St Catherine's School

Year: 11
Subject: Mathematics
Time Allowed: 55 minutes
Date: 10 June 2004

Instructions

- All questions are to be attempted.
- Marks may be deducted for careless or badly presented work.
- Answer all questions in the spaces provided.
- Show all your working.

GOOD LUCK !

TEACHER'S USE ONLY Total Marks	
Functions (Q1)	
Plane Geometry (Q2)	
Coordinate Geometry (Q3)	
TOTAL	

NAME:___
TEACHER:_

NAME:__
TEACHER:_

Q1 Functions, Graphs, Regions /14

(i) If $f(x) = 3 + 2x - x^2$, find

a. $f(2)$

b. $f(-x)$

(ii) If $f(x) = \begin{cases} x^2 & \text{for } x < 0 \\ x+1 & \text{for } x \geq 0 \end{cases}$

a. Find

(i) $f(-4)$

(ii) $f(0)$

b. Sketch the function.

(iii) On separate number planes, draw neat sketches, showing important features, of the following:

a. $y = (x-1)^2$

b. $y = \sqrt{1-x^2}$

NAME:
TEACHER:

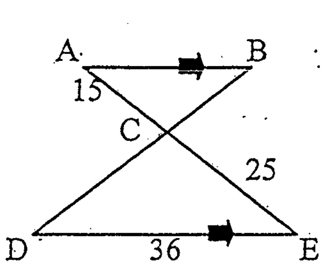
c. $y = 1 - x^2$

$y=0$

(iv) a. On one number plane, sketch $y \geq x^2 + 4x$ and $y = x - 2$. Solve simultaneously to find the points of intersection.

Q2 Plane Geometry /17

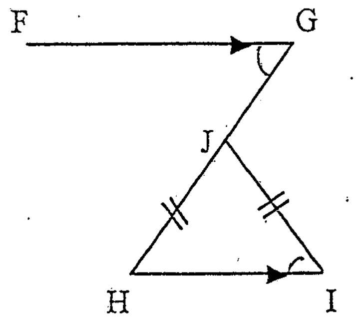
(i) a. Prove that $\triangle ABC$ is similar to $\triangle CDE$, given AB $\parallel$ DE, AC=15, CE=25, DE=36



b. Hence, find AB.

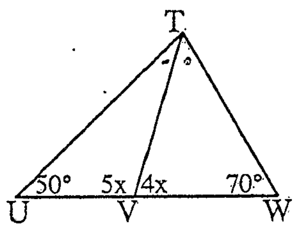
NAME:_ł_
TEACHER:_

(ii) In the diagram below, FG∥ HI and JH=JI. Prove that $\angle FGH = \angle JIH$, giving reasons.



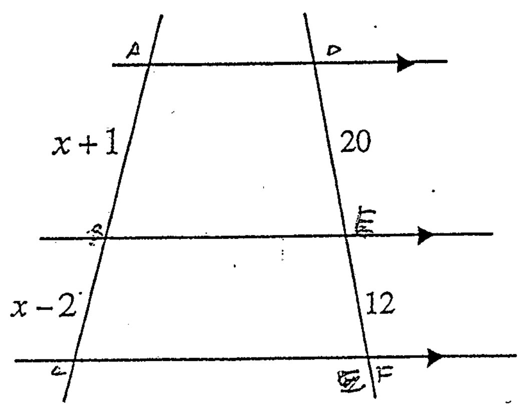
(iii) In the diagram below

a. Find x giving reasons.



b. Prove that TV bisects $\angle UTW$.

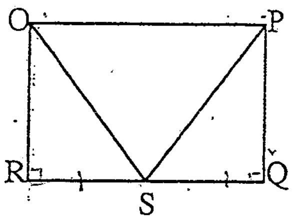
(iv) Find the value of x in the following, giving reasons.



