St Catherine's School

Year: 11
Subject: Extension 1 Mathematics
Time allowed: 55 minutes

Assessment Task No: 1
Date: March 2005

Student Name:
Teacher's Name:

Directions to candidates:

- All questions are to be attempted.
- Marks may be deducted for careless or badly arranged work
- All necessary **working** must be shown in every question.
- Approved calculators and rulers are required.

TEACHERS' USE ONLY

Q 1 - 4	Total 12½	/22
Q 5 - 8	Total 11	/15
	TOTAL 23½	/37

START A NEW PAGE

1. Express as a fraction in simplest form: $0.45\dot{3}$ 2marks

2. Factorise
$(x+y)^2-(w-p)^2$
2marks

3. Solve for x:
(a) $(x-2)(x+3)>0$
(b) $\frac{1}{x-4}\leq 2$
(c) $|x-2|=3$
(d) $|x-1|=2x+3$ 10m

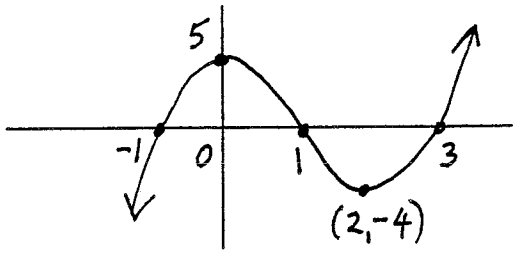
4. Sketch the following functions showing distinguishing features, and state the domain and range for each:
(a) $y=|x|-1$
(b) $y=9-x^2$
(c) $y=\sqrt{9-x^2}$
(d) $y=-\frac{3}{x}$

8 marks

5. State the domain of the following functions:
(a) $y=\frac{1}{x^2-9}$
(b) $y=|x+5|+\frac{1}{\sqrt{x^2-9}}$

3 marks

6. Consider the graph of $y = f(x)$ as shown in the diagram.



Use your knowledge of graph effects to sketch the following on the axes provided over page:

(a) $y = f(x) + 2$

(b) $y = | f(x) |$

(c) $y = f(x+1)$

5 marks

7. (a) Factorise $a^6 - b^6$ by first using a difference of two squares.

(b) Factorise $a^6 - b^6$ by first using a difference of two cubes.

(c) Hence show that

$$a^4 + a^2b^2 + 1 = \left(a^2 + ab + 1\right)\left(a^2 - ab + 1\right)$$

3 marks

8. (a) Expand $\left(x + \frac{1}{x}\right)^2$.

(b) If $x = \sqrt{3} - \sqrt{2}$,

(i) evaluate $x + \frac{1}{x}$

(ii) hence evaluate $x^2 + \frac{1}{x^2}$.

4 marks

END

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1. Let $x = 0.4535353\ldots$

$100x = 45.3535\ldots$

$-\quad x = 0.4535\ldots$

$99x = 44.900$

$x = \frac{44.9}{99}$ (2)

$= \frac{449}{990}$

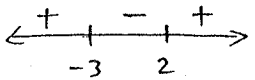
2. $(x+y)^2 - (w-p)^2$ (2)

$= (x+y-(w-p))(x+y+w-p)$

$= (x+y-w+p)(x+y+w-p)$

a) $(x-2)(x+3) > 0$

Test around critical values:



(2)

$\therefore\ x < -3$ or $x > 2$

b) $\frac{1}{x-4} \leq 2$, $x \neq 4$

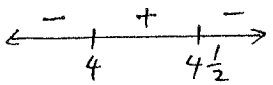
$\frac{1}{x-4} - 2 \leq 0$ (3)

$\frac{1-2(x-4)}{x-4} \leq 0$

$\frac{1-2x+8}{x-4} \leq 0$

$\frac{9-2x}{x-4} \leq 0$, $x \neq 4$

Test around critical values:



$\therefore\ x < 4$ or $x \geq 4\frac{1}{2}$

3 c) $|x-2| = 3$ (2)

$\therefore\ x - 2 = 3$ or $x - 2 = -3$

$x = 5$ or $x = -1$

d) $|x-1| = 2x+3$

$2x+3 \geq 0$

$\therefore\ x \geq -\frac{3}{2}$ and $x - 1 = 2x + 3$

$-4 = x$ ✗

$\therefore$ No soln

or $x \geq -\frac{3}{2}$ and $-(x-1) = 2x+3$

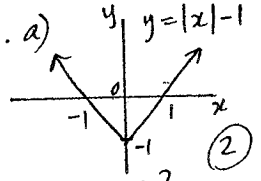
$-x + 1 = 2x + 3$

$-2 = 3x$

$x = -\frac{2}{3}$ ✓

$\therefore$ only solution $x = -\frac{2}{3}$. (3)

