{1}{x}\right) = 2.$$

$$x^2 - 3 = 2x.$$

$$x^2 - 2x - 3 = 0$$

$$(x-3)(x+1) = 0$$

$x = 3$ or -1

if $x = 3$, $y = \frac{1}{3}$

if $x = -1$, $y = -1$

3

(7) $f(x) = \dfrac{x}{x^2+1}$

$$f(-x) = \frac{-x}{(-x)^2+1}$$

$$= \frac{-x}{x^2+1}$$

$$= -f(x)$$

$\therefore f(x)$ is odd.

2

(8) $|x+1| = 2x - 1.$

$x + 1 = 2x - 1$ or $-(x+1) = 2x - 1$

$x = 2$ $\qquad -x - 1 = 2x - 1$

$x = 0$

Test $x = 2$

LHS $= |3|$

$= 3$

RHS $= 2(2) - 1$

$= 3$

$= $ LHS

$\therefore x = 2$ is a soln.

Test $x = 0$.

LHS $= |0+1|$

$= 1$

RHS $= 2(0) - 1$

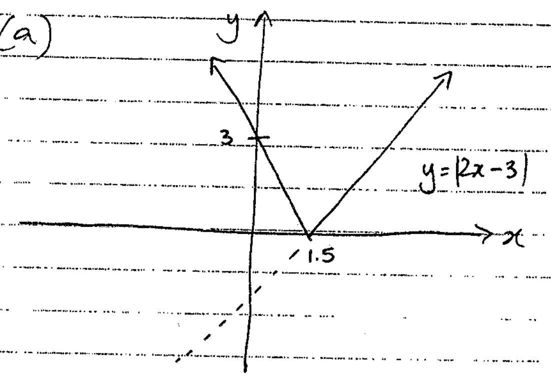
$= -1$

$\neq$ LHS

$\therefore x = 0$ is not a soln.

$\therefore x = 2$ only

(9) (a)



sketching

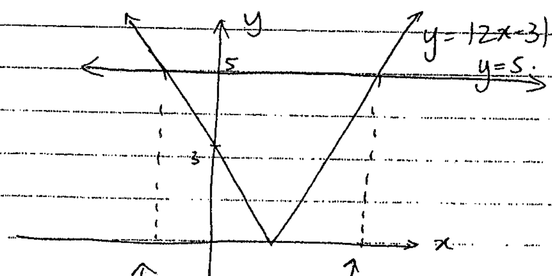
$y = 2x - 3$

if $x = 0$, $y = -3$

if $y = 0$, $2x = 3$

$x = 1.5$

(b)



finding these intercepts

$2x - 3 = 5$ or $-(2x-3) = 5$

$2x = 8$ $\qquad 2x - 3 = -5$

4 $\qquad 2x = -2$

$x = -1$

From diagram $-1 < x < 4$

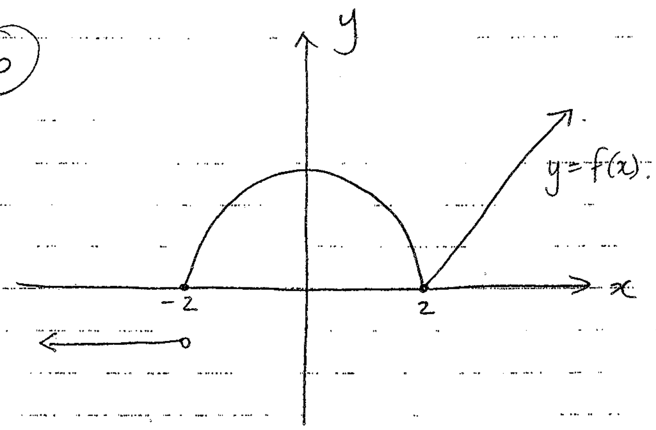
OR

$-5 < 2x - 3 < 5$

$-2 < 2x < 8$

$-1 < x < 4$

10



11 (a) $y = 3 + 2x - x^2$

$= -(x^2 - 2x - 3)$

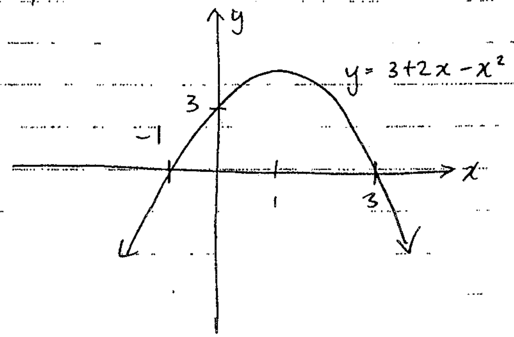
$= -(x-3)(x+1)$

if $x = 0$, $y = 3$

if $y = 0$ $\quad -(x-3)(x+1) = 0$

$x = -1$ or 3

$a < 0$ parabola is concave down.



(b) D: all real x.

For range, find vertex.

$x = \frac{-1+3}{2} = 1 \qquad y = 3 + 2(1) - (1)^2$
$= 4$

$\therefore$ R : $y \leq 4$

(c) $y = \dfrac{1}{\sqrt{3 + 2x - x^2}}$

now $3 + 2x - x^2 > 0$

from graph in (a) $-1 < x < 3$

$\therefore$ D: $-1 < x < 3$

For y, $1 > 0$ & $\sqrt{3+2x-x^2} > 0$

$\therefore y > 0$

Also, from (b)

the max value of $3 + 2x - x^2$ is 4.

$\therefore$ the minimum value of y is

$\dfrac{1}{\sqrt{4}}$ which is $\frac{1}{2}$.

$\therefore$ R: $y \geq \frac{1}{2}$

OR/ $3 + 2x - x^2 \leq 4$ from (b)

$\therefore \sqrt{3+2x-x^2} \leq 2$

$\therefore \dfrac{1}{\sqrt{3+2x-x^2}} \geq \dfrac{1}{2}$

$\therefore y \geq \frac{1}{2}$

So R: $y \geq \frac{1}{2}$