NAME:___
TEACHER:_

Q3 Coordinate Geometry /14

(i) In rectangle OPQR, S is the midpoint of QR. Prove OS = PS



(ii) A(2,2), B(6,5), C(0,5) are the vertices of parallelogram ABCD.

a. Find the midpoint of AC.

b. Hence find vertex D.

(iii) a. Sketch the lines AB and CD given the points A(-6,-1), B(4,5), C(0,-1) and D(3,6).

b. Find the gradient of each line.

c. Find the angle of inclination of each line to the positive direction of the x-axis.

22/6/04

SOLUTIONS

NAME:________________________________

TEACHER:______________________________

Q1 Functions, Graphs, Regions /14

(i) If $f(x) = 3 + 2x - x^2$, find

a. $f(2) = 3+2(2)-(2)^2$ — 2

$= 3+4-4$

$f(2) = 3$

b. $f(-x) = 3+2(-x)-(-x)^2$ — 2

$f(-x) = 3-2x-x^2$

(ii) If $f(x) = \begin{cases} x^2 & \text{for } x < 0 \\ x+1 & \text{for } x \geq 0 \end{cases}$

a. Find — 2

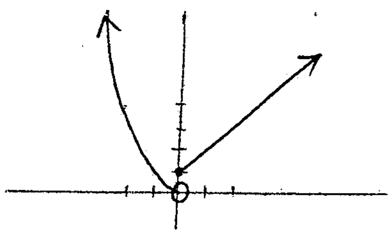
(i) $f(-4) = (-4)^2$

$= 16$

(ii) $f(0) = 0+1$

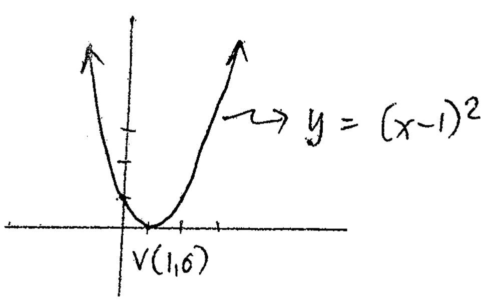
$= 1$

b. Sketch the function. — 2



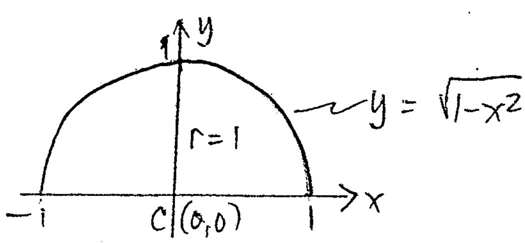
(iii) On separate number planes, draw neat sketches, showing important features, of the following:

a. $y = (x-1)^2$



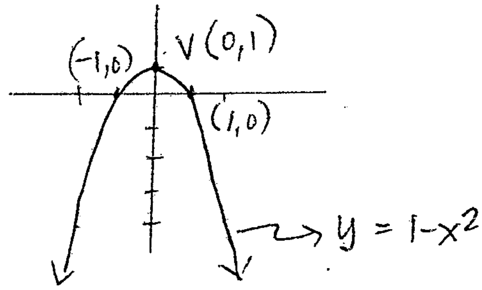
2
1 for features
1 for sketch

b. $y = \sqrt{1-x^2}$ — 2

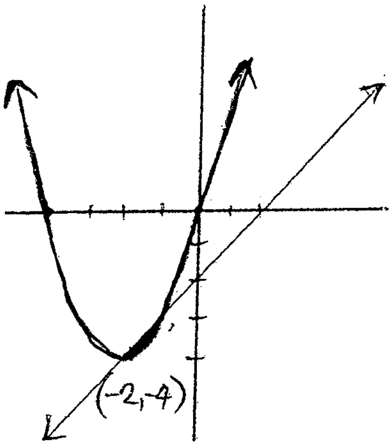


NAME:_______________________
TEACHER:_____________________

c. $y = 1 - x^2$ 2



(iv) a. On one number plane, sketch $y \geq x^2 + 4x$ and $y = x - 2$. Solve simultaneously to find the points of intersection. 4