4. a) $y = |x| - 1$

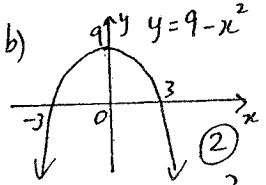


(2)

$D = \{x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R} : y \geq -1\}$

b) $y = 9 - x^2$

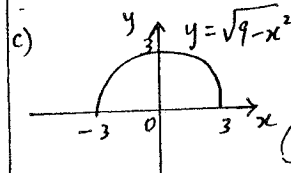


(2)

$D = \{\text{all } x \in \mathbb{R}\}$

$R = \{y \in \mathbb{R} : y \leq 9\}$

c) $y = \sqrt{9 - x^2}$

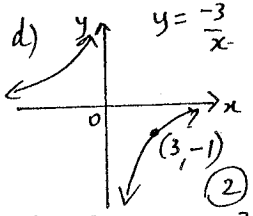


(2)

$D = \{x \in \mathbb{R} : -3 \leq x \leq 3\}$

$R = \{y \in \mathbb{R} : 0 \leq y \leq 3\}$

d) $y = \frac{-3}{x}$



(2)

$D = \{x \in \mathbb{R} : x \neq 0\}$

$R = \{y \in \mathbb{R} : y \neq 0\}$

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5. a) $y = \frac{1}{x^2 - 9}$ $D = \{x \in \mathbb{R} : x \neq 3, -3\}$

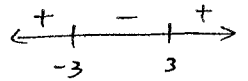
$= \frac{1}{(x+3)(x-3)}$ (1)

b) $y = |x+5| + \frac{1}{\sqrt{x^2-9}}$

$= |x+5| + \frac{1}{\sqrt{(x-3)(x+3)}}$ (2)

Restriction: $(x-3)(x+3) > 0$

Test around critical values:

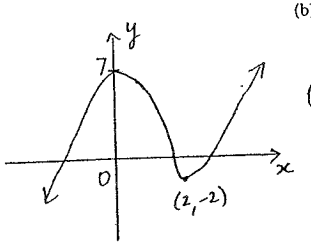


$\therefore\ D = \{x \in \mathbb{R} : x < -3 \text{ or } x > 3\}$

6.

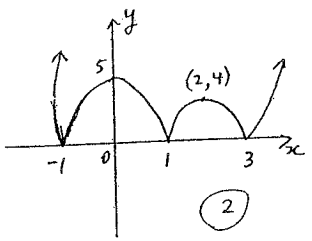
ANSWERS TO QUESTION 6 NAME: Solutions

(a)



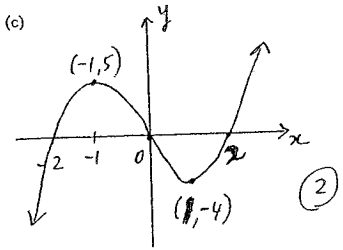
(1)

(b)



(2)

(c)



(2)

7 a) $a^6 - b^6 = (a^3 - b^3)(a^3 + b^3)$ (1)

$= (a-b)(a^2+ab+b^2)(a+b)(a^2-ab+b^2)$

b) $a^3 - b^3 = (a^2 - b^2)(a^4 + a^2b^2 + b^4)$ (1)

$= (a-b)(a+b)(a^4+a^2b^2+b^4)$

c) $\therefore a^4 + a^2b^2 + b^4 = (a^2+ab+b^2)(a^2-ab+b^2)$

(1) (since equals of equals are equals)

8. a) $\left(x + \frac{1}{x}\right)^2 = x^2 + 2 + \frac{1}{x^2}$ (1)

b) i) $x + \frac{1}{x} = \sqrt{3} - \sqrt{2} + \frac{1}{\sqrt{3}-\sqrt{2}} \times \frac{\sqrt{3}+\sqrt{2}}{\sqrt{3}+\sqrt{2}}$

$= \sqrt{3} - \sqrt{2} + \frac{\sqrt{3}+\sqrt{2}}{3-2}$ (2)

$= 2\sqrt{3}$

ii) $\therefore x^2 + \frac{1}{x^2} = \left(x + \frac{1}{x}\right)^2 - 2$

$\therefore = (2\sqrt{3})^2 - 2$ (1)

$= 10$