$x^2 + 4x = x - 2$

$x^2 + 4x - x + 2 = 0$

$x^2 + 3x + 2 = 0$

$(x+2)(x+1) = 0$

$x = -2 \qquad x = -1$

$y = -4 \qquad y = -3$

Pts of ∩: $(-2,-4)$ $(-1,-3)$

~~b. Shade the region for which the above inequation and equation hold simultaneously.~~ 1

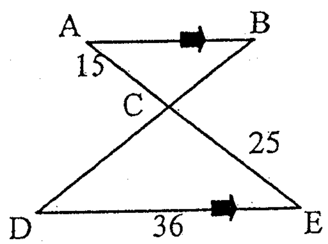
~~Test $(-2,-3)$: $-3 \geq (-2)^2 + 4(-2)$~~

~~$-3 \geq 4 - 8$~~

~~$-3 \geq -4$ yes~~

Q2 Plane Geometry /17

(i) a. Prove that $\triangle ABC$ is similar to $\triangle CDE$, given $AB \parallel DE$, AC=15, CE=25, DE=36 4



In $\triangle ABC$ & $\triangle DCE$,

$\angle ACB = \angle DCE$ (vert. opp $\angle$s =)

$\angle ABC = \angle CDE$ (alt $\angle$s of ∥ lines =)

$\angle BAC = \angle CED$ (alt $\angle$s of ∥ lines =)

∴ $\triangle ABC \,|||\, \triangle CDE$ (all corresp. $\angle$s =)

b. Hence, find AB. 2

$\frac{AB}{36} = \frac{15}{25}$

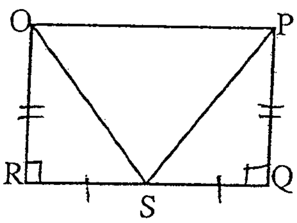
$AB = \frac{15 \times 36}{25}$

$AB = 21.6$

NAME:__________________
TEACHER:________________

Q3 Coordinate Geometry /14

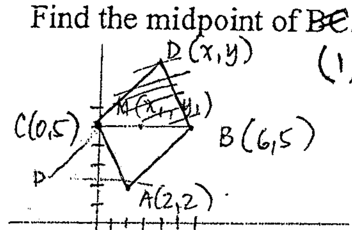
(i) In rectangle OPQR, S is the midpoint of QR. Prove OS = PS   4



In ΔORS & ΔSPQ,
$RS = SQ$ (S is midpt of QR - given)
$OR = PQ$ (opp sides of rect =)
$\angle R = \angle Q = 90^\circ$ (all $\angle$s of rect are rt $\angle$s)
$\Delta ORS \equiv \Delta SPQ$ (by SAS)
$\therefore OS = PS$ (corresp. sides of cong Δs =)

(ii) A(2,2), B(6,5), C(0,5) are the vertices of parallelogram ABCD.

a. Find the midpoint of ~~BC~~ AC   2



$(1, \frac{7}{2})$

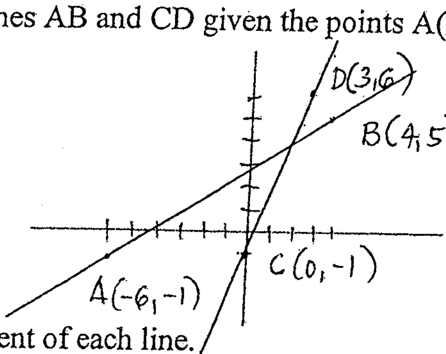
$M(x_1, y_1)$
$x_1 = \frac{6+0}{2} = 3$
$y_1 = \frac{5+5}{2} = 5$
$\therefore (3,5)$

b. Hence find vertex D.   2

$\frac{6+x}{2} = 1$ $\quad \frac{5+y}{2} = \frac{7}{2}$
$6 + x = 2$ $\quad y = 2$
$x = -4$
$(-4, 2)$

$D(x,y)$
$\frac{y+2}{2} = 5$ $\quad \frac{x+2}{2} = 3$
$y + 2 = 10$ $\quad x + 2 = 6$
$y = 8$ $\quad x = 4$
$\therefore D(4,8)$

(iii) a. Sketch the lines AB and CD given the points A(-6,-1), B(4,5), C(0,-1) and D(3,6).   2



b. Find the gradient of each line.   2

$m_{AB} = \frac{y_2 - y_1}{x_2 - x_1}$
$= \frac{5-(-1)}{4-(-6)} = \frac{5+1}{4+6}$
$= \frac{3}{5}$

$m_{CD} = \frac{6-(-1)}{3-(0)} = \frac{6+1}{3}$
$= \frac{7}{3}$

c. Find the inclination of each line to the positive direction of the x-axis.   2

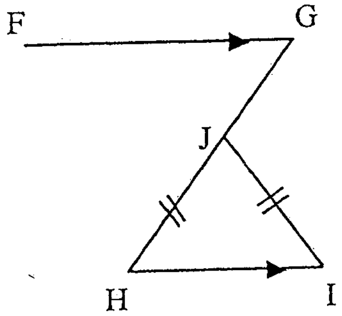
$\theta_{AB} = \tan^{-1}\left(\frac{3}{5}\right)$
$= 30^\circ 58'$ or $30.96'$

$\theta_{CD} = \tan^{-1}\left(\frac{7}{3}\right)$
$= 66^\circ 48'$ or 66.8°

------ End of Test -----

NAME:____________________
TEACHER:____________________

(ii) In the diagram below, FG∥ HI and JH=JI. Prove that $\angle FGH = \angle JIH$. giving reasons. 3



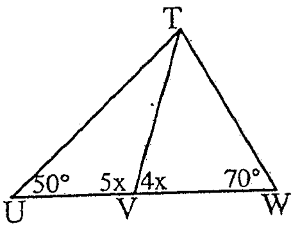
$\angle JHI = \angle JIH$ (base ∠s of isos Δ =)

$\angle FGH = \angle JHI$ (alt ∠s of ∥ lines =)

$\therefore \angle FGH = \angle JIH$

(iii) In the diagram below 2

a. Find **x** giving reasons.



$5x + 4x = 180°$ (supplementary ∠s add up to 180°)

$9x = 180°$

$x = 20°$

b. Prove that TV bisects $\angle UTW$. 3

In Δ UTV, $180 = 50 + 5 \times 20 + \angle UTV$

$\angle UTV = 180 - 50 - 100$

$= 30°$

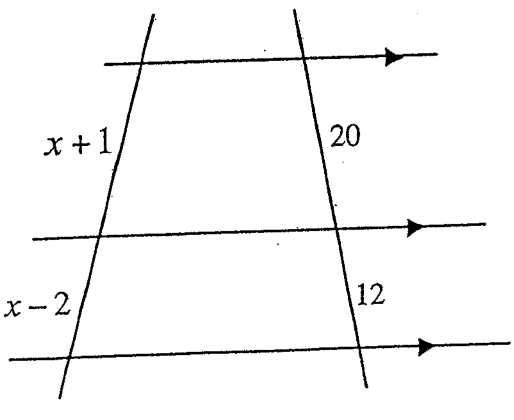
In Δ VTW, $180 = 4 \times 20 + 70 + \angle VTW$

$\angle VTW = 180 - 80 - 70$

$= 30°$

$\angle UTV = \angle VTW$, therefore TV bisects $\angle UTW$

(iv) Find the value of x in the following, giving reasons. 3



$\frac{x+1}{x-2} = \frac{20}{12}$ (family of ∥ lines cuts intercepts in proportion)

$12(x+1) = 20(x-2)$

$12x + 12 = 20x - 40$

$20x - 12x = 12 + 40$

$8x = 52$

$x = \frac{13}{2